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Utkin 정리의 단일입력 불확실 선형 시스템에 대한 증명

(A Poof of Utkin's Theorem for a SI Uncertain Linear Case)

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요 약

본 연구에서는 불확실 단일 입력 시스템의 경우에 대하여 Utkin 정리의 증명을 제시한다. 소위 두가지의 대각화 방법(Diagonalization Method)이라 불리는 Utkin 정리의 두 변환 방법에 대한 불변 정리를 비교적으로 분명히 증명한다. 슬라이딩 모드의 수식 즉 슬라이딩 면은 두가지 대각화 변환에 대하여 변화 없고 두 인가된 제어입력은 같은 이득을 갖는다. 두가지 대각화 방법에 의하여 같은 결과를 얻는다. 설계 예와 시뮬레이션 연구를 통하여 제안된 결과의 효용성을 입증한다.

Abstract

In this note, a proof of Utkin's theorem is presented for SI(Single input) uncertain linear systems. The invariance theorem with respect to the two transformation methods so called the two diagonalization methods is proved clearly and comparatively for SI uncertain linear systems. With respect to the sliding surface transformation, the equation of the sliding mode i.e., the sliding surface is invariant. The control inputs by the two transformation methods both have the same gains. By means of the two transformation methods, the same results can be obtained. Through an illustrative example and simulation study, the usefulness of the main results is verified.

Keywords : variable structure system, sliding mode control, proof of Ukin's Theorem, diagonalization methods

Ⅰ. Introduction

The sliding mode control(SMC) can provide the effective means to the control of uncertain dynamical systems under parameter variations and external disturbances[1~3]. One of its essential advantages is the robustness of the controlled system to matched parameter uncertainties and matched external disturbances in the sliding mode on the predetermined sliding surface, $s=0$[4]. The proper design of the sliding surface can determine the almost output dynamics and its performances[5]. Many design algorithms including the linear(optimal control[6, 9], geometric approach[7], pole assignment[8], eigenstructure assignment[9], differential geometric approach[12], Lyapunov approach[15~16], integral augmentation[5, 13, 20], Ackermann's formula[17], and dynamic sliding surface[18]) and nonlinear[14, 19] techniques are reported. In these sliding surface design methods so far, however the existence condition of the sliding mode on the predetermined sliding surface is not proved.

To take the advantages of the sliding mode on the predetermined sliding surface, the precise existence condition of the sliding mode, $s \cdot \dot{s} < 0$ for the linear SI case and $s_i \cdot \dot{s}_i < 0,\ i = 1,2,\ldots m$ for the linear MI(Multi Input) case should be satisfied where s is the sliding surface and s_i is the sub-manifold of the

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sliding surface. Therefore the existence condition of the sliding mode must be proved. Even for a linear SI case, that proof is hardly reported. For a linear MI case, a few design methods were studied, those are hierarchical control methodology[1, 3], diagonalization methods[1, 3], simplex algorithm[10], Lyapunov approach[15~16], and so on. Until now in MIMO(Multi input multi output) VSS(Variable structure system)s, it is difficult to prove the precise existence condition of the sliding mode on the predetermined sliding surface theoretically, but in [15~16, 20], only the results of the derivative of the Lyapunov candidate function is negative, i.e. $\dot{V}<0$ is obtained when $V=1/2s^Ts$. Without the proofs, Utkin presented the two methodologies to prove the existence condition of the sliding mode on the sliding surface[1]. It is so called the invariance theorem, that is the equation of the sliding mode is invariant with respect to the two nonlinear transformations. Those are the control input transformation and sliding surface transformation, so called the two diagonalization methods. The essential feature of both methods is conversion of a multi-input design problem into m single-input design problems[2]. Those were only reviewed in [2]. DeCarlo, Zak, and Matthews tried to prove Utkin's invariance theorem. But, the proofs are not clear. In [16], Su, Drakunov, and Ozguner mentioned the sliding surface transformation, which would diagonalize the control coefficient matrix to the dynamics for the sliding surface s. But they did not prove the precise existence condition of the sliding mode on the predetermined sliding surface.

