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PERMUTING TRI-f-DERIVATIONS IN LATTICES

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Abstract. The aim of this paper is to introduce the notion of permuting tri-f-derivations in lattices and to study some properties of permuting tri-f-derivations.

1. Introduction

Lattices play an important role in many fields such as information theory, information retrieval, information access conrols and cryptanalysis ([2], [6], [20]). Recently the properties of lattices were widely researched ([1], [2], [5], [10], [12], [20], [22]). In the theory of rings and near rings, the properties of derivations are an important topic to study ([3], [4], [19]). In [21], G. Szász introduced the notion of derivation on a lattice and discussed some related properties. Y. B. Jun and X. L. Xin [13] applied the notion of derivation in ring, near ring and lattice theory to BCI-algebras. In [24], J. Zhan and Y. L. Liu introduced the notion of left-right (or right-left) f-derivation of a BCI algebra and investigated some properties.

Recently, the notion of f-derivation, symmetric bi-derivations and permuting tri-derivations in lattices are introduced and proved some results ([7], [9], and [18]). The goal of this paper is to introduce the notion of permuting tri-f-derivations in lattices and to study some properties of permuting tri-f-derivations.

2. Preliminaries

Definition 1 ([5]). Let L be a nonempty set endowed with operations $\wedge$ and $\vee$. By a lattice $(L, \wedge, \vee)$, we mean a set L satisfying the following conditions:
(i) $x \wedge x = x$, $x \vee x = x$,
(ii) $x \wedge y = y \wedge x$, $x \vee y = y \vee x$,
(iii) $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, $(x \vee y) \vee z = x \vee (y \vee z)$,
(iv) $(x \wedge y) \vee x = x$, $(x \vee y) \wedge x = x$ for all $x, y, z \in L$.

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Definition 2 ([5])**.** Let $(L, \wedge, \vee)$ be a lattice. A binary relation $\leq$ is defined by $x \leq y$ if and only if $x \wedge y = x$ and $x \vee y = y$.

Lemma 1 ([22])**.** *Let $(L, \wedge, \vee)$ be a lattice. Define the binary relation $\leq$ as the Definition 2. Then $(L, \leq)$ is a poset and for any $x, y \in L$, $x \wedge y$ is the g.l.b. of $\{x, y\}$ and $x \vee y$ is the l.u.b. of $\{x, y\}$.*

Definition 3 ([5])**.** A lattice L is distributive if the identity (i) or (ii) holds:
(i) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$,
(ii) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.
In any lattice, the conditions (i) and (ii) are equivalent.

Definition 4 ([1])**.** A lattice L is modular if the identity (i) holds:
(i) If $x \leq z$, then $x \vee (y \wedge z) = (x \vee y) \wedge z$.

Definition 5. Let L be a lattice. A mapping $D : L \times L \times L \to L$ is called permuting if it is satisfies following conditions $D(x, y, z) = D(x, z, y) = D(y, x, z) = D(y, z, x) = D(z, x, y) = D(z, y, x)$ for all $x, y, z \in L$.

A mapping $d : L \to L$ defined by $d(x) = D(x, x, x)$ is called the trace of D, where D is a permuting mapping.

Definition 6. Let L be a lattice. A permuting mapping tri-derivation if

$$D(x \wedge w, y, z) = (D(x, y, z) \wedge w) \vee (x \wedge D(w, y, z))$$

for all $x, y, z, w \in L$.

It is obvious that D is a permuting tri-derivation then D satisfies the relations $D(x, y \wedge w, z) = (D(x, y, z) \wedge w) \vee (y \wedge D(x, w, z))$ and $D(x, y, z \wedge w) = (D(x, y, z) \wedge w) \vee (z \wedge D(x, y, w))$ for all $x, y, z, w \in L$.

3. The permuting tri-f-derivations in lattices

The following definitions introduce the notion of permuting tri-f-derivation for a lattice.

Definition 7. Let L be a lattice. A permuting mapping $D : L \times L \times L \to L$ is called permuting tri-f-derivation if there exists a function $f : L \to L$ such that

$$D(x \wedge w, y, z) = (D(x, y, z) \wedge f(w)) \vee (f(x) \wedge D(w, y, z))$$

for all $x, y, z, w \in L$.

