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# Balanced Howell Rotations를 이용한 동적 라우팅 정보 생성

# ( Generation of Dynamic Routing Information by using Balanced Howell Rotations )

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## 요 약

통신 노드들이 이동 중인 mobile ad hoc networks에서, 통신처리율에 따라 노드 쌍들의 순위를 결정하면, 이를 바탕으로 전체 네트워크의 처리율을 향상시키는 동적인 라우팅을 할 수 있다. Balanced Howell rotations는 브리지 게임을 위한 토너먼트 구성 방법의 하나이다. 본 논문에서는 mobile ad hoc networks의 전반적인 통신 처리율 향상을 위해 balanced Howell rotations를 활용할 수 있음을 설명한다. 그리고 balanced Howell rotations가 존재할 수 있는 조건을 제시하고, 이를 증명한다.

## Abstract

In mobile ad hoc networks where the communication nodes are moving around, one may perform dynamic routing that can increase the total communication throughput of the network, by determining the ranks of pairs of nodes according to their communication throughput. The balanced Howell rotation is a tournament design scheme for bridge games. This paper explains that the balanced Howell rotation can be applied to enhance the overall communication throughput of mobile ad hoc networks, and proposes and proves the condition under which the balanced Howell rotations may exist.

**Keywords :** mobile ad hoc networks, inter-node communication, balanced Howell rotations, Number theory, twin prime powers

## Ⅰ. Introduction

Mobile ad hoc networks are communication environments that change dynamically in time. For the nodes are moving around during the communications, the network should be able to handle problems like dynamic routing. In such an environment, it can be a help to know which pairs of nodes are in better conditions of communications. So, one may need to design a fast procedure to find the wanted pairs effectively. For such a procedure, one may introduce the use of balanced Howell rotation (BHR). When communications are going on over the network, one may perform a tournament, during a given time interval, to determine the ranks of pairs of nodes according to their communication throughput. In routing, by constructing communication paths with the results of the tournament, one may expect to improve the overall communication throughput of the network.

In the main part of this paper, the mathematical properties that ensure BHR can be constructed are proposed and proved. Consider a tournament of $n=2k$ teams and $n-1$ boards. On each board $i$, the $n$ teams are divided into $n$ ordered pairs

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$(a_{ij}, b_{ij})$, $j = 1,2,\cdots,k$. The two teams $a_{ij}$ and $b_{ij}$ are said to oppose each other on board $i$ and two teams $a_{ij}$ and $a_{ik}$(or $b_{ij}$ and $b_{ik}$) are said to compete with each other. A balanced Howell rotation of order $n$, denoted by $BHR(n)$, is such a tournament satisfying the following conditions.

(a) Each team opposes every other team exactly once.

(b) Each team competes with every other team exactly $k-1$ times.

The balanced Howell rotation has applications in the bridge tournament. Parker and Mood[1] first bought this design to one's attention. They proved that if $BHR(n)$ exists then $n \equiv 0 (\mathrm{mod}\, 4)$. So far, the existence of $BHR(n)$ has been known for the following $n$:

(1) $n-1$ is a prime power (Berlekamp and Hwang [2]).

(2) $n/2-1$ is an odd prime power at least 7 (Schellenberg[3], Hanner[4]).

(3) $n = PQ+1$where $P = p^{\alpha}$ and$Q = q^{\beta}$ are two prime powers with $P+2 = Q$ and $q \neq 3$ (Du and Hwang[5]).

In this note, we study $BHR(PQ+1)$ in the case $q \neq 3$ and prove that $BHR(PQ+1)$ always exists for $PQ < 10^{20}$.

Strong starters and skew starters[6~7] are in wide use for combinatorial designs. Particularly, skew balanced starters and symmetric skew balanced starters[6~7] can be used in constructing completely balanced Howell rotations. Du and Hwang[5] has proved, with the properties of a Galois domain which is a direct sum of two Galois fields, that $BHR(n)$ exists when $n = p^r q^s$ where $p$ and $q \neq 3$ are primes and $q^s = p^r + 2$. Du and Hwang[6] has given an approach to construct symmetric skew balanced starters on $n$, where $n$ is of the form: $n = 2^m k + 1$ a prime power with $k$ odd. Also, showed that, for $n$ of the above form with $k > 9 \cdot 2^{3m}$, there exists a symmetric skew balanced starter. Note that a symmetric skew balanced starter on $n$, where $n$ is odd, can be used to construct complete balanced Howell rotations for $n$ and $2(n+1)$, and also for $n+1$, where $n \equiv 3(\mathrm{mod}\, 4)$. As well, let $n = 2^m k + 1$ be an odd prime power where $m \geq 2$ and $k$ is an odd number. The existence of symmetric skew balanced starters for $GF(n)$ is proved, by Du and Hwang[7], for $m \geq 2$ and $k \geq 3$.

The BHR, when multiple teams are playing games, is defined to determine the winner reasonably. In each game of a tournament, situations should be determined by the players, and the results should not be predicted by any means. In terms of equity, all players should be assigned to play the same number of games. The BHR, satisfying such fundamental matters, is defined so that players should have equal opportunities. In the next section, an example of BHR is shown to explain the meanings of the words and components, and the relationships among those. Section Ⅲ explains how the BHR can be applied to determine the ranks of pairs according to their throughput over the network. In section Ⅳ, some mathematical conditions, under which BHR can be constructed, are suggested and proved. Section Ⅴ concludes with the mention on the effect of the results and further study.