Until now, for SI uncertain linear systems, a rigorous proof of Utkin's theorem is not reported.

In this paper, a proof of Utkin's theorem is presented for a SI uncertain linear case. The invariance theorem with respect to the two transformation methods so called the two diagonalization methods is proved clearly and comparatively for uncertain SI linear systems. A design example and simulation study shows usefulness of the main results.

Ⅱ. Main Results of Proof Utkin's Theorem

The invariant theorem of Utkin's is as follows[1~2]:

Theorem 1: The equation of the sliding mode is invariant with respect to the two nonlinear transformations, i.e. the sliding surface transformation and control input transformation:

$$\begin{aligned} s^*(x) &= H_s(x,t)\cdot s(x) \\ u^*(x) &= H_u(x,t)\cdot u(x) \end{aligned} \tag{1}$$

where $H_s(x,t)$ and $H_u(x,t)$ are the transformation matrices for $\det H_s \neq 0$ and $\det H_u \neq 0$.

For a SI(single input) uncertain linear system:

$$\dot{x}=(A_0+\Delta A)x+(B_0+\Delta B)u+\Delta D(t) \tag{2}$$

where $x\in R^n$ is the state, $u\in R^1$ is the control input, $A_0\in R^{n\times n}$ is the nominal system matrix, $B_0\in R^{n\times 1}$ is the nominal input matrix, $\triangle A$ and $\triangle B$ are the system matrix uncertainty and input matrix uncertainty, those are bounded, and $\Delta D(t)$ is bounded external disturbance, respectively.

The conventional sliding surface is a linear combination of the full state variable as

$$s=C\cdot x=\sum_{i=1}^{n}c_ix_i \tag{3}$$

where C is a non zero element coefficient matrix for the sliding surface.

The VSS control input is as follows:

$$u_1=-K\cdot x-\Delta K\cdot x-G\cdot sign(s) \tag{4}$$

where K is a constant gain, ΔK is a state dependent switching gain, G is a switching gain.

1. Control input transformation[1~2]

$$\begin{aligned} u^* &= (CB_0)^{-1}u_1, \quad H_u=(CB_0)^{-1} \\ &= (CB_0)^{-1}[-Kx-\Delta Kx-Gsign(s)] \end{aligned} \tag{5}$$

where the control input transformation matrix is selected as $H_u=(CB_0)^{-1}$ for the SI uncertain linear case. In [1] and [2], the proofs for the nonlinear control input transformation are not clear. The real dynamics of s, i.e. the time derivative of s is as follows:

$$\begin{aligned}\dot{s}&= C\dot{x}\\ &= C(A_0+\Delta A)x + C(B+\Delta B_0)u^* + C\Delta D(t)\\ &= C(A_0+\Delta A)x + (I+\Delta I)u_1 + C\Delta D(t),\\ &\qquad \Delta I = C\Delta B(CB_0)^{-1}\\ &= C(A_0+\Delta A)x\\ &\quad + (I+\Delta I)(-Kx-\Delta Kx - Gsign(s))\\ &\quad + C\Delta D(t)\\ &= CA_0x - Kx + C\Delta Ax - \Delta IKx\\ &\quad -(I+\Delta I)\Delta Kx + C\Delta D(t)\\ &\quad -(I+\Delta I)Gsign(s)\end{aligned} \tag{6}$$

By letting the constant gain

$$K = CA_0 \tag{7}$$

then the real dynamics of s becomes

$$\begin{aligned}\dot{s}&= [C\Delta A - \Delta IK]x - (I+\Delta I)\Delta Kx\\ &\quad + C\Delta D(t) - (I+\Delta I)Gsign(s)\end{aligned} \tag{8}$$