Example 1. Let L be a lattice of following Figure 1 and define mappings D and f on L by

$$D(x,y,z)=\begin{cases} 2, & (x,y,z)=(0,0,0) \\ 2, & (x,y,z)=(0,0,1) \text{ or } (0,1,0) \text{ or } (1,0,0) \\ 2, & (x,y,z)=(0,0,2) \text{ or } (0,2,0) \text{ or } (2,0,0) \\ 1, & (x,y,z)=(1,1,1) \\ 0, & (x,y,z)=(2,2,2) \\ 1, & (x,y,z)=(0,1,1) \text{ or } (1,0,1) \text{ or } (1,1,0) \\ 0, & (x,y,z)=(0,2,2) \text{ or } (2,0,2) \text{ or } (2,2,0) \\ 2, & (x,y,z)=(0,1,2) \text{ or } (1,0,2) \text{ or } (1,2,0) \\ & \text{or } (2,1,0) \text{ or } (2,0,1) \\ 2, & (x,y,z)=(1,1,2) \text{ or } (1,2,1) \text{ or } (2,1,1) \\ 0, & (x,y,z)=(1,2,2) \text{ or } (2,1,2) \text{ or } (2,2,1) \end{cases}$$

and

$$f(x)=\begin{cases} 2, x=0 \\ 1, x=1 \\ 0, x=2. \end{cases}$$

We can see that D is a permuting tri f-derivation on L. But D is not permuting tri-derivation. Because

$$D(0\wedge 1,0,2)=D(0,0,2)=2$$

also

$$\begin{aligned} D(0\wedge 1,0,2) &= (D(0,0,2,)\wedge 1)\vee(0\wedge D(1,0,2)) \\ &= (2\wedge 1)\vee(0\wedge 2) \\ &= 1\vee 0=1. \end{aligned}$$

$$0\to 1\to 2$$

Figure 1

Proposition 1. *Let L be a lattice and D be a permuting tri-f-derivation on L. Then the following identities hold for all $x,y,z,w\in L$:*

(i) *$D(x,y,z)\le f(x)$, $D(x,y,z)\le f(y)$ and $D(x,y,z)\le f(z)$.*

(ii) *$D(x,y,z)\wedge D(w,y,z)\le D(x\wedge w,y,z)\le D(x,y,z)\vee D(w,y,z)$.*

(iii) *$D(x\wedge w,y,z)\le f(x)\vee f(w)$.*

(iv) *If L has a least element 0, then $f(0)=0$ implies $D(0,y,z)=0$ for all $y,z\in L$.*

Proof. (i) Since $x\wedge x=x$ for all $x\in L$, we have

$$\begin{aligned} D(x,y,z) &= D(x\wedge x,y,z) \\ &= (D(x,y,z)\wedge f(x))\vee(f(x)\wedge D(x,y,z) \\ &= D(x,y,z)\wedge f(x). \end{aligned}$$

Therefore $D(x,y,z)\le f(x)$ for all $x,y,z\in L$. Similarly, we see that $D(x,y,z)$ $\le f(y)$ and $D(x,y,z)\le f(z)$ for all $x,y,z\in L$.

(ii) Since $D(x,y,z)\le f(x)$ and $D(w,y,z)\le f(w)$, from (i), we have

$$D(x,y,z)\wedge D(w,y,z)\le f(x)\wedge D(w,y,z)$$

and

$$D(x,y,z)\wedge D(w,y,z)\leq f(w)\wedge D(x,y,z)$$

for all $x,y,z,w\in L$. Hence

$$\begin{aligned}D(x,y,z)\wedge D(w,y,z)&\leq (f(x)\wedge D(w,y,z))\vee (f(w)\wedge D(x,y,z))\\&=D(x\wedge w,y,z).\end{aligned}$$

Furthermore, since $f(x)\wedge D(w,y,z)\leq D(w,y,z)$ and $f(w)\wedge D(x,y,z)\leq D(x,y,z)$, we get

$$(f(x)\wedge D(w,y,z))\vee (f(w)\wedge D(x,y,z))\leq D(x,y,z)\vee D(w,y,z).$$

That is, $D(x\wedge w,y,z)\leq D(x,y,z)\vee D(w,y,z)$.

(iii) Since $D(x,y,z)\wedge f(w)\leq f(w)$ and $f(x)\wedge D(w,y,z)\leq f(x)$, we get

$$(D(x,y,z)\wedge f(w))\vee (f(x)\wedge D(w,y,z))\leq f(x)\vee f(w).$$

That is, $D(x\wedge w,y,z)\leq f(x)\vee f(w)$.