If one takes the switching gain as design parameters

$$\triangle k_j = \begin{cases} \geq \dfrac{\max\{C\triangle A - \triangle ICA_0\}_j}{\min\{I+\triangle I\}} & sign(sx_j)>0\\ \leq \dfrac{\min\{C\triangle A - \triangle ICA_0\}_j}{\min\{I+\triangle I\}} & sign(sx_j)<0\end{cases}$$

$$j=1,2,\ldots,n \tag{9}$$

$$G = \begin{cases} \geq \dfrac{\max\{C\Delta D(t)\}}{\min\{I+\triangle I\}} & sign(s)>0\\ \leq \dfrac{\min\{C\Delta D(t)\}}{\min\{I+\triangle I\}} & sign(s)<0\end{cases} \tag{10}$$

then one can obtain the following equation

$$s\cdot\dot{s}<0 \tag{11}$$

The existence condition of the sliding mode is proved for the SI uncertain linear system. The equation of the sliding mode, i.e. the sliding surface is invariant to the control input transformation

2. Sliding surface transformation[1~2, 16]

$$s^* = (CB_0)^{-1}\cdot s, \quad H_s(x,t) = (CB_0)^{-1} \tag{12}$$

The sliding surface transformation matrix is selected as $H_s(x,t)=(CB_0)^{-1}$. In [2], the proof is not sufficient. Now, the VSS control input for the new sliding surface is taken as follows:

$$u_2 = -K\cdot x - \Delta K\cdot x - G\cdot sign(s^*) \tag{13}$$

The real dynamics of the sliding surface, i.e. the time derivative of s^* becomes

$$\begin{aligned}\dot{s}^*&= (CB_0)^{-1}\dot{s} = (CB_0)^{-1}C\dot{x}\\ &= (CB_0)^{-1}C(A_0+\Delta A)x\\ &\quad + (CB_0)^{-1}C(B_0+\Delta B)u_2\\ &\quad + (CB_0)^{-1}C\Delta D(t)\\ &= (CB_0)^{-1}C(A_0+\Delta A)x + (I+\Delta I')u_2\\ &\quad + (CB_0)^{-1}C\Delta D(t),\\ &\qquad \Delta I' = (CB_0)^{-1}C\Delta B\\ &= (CB_0)^{-1}C(A_0+\Delta A)x\\ &\quad + (I+\Delta I')(-Kx - \Delta Kx - Gsign(s^*))\\ &\quad + (CB_0)^{-1}C\Delta D(t)\\ &= (CB_0)^{-1}CA_0x - Kx + (CB_0)^{-1}C\Delta Ax\\ &\quad -\Delta I'Kx - (I+\Delta I')\Delta Kx\\ &\quad + (CB_0)^{-1}C\Delta D(t) - (I+\Delta I')Gsign(s^*)\end{aligned} \tag{14}$$

By letting gain

$$K = (CB)^{-1}CA_0 \tag{15}$$

then the real dynamics of s^* becomes

$$\begin{aligned}\dot{s}^*&= [(CB_0)^{-1}C\Delta A - \Delta I'K]x\\ &\quad -(I+\Delta I')\Delta Kx\\ &\quad + (CB_0)^{-1}C\Delta D(t)\\ &\quad -(I+\Delta I')Gsign(s^*)\end{aligned} \tag{16}$$

In [16], without uncertainty and disturbance, it is mentioned that the sliding surface transformation would diagonalize the control coefficient matrix to the dynamics for s and the $\dot{V}(x)<0$ is proved when $V(x)=x^TPx>0$.

If one takes the switching gains as follows:

$$\triangle k_j = \begin{cases} \geq \dfrac{\max\left\{ \begin{array}{l} (CB_0)^{-1} C \triangle A \\ - \triangle I' (CB_0)^{-1} CA_0 \end{array} \right\}_j}{\min\{I + \triangle I'\}} & sign(s^* x_j) > 0 \\ \leq \dfrac{\min\left\{ \begin{array}{l} (CB_0)^{-1} C \triangle A \\ - \triangle I' (CB_0)^{-1} CA_0 \end{array} \right\}_j}{\min\{I + \triangle I'\}} & sign(s^* x_j) < 0 \end{cases}$$

$$j = 1, 2, \ldots, n \tag{17}$$

$$G = \begin{cases} \geq \dfrac{\max\{(CB_0)^{-1} C \Delta D(t)\}}{\min\{I + \triangle I'\}} & sign(s^*) > 0 \\ \leq \dfrac{\min\{(CB_0)^{-1} C \Delta D(t)\}}{\min\{I + \triangle I'\}} & sign(s^*) < 0 \end{cases} \tag{18}$$

then

$$s^* \cdot \dot{s}^* < 0. \tag{19}$$

If the sliding mode equation $s^* = 0$, then $s = 0$ since $CB_0 \neq 0$. The inverse augment also holds, therefore the two sliding surfaces both are equal i.e. $s = s^* = 0$, which completes the proof of Theorem 1.

The sliding mode equation i.e. the sliding surface $s = 0$ is the same as that of $s^* = 0$. To compare the control inputs, u_1 and u_2, the form is the same but the gains of u_1 are multiplied by (CB_0). To compare the control input, u^* and u_2, the form and the gain is the same. The two methods both equivalently diagonalize the system, so those are called the two diagonalization methods.

III. Design Example and Simulation Studies

Consider a third order uncertain system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 \pm 0.3 & 2 \pm 0.6 & 3 \pm 0.9 \end{bmatrix} x(t) \\ &+ \begin{bmatrix} 0 \\ 2 \pm 0.1 \\ 3 \pm 0.2 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ \pm 5.0 \end{bmatrix} \end{aligned} \tag{20}$$

where the nominal parameter A_0 and B_0 and matched uncertainties $\triangle A$, $\triangle B$ are

$$A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix},$$

$$\triangle A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \pm 0.3 & \pm 0.6 & \pm 0.9 \end{bmatrix}, \quad \triangle B = \begin{bmatrix} 0 \\ \pm 0.1 \\ \pm 0.2 \end{bmatrix},$$

$$\Delta D(t) = \begin{bmatrix} 0 \\ 0 \\ \pm 5.0 \end{bmatrix} \tag{21}$$

The stable coefficient of the sliding surface is determined as

$$C = [1 \;\; 1 \;\; 5] \tag{22}$$

3.1 Control input transformation

$$\begin{aligned} u^* &= (CB_0)^{-1} u_1, \\ H_u &= (CB_0)^{-1} = [1 \;\; 1 \;\; 5] \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} = 17^{-1} \\ &= 17^{-1} [-Kx - \Delta Kx - Gsign(s)] \end{aligned} \tag{23}$$

Then, the real dynamics of s, i.e. the time derivative of s is as follows:

$$\begin{aligned} \dot{s} &= C\dot{x} \\ &= CA_0 x - Kx + C\Delta Ax - \Delta IKx \\ &\quad - (I + \Delta I)\Delta Kx + C\Delta D(t) \\ &\quad - (I + \Delta I) Gsign(s) \end{aligned} \tag{24}$$

where

$$\begin{aligned} \Delta I &= C\Delta B (CB_0)^{-1} \\ &= [1 \;\; 1 \;\; 5] \begin{bmatrix} 0 \\ \pm 0.1 \\ \pm 0.2 \end{bmatrix} 17^{-1} \\ &= \pm 1.1/17 = \pm 0.065 \end{aligned} \tag{25}$$

By letting the constant gain

$$K = CA_0 = [1 \;\; 1 \;\; 5] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} = [5 \;\; 11 \;\; 16] \tag{26}$$

then the real dynamics of s becomes

$$\dot{s} = [C\Delta A - \Delta IK]x - (I + \Delta I)\Delta Kx + C\Delta D(t) - (I + \Delta I)Gsign(s) \tag{27}$$