(iv) Since 0 is the least element of L. We have

$$\begin{aligned}D(0,y,z)&=D(0\wedge 0,y,z)\\&=(D(0,y,z)\wedge f(0))\vee (f(0)\wedge D(0,y,z))\\&=0\vee 0=0\end{aligned}$$

for all $y,z\in L$ □

Corollary 1. *Note that,*

$$\begin{aligned}D(x,x,x)=D(x\wedge x,x,x)&=(D(x,x,x)\wedge f(x))\vee (f(x)\wedge D(x,x,x))\\&=D(x,x,x)\wedge f(x)\end{aligned}$$

for all $x\in L$. That is, $d(x)\leq f(x)$ for all $x\in L$.

Definition 8. Let L be a lattice and D be a permuting tri-f-derivation on L.

(i) If $x\leq w$ implies $D(x,y,z)\leq D(w,y,z)$, then D is called an isotone permuting tri-f-derivation.

(ii) If D is one-to-one, D is called a monomorfic permuting tri-f-derivation.

(iii) If D is onto, D is called an epic permuting tri-f-derivation.

Proposition 2. *Let L be lattice, D be a permuting tri-f-derivation on L and 1 be the greatest element of L. Then the following identities hold:*

(i) *If $f(x)\leq D(1,y,z)$, then $D(x,y,z)=f(x)$.*

(ii) *If $f(x)\geq D(1,y,z)$ and $f(1)=1$, then $D(x,y,z)\geq D(1,y,z)$.*

Proof. (i) Since

$$\begin{aligned}D(x,y,z)&=D(x\wedge 1,y,z)\\&=(D(x,y,z)\wedge f(1))\vee (f(x)\wedge D(1,y,z))\\&=D(x,y,z)\vee f(x),\end{aligned}$$

we get $f(x)\leq D(x,y,z)$. From Proposition 1(i), we obtain $D(x,y,z)=f(x)$.

(ii) Since

$$\begin{aligned} D(x,y,z) &= D(x\wedge 1,y,z) \\ &= (D(x,y,z)\wedge f(1))\vee(f(x)\wedge D(1,y,z)) \\ &= D(x,y,z)\vee D(1,y,z), \end{aligned}$$

we get $D(1,y,z)\leq D(x,y,z)$ for all $x,y,z\in L$. □

Proposition 3. *Let L be a lattice and D be a permuting tri-f-derivation on L. If f is an increasing function, then $w\leq x$ and $D(x,y,z)=f(x)$ imply that $D(w,y,z)=f(w)$.*

Proof. Suppose $w\leq x$, then $x\wedge w=w$. Thus

$$\begin{aligned} D(w,y,z) &= D(x\wedge w,y,z) \\ &= (D(x,y,z)\wedge f(w))\vee(f(x)\wedge D(w,y,z)) \\ &= (f(x)\wedge f(w))\vee(f(x)\wedge D(w,y,z)) \\ &= f(w)\vee(f(x)\wedge D(w,y,z)) \\ &= f(w)\vee D(w,y,z) \\ &= f(w). \end{aligned}$$

□

Proposition 4. *Let L be a lattice and D be a permuting tri-f-derivation on L. Then for any $x,y,z,w\in L$ the followings hold:*

(i) *If D is isotone, then*

$$D(x,y,z)=D(x,y,z)\vee(D(x\vee w,y,z)\wedge f(x)).$$

(ii) *If $f(x\vee w)=f(x)\vee f(w)$, then*

$$D(x,y,z)=D(x,y,z)\vee(D(x\vee w,y,z)\wedge f(x)).$$

(iii) *If f is an increasing function, then*

$$D(x,y,z)=D(x,y,z)\vee(f(x)\wedge D(x\vee w,y,z)).$$

Proof. (i) Since D is an isotone permuting tri-f-derivation then we have

$$\begin{aligned} D(x,y,z) &= D((x\vee w)\wedge x,y,z) \\ &= (D(x\vee w,y,z)\wedge f(x))\vee(f(x\vee w)\wedge D(x,y,z)) \\ &= (D(x\vee w,y,z)\wedge f(x))\vee D(x,y,z). \end{aligned}$$

(ii) Since $D(x,y,z)\leq f(x)\leq f(x)\vee f(w)$, we get

$$\begin{aligned} D(x,y,z) &= D((x\vee w)\wedge x,y,z) \\ &= (D(x\vee w,y,z)\wedge f(x))\vee(f(x\vee w)\wedge D(x,y,z)) \\ &= (D(x\vee w,y,z)\wedge f(x))\vee D(x,y,z). \end{aligned}$$

(iii) Since f is an increasing function and $x \leq x \vee y$ then $f(x) \leq f(x \vee y)$ and so;

$$\begin{aligned} D(x,y,z) &= D((x \vee w) \wedge x, y, z) \\ &= (D(x \vee w, y, z) \wedge f(x)) \vee (f(x \vee w) \wedge D(x,y,z)) \\ &= (D(x \vee w, y, z) \wedge f(x)) \vee D(x,y,z). \end{aligned}$$

□

Proposition 5. *Let L be a lattice, D be an isotone permutating tri-f-derivation and f be a decreasing function. If $D(x,y,z) = f(x)$ and $D(w,y,z) = f(w)$, then $D(x \vee w, y, z) = f(x) \vee f(w)$.*

Proof. Since $x \leq x \vee w$, $w \leq x \vee w$ and D is isotone, we get $D(x,y,z) \leq D(x \vee w, y, z)$ and $D(w,y,z) \leq D(x \vee w, y, z)$. Hence $f(x) \vee f(w) \leq D(x \vee w, y, z)$. Also, $D(x \vee w, y, z) \leq f(x \vee w) \leq f(x) \vee f(w)$. Therefore $D(x \vee w, y, z) = f(x) \vee f(w)$. □

Theorem 1. *Let L be a lattice with greatest element 1 and D be a permuting tri-f-derivation on L and $f(x \wedge y) = f(x) \wedge f(y)$. The following conditions are equivalent*:

(i) *D is an isotone permuting tri-f-derivation.*
(ii) $D(x,y,z) \vee D(w,y,z) \leq D(x \vee w, y, z)$.
(iii) $D(x,y,z) = f(x) \wedge D(1,y,z)$.
(iv) $D(x \wedge w, y, z) = D(x,y,z) \wedge D(w,y,z)$.

Proof. (i) $\Longrightarrow$ (ii) Suppose that D is an isotone permuting tri-f-derivation. Since $x \leq x \vee w$ and $w \leq x \vee w$, we have $D(x,y,z) \leq D(x \vee w, y, z)$ and $D(w,y,z) \leq D(x \vee w, y, z)$. Therefore, $D(x,y,z) \vee D(w,y,z) \leq D(x \vee w, y, z)$.

(ii) $\Longrightarrow$ (i) Suppose that $D(x,y,z) \vee D(w,y,z) \leq D(x \vee w, y, z)$ and $x \leq w$, Since

$$\begin{aligned} D(x,y,z) &\leq D(x,y,z) \vee D(w,y,z) \leq D(x \vee w, y, z) \\ &= D(w,y,z), \end{aligned}$$

we get D is isotone.

(i) $\Longrightarrow$ (iii) Suppose that D is an isotone permuting tri-f-derivation. Since $D(x,y,z) \leq D(1,y,z)$, we get $D(x,y,z) \leq f(x) \wedge D(1,y,z)$ by Proposition 1(i). From Proposition 4(i), for $w = 1$ we get

$$\begin{aligned} D(x,y,z) &= (D(1,y,z) \wedge f(x)) \vee D(x,y,z) \\ &= D(1,y,z) \wedge f(x). \end{aligned}$$

(iii) $\Longrightarrow$ (iv) From (iii),

$$\begin{aligned} D(x \wedge w, y, z) &= f(x \wedge w) \wedge D(1,y,z) \\ &= f(x) \wedge f(w) \wedge D(1,y,z) \\ &= (f(x) \wedge D(1,y,z)) \wedge (f(w) \wedge D(1,y,z)) \\ &= D(x,y,z) \wedge D(w,y,z). \end{aligned}$$

(iv) $\Longrightarrow$ (i) Let $D(x \wedge w, y, z) = D(x, y, z) \wedge D(w, y, z)$ and $x \leq w$. Then, we get $D(x, y, z) = D(x \wedge w, y, z) = D(x, y, z) \wedge D(w, y, z)$. Therefore, $D(x, y, z) \leq D(w, y, z)$. □

Theorem 2. *Let L be a modular lattice and D be a permuting tri-f-derivation on L. Then,*

(i) *D is an isotone if and only if $D(x \wedge w, y, z) = D(x, y, z) \wedge D(w, y, z)$.*