If one takes the switching gain as design parameters

$$\Delta k_1 = \begin{cases} 1.98 & \text{if } sx_1 > 0 \\ -1.98 & \text{if } sx_1 < 0 \end{cases}$$

$$\Delta k_2 = \begin{cases} 3.98 & \text{if } sx_2 > 0 \\ -3.98 & \text{if } sx_2 < 0 \end{cases}$$

$$\Delta k_3 = \begin{cases} 5.93 & \text{if } sx_3 > 0 \\ -5.93 & \text{if } sx_3 < 0 \end{cases}$$

$$G_3 = \begin{cases} 26.74 & \text{if } s > 0 \\ -26.74 & \text{if } s < 0 \end{cases} \tag{28}$$

then one can obtain the following equation

$$s \cdot \dot{s} < 0 \tag{29}$$

The existence condition of the sliding mode is proved. The equation of the sliding mode, i.e. the sliding surface is invariant to the control input transformation.

The simulation is carried out under 1[msec] sampling time and with $x(0) = [1 \;\; .5 \;\; 0]^T$ initial state. Fig. 1 shows the three output responses, X_1, X_2 and X_3 by u^* with s.

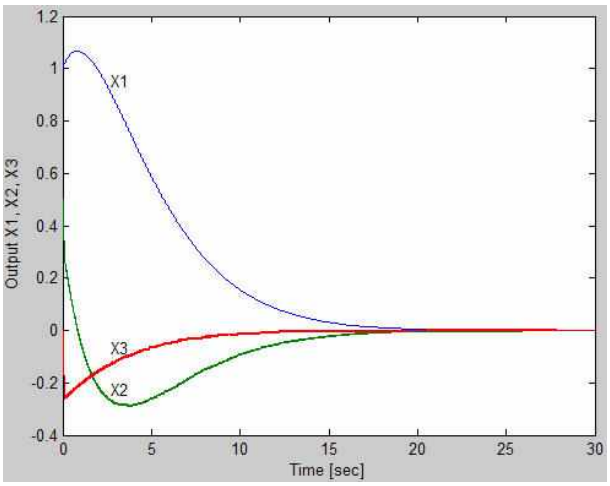

그림 1. 제어입력 변환에 의한 출력응답 X_1, X_2, and X_3

Fig. 1. Output responses, X_1, X_2, and X_3 by the control input transformation.

3.2 Sliding surface transformation

$$s^* = (CB_0)^{-1} \cdot s, \quad H_s(x,t) = (CB_0)^{-1} = 17^{-1} \tag{30}$$

Now, the VSS control input is taken as follows:

$$u_2 = -K \cdot x - \Delta K \cdot x - G \cdot sign(s^*) \tag{31}$$

The real dynamics of the sliding surface, i.e. the time derivative of s^* becomes

$$\begin{aligned} \dot{s}^* &= (CB_0)^{-1}\dot{s} = (CB_0)^{-1}C\dot{x} \\ &= (CB_0)^{-1}CA_0x - Kx + (CB_0)^{-1}C\Delta Ax \\ &\quad - \Delta IKx - (I + \Delta I)\Delta Kx \\ &\quad + (CB_0)^{-1}C\Delta D(t) \\ &\quad - (I + \Delta I)Gsign(s^*) \end{aligned} \tag{32}$$

By letting gain

$$\begin{aligned} K &= (CB_0)^{-1}CA_0 = 17^{-1}[5 \;\; 11 \;\; 16] \\ &= [0.294 \;\; 0.647 \;\; 0.942] \end{aligned} \tag{33}$$

then the real dynamics of s^* becomes

$$\begin{aligned} \dot{s}^* &= [(CB_0)^{-1}C\Delta A - \Delta I' K]x - (I + \Delta I')\Delta Kx \\ &\quad + (CB_0)^{-1}C\Delta D(t) - (I + \Delta I')Gsign(s^*) \end{aligned} \tag{34}$$

$$\begin{aligned} \Delta I' &= (CB_0)^{-1}C\Delta B = 17^{-1}[1 \;\; 1 \;\; 5]\begin{bmatrix} 0 \\ \pm 0.1 \\ \pm 0.2 \end{bmatrix} \\ &= \pm 1.1/17 = \pm 0.065 \end{aligned} \tag{35}$$