(ii) *If D is an isotone and $f(x \vee w) = f(x) \vee f(w)$, then $D(x, y, z) = f(x)$ implies $D(x \vee w, y, z) = D(x, y, z) \vee D(w, y, z)$.*

Proof. (i) Let D be isotone. Since $x \wedge w \leq x$ and $x \wedge w \leq w$, we get $D(x \wedge w, y, z) \leq D(x, y, z) \wedge D(w, y, z)$. Then,

$$\begin{aligned} D(x, y, z) \wedge D(w, y, z) &= (D(x, y, z) \wedge D(w, y, z)) \wedge (f(x) \wedge f(w)) \\ &= (D(x, y, z) \wedge f(w)) \wedge (f(x) \wedge D(w, y, z)) \\ &\leq (D(x, y, z) \wedge f(w)) \vee (D(w, y, z) \wedge f(x)) \\ &= D(x \wedge w, y, z). \end{aligned}$$

Thus, we obtain $D(x \wedge w, y, z) = D(x, y, z) \wedge D(w, y, z)$.

Conversely let $D(x \wedge w, y, z) = D(x, y, z) \wedge D(w, y, z)$ and $x \leq w$. Since $D(x, y, z) = D(x \wedge w, y, z) = D(x, y, z) \wedge D(w, y, z)$, we obtain $D(x, y, z) \leq D(w, y, z)$.

(ii) Suppose that D is isotone and $D(x, y, z) = f(x)$. From Proposition 4 and since L is a modular lattice, we get

$$\begin{aligned} D(w, y, z) &= D(w, y, z) \vee (f(w) \wedge D(x \vee w, y, z)) \\ &= f(w) \wedge D(x \vee w, y, z). \end{aligned}$$

Therefore, we get

$$\begin{aligned} D(x, y, z) \vee D(w, y, z) &= D(x, y, z) \vee (f(w) \wedge D(x \vee w, y, z)) \\ &= (D(x, y, z) \vee f(w)) \wedge D(x \vee w, y, z) \\ &= (f(x) \vee f(w)) \wedge D(x \vee w, y, z) \\ &= f(x \vee w) \wedge D(x \vee w, y, z) \\ &= D(x \vee w, y, z) \end{aligned}$$

by hypothesis. □

Theorem 3. *Let L be a distributive lattice and D be a permuting tri-f-derivation on L where $f(x \vee w) = f(x) \vee f(w)$. Then the followings hold:*

(i) *If D is isotone, then $D(x \wedge w, y, z) = D(x, y, z) \wedge D(w, y, z)$.*

(ii) *If D is isotone if and only if $D(x \vee w, y, z) = D(x, y, z) \vee D(w, y, z)$.*

Proof. (i) Since D is isotone $D(x \wedge w, y, z) \leq D(x, y, z) \wedge D(w, y, z)$. From Proposition 1, we have

$$
\begin{aligned}
D(x,y,z) \wedge D(w,y,z) &= (D(x,y,z) \wedge f(x)) \wedge (f(w) \wedge D(w,y,z)) \\
&= (D(x,y,z) \wedge f(w)) \wedge (f(x) \wedge D(w,y,z)) \\
&\leq (D(x,y,z) \wedge f(w)) \vee (f(x) \wedge D(w,y,z)) \\
&= D(x \wedge w, y, z).
\end{aligned}
$$

That is, $D(x \wedge w, y, z) = D(x,y,z) \wedge D(w,y,z)$.

(ii) Let D be isotone. From (i), we have $D(x \wedge w, y, z) = D(x,y,z) \wedge D(w,y,z)$. From Proposition 1 and Proposition 4, we obtain

$$
\begin{aligned}
D(w,y,z) &= D(w,y,z) \vee (f(w) \wedge D(x \vee w, y, z)) \\
&= (D(w,y,z) \vee f(w)) \wedge (D(w,y,z) \vee D(x \vee w, y, z)) \\
&= f(w) \wedge D(x \vee w, y, z).
\end{aligned}
$$

Similarly

$$D(x,y,z) = f(x) \wedge D(x \vee w, y, z).$$

Therefore, we get

$$
\begin{aligned}
D(x,y,z) \vee D(w,y,z) &= (f(x) \wedge D(x \vee w, y, z)) \vee (f(w) \wedge D(x \vee w, y, z)) \\
&= (f(x) \vee f(w)) \wedge D(x \vee w, y, z) \\
&= f(x \vee w) \wedge D(x \vee w, y, z) \\
&= D(x \vee w, y, z).
\end{aligned}
$$

Conversely, let $D(x \vee w, y, z) = D(x,y,z) \vee D(w,y,z)$ and $x \leq w$. Since $D(w,y,z) = D(x \vee w, y, z) = D(x,y,z) \vee D(w,y,z)$, we get $D(x,y,z) \leq D(w,y,z)$. □

References

[1] R. Balbes and P. Dwinger, *Distributive Lattices*, University of Missouri Press, Columbia, Mo., 1974.