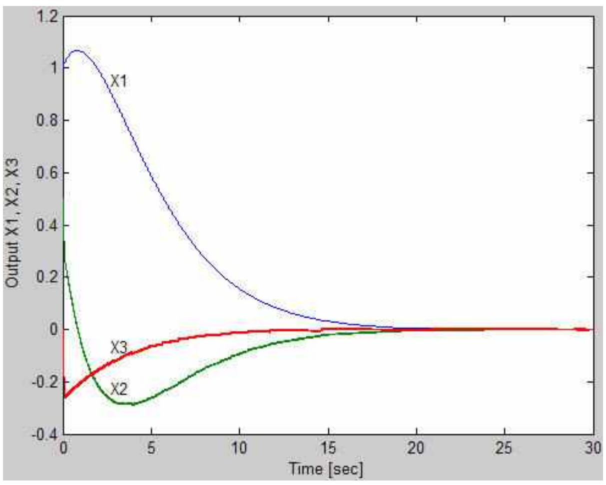

그림 2. 슬라이딩면 변환에 의한 출력응답 X_1, X_2, and X_3

Fig. 2. Output responses, X_1, X_2, and X_3 by the sliding surface transformation

If one takes the switching gains as follows:

$$\Delta k_1 = \begin{cases} 0.115 & \text{if } s^* x_1 > 0 \\ -0.115 & \text{if } s^* x_1 < 0 \end{cases}$$

$$\Delta k_2 = \begin{cases} 0.234 & \text{if } s^* x_2 > 0 \\ -0.234 & \text{if } s^* x_2 < 0 \end{cases}$$

$$\Delta k_3 = \begin{cases} 0.349 & \text{if } s^* x_3 > 0 \\ -0.349 & \text{if } s^* x_3 < 0 \end{cases}$$

$$G_3 = \begin{cases} 1.573 & \text{if } s > 0 \\ -1.573 & \text{if } s < 0 \end{cases} \tag{36}$$

then

$$s^* \cdot \dot{s}^* < 0 \tag{37}$$

if $s^* = 0$, then $s = 0$. The inverse augment also holds. The switching gains in (36) can be obtained also from (28) by multiplying $(CB_0)^{-1} = 17^{-1}$.

The simulation is carried out under 1[msec] sampling time and with $x(0) = [1 \ \ .5 \ \ 0]^T$ initial condition. Fig. 2 shows the three output responses, X_1, X_2 and X_3 by u_2 with s^*. Those are almost identical to Fig. 1 because the sliding surface $s = 0 = s^*$ is equal and the continuous gains and discontinuous gains of the two controls, u^* and u_2, both are equal.

Ⅳ. Conclusions

In this note, the invariant theorem of Utkin is rigorously proved for SI uncertain linear systems. The invariance theorem of the two diagonal methods i.e., the control input transformation and sliding surface transformation is proved clearly and comparatively. Therefore, the equation of the sliding mode, i.e., the sliding surface is invariant with respect to the two diagonalization methods. These two methods diagonalize the input system of the real sliding dynamics of the sliding surface s or s^* so that the existence condition of the sliding mode on the predetermined sliding surface is easily proved. During the proof of Utkin's theorem, the design rules of the two control inputs are proposed. Through an illustrative example and simulation study, the effectiveness of the proposed main results is verified. The same results of the outputs by the two diagonalization methods are obtained. The equation of the sliding mode, i.e., the sliding surface is invariant with respect to the two diagonalization methods.

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