[2] A. J. Bell, *The co-information lattice*, in: 4th International Symposium on Independent Component Analysis and Blind Signal Separation (ICA2003), Nara, Japan, 2003, 921–926.

[3] H. E. Bell and L. C. Kappe, *Rings in which derivations satisfy certain algebraic conditions*, Acta Math. Hungar. **53** (1989), no. 3-4, 339–346.

[4] H. E. Bell and G. Mason, *On derivations in near-rings*, Near-rings and near-fields (Tubingen, 1985), 31–35, North-Holland Math. Stud., 137, North-Holland, Amsterdam, 1987.

[5] G. Birkhoof, *Lattice Theory*, American Mathematical Society, Colloquium, 1940.

[6] C. Carpineto and G. Romano, *Information retrieval through hybrid navigation of lattice representations*, Int. J. Human-Computers Studies **45** (1996), 553–558.

[7] Y. Çeven, *Symmetric bi-derivations of lattices*, Quaest. Math. **32** (2009), no. 2, 241–245.

[8] Y. Çeven and M. A. Öztürk, *On the trace of a permuting tri-additive mapping in left s-unital rings*, Int. J. Pure Appl. Math. **23** (2005), no. 4, 465–474.

[9] ———, *On f-derivations of lattices*, Bull. Korean Math. Soc. **45** (2008), no. 4, 701–707.

[10] C. Degang, Z. Wenxiu, D. Yeung, and E. C. C. Tsang, *Rough approximations on a complete completely distributive lattice with applications to generalized rough sets*, Inform. Sci. **176** (2006), no. 13, 1829–1848.

[11] L. Ferrari, *On derivations of lattices*, Pure Math. Appl. **12** (2001), no. 4, 365–382.

[12] A. Honda and M. Grabish, *Entropy of capacities on lattices and set systems*, Inform. Sci. **176** (2006), no. 23, 3472–3489.
[13] Y. B. Jun and X. L. Xin, *On derivations of BCI-algebras*, Inform. Sci. **159** (2004), no. 3-4, 167–176.
[14] F. Karaçal, *On the direct decomposability of strong negations and S-implication operators on product lattices*, Inform. Sci. **176** (2006), no. 20, 3011–3025.
[15] D. Özden, M. A. Öztürk, and Y. B. Jun, *Permuting tri-derivations in prime and semi-prime gamma rings*, Kyungpook Math. J. **46** (2006), no. 2, 153–167.
[16] M. A. Öztürk, *Permuting tri-derivations in prime and semi-prime rings*, East Asian Math. J. **15** (1999), no. 2, 177–190.
[17] M. A. Öztürk, Y. Çeven, and Y. B. Jun, *Generalized derivations of BCI-algebras*, Honam Math. J. **31** (2009), no. 4, 601–609.
[18] M. A. Öztürk, H. Yazarlı, and K. H. Kim, *Permuting tri-derivations in lattices*, Quaest. Math. **32** (2009), no. 3, 415–425.
[19] E. Posner, *Derivations in prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1093–1100.
[20] R. S. Sandhu, *Role hierarchies and constraints for lattice-based access controls*, in: Proceedings of the 4th European Symposium on Research in Computer Security, Rome, Italy, 1996, 65–79.
[21] G. Szász, *Derivations of lattices*, Acta Sci. Math. (Szeged) **37** (1975), 149–154.
[22] X. L. Xin, T. Y. Li, and J. H. Lu, *On derivations of lattices*, Inform. Sci. **178** (2008), no. 2, 307–316.
[23] H. Yazarlı, M. A. Öztürk, and Y. B. Jun, *Tri-additive maps and permuting tri-derivations*, Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. Stat. **54** (2005), no. 1, 1–8.
[24] J. Zhan and Y. L. Liu, *On f-derivations of BCI-algebras*, Int. J. Math. Math. Sci. **2005** (2005), no. 11, 1675–1684.

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