## Ⅱ. Related Work

'Games on Four Tables' (Table 1) is an example of BHR, by General Gruenther[1], which is reproduced here for an illustration purpose, and by which eight teams can play duplicate bridge game on four tables. As shown below, to run the seven boards, seven rounds are going on concurrently on the four tables. 'Board and Games' (Table 2) is from Table 1, where there are four games in each board. As shown in Table 2, the contestants of each team are determined in each board. For example, in board 1 the 8th team plays a game with the 1st team. However, the 1st team is not the contestant of the 8th team. The 8th team gets a relative score to those of 3rd, 4th, 5th teams, which are on the left side in the same board.

표 1. 테이블별 시합
Table 1. Games on Four Tables.

| round | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| table1 | teams | 8<br>1 | 8<br>2 | 8<br>3 | 8<br>4 | 8<br>5 | 8<br>6 | 8<br>7 |
| | board | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| table2 | teams | 6<br>3 | 7<br>4 | 1<br>5 | 2<br>6 | 3<br>7 | 4<br>1 | 5<br>2 |
| | board | 4 | 5 | 6 | 7 | 1 | 2 | 3 |
| table3 | teams | 2<br>7 | 3<br>1 | 4<br>2 | 5<br>3 | 6<br>4 | 7<br>5 | 1<br>6 |
| | board | 6 | 7 | 1 | 2 | 3 | 4 | 5 |
| table4 | teams | 4<br>5 | 5<br>6 | 6<br>7 | 7<br>1 | 1<br>2 | 2<br>3 | 3<br>4 |
| | board | 7 | 1 | 2 | 3 | 4 | 5 | 6 |

표 2. 보드별 시합
Table 2. Board and Games.

| board | games | | | |
|---|---|---|---|---|
| 1 | 8,1 | 3,7 | 4,2 | 5,6 |
| 2 | 8,2 | 4,1 | 5,3 | 6,7 |
| 3 | 8,3 | 5,2 | 6,4 | 7,1 |
| 4 | 8,4 | 6,3 | 7,5 | 1,2 |
| 5 | 8,5 | 7,4 | 1,6 | 2,3 |
| 6 | 8,6 | 1,5 | 2,7 | 3,4 |
| 7 | 8,7 | 2,6 | 3,1 | 4,5 |

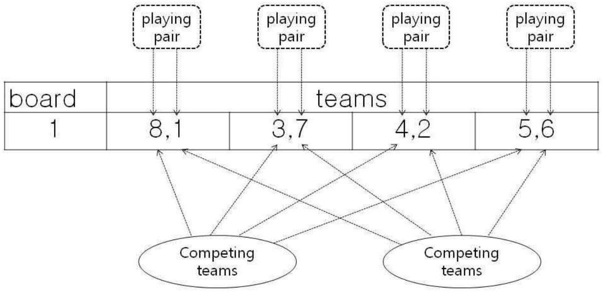

그림 1. 경기상대와 경쟁자
Fig. 1. Partner and Contestant.

That is, the 8th team directly contests with 3rd, 4th, 5th teams in board 1. The play partner and the contestants are determined by Table 2, in such a way.

With this method of scoring, the possibility of scoring by a lucky chance can be reduced much. One of the important points, from this example, is that each team contests equally three times with other different teams.

표 3. 보드별 경쟁자 지정
Table 3. contestants in each board.

| team<br>s | contesting teams in each board | | | | | | |
|---|---|---|---|---|---|---|---|
| | board<br>1 | board<br>2 | board<br>3 | board<br>4 | board<br>5 | board<br>6 | board<br>7 |
| team<br>1 | 7<br>2<br>6 | 2<br>3<br>7 | 3<br>2<br>4 | 8<br>6<br>7 | 8<br>7<br>2 | 8<br>2<br>3 | 7<br>6<br>5 |
| team<br>2 | 1<br>7<br>6 | 1<br>3<br>7 | 3<br>4<br>1 | 4<br>3<br>5 | 8<br>7<br>1 | 8<br>1<br>3 | 8<br>3<br>4 |
| team<br>3 | 8<br>4<br>5 | 2<br>1<br>7 | 2<br>4<br>1 | 4<br>5<br>2 | 5<br>4<br>6 | 8<br>1<br>2 | 8<br>2<br>4 |
| team<br>4 | 8<br>3<br>5 | 8<br>5<br>6 | 3<br>2<br>1 | 3<br>5<br>2 | 5<br>6<br>3 | 6<br>5<br>7 | 8<br>2<br>3 |
| team<br>5 | 8<br>3<br>4 | 8<br>4<br>6 | 8<br>6<br>7 | 4<br>3<br>2 | 4<br>6<br>3 | 6<br>7<br>4 | 7<br>6<br>1 |
| team<br>6 | 1<br>7<br>2 | 8<br>4<br>5 | 8<br>5<br>7 | 8<br>7<br>1 | 5<br>4<br>3 | 5<br>7<br>4 | 7<br>1<br>5 |
| team<br>7 | 1<br>2<br>6 | 2<br>1<br>3 | 8<br>5<br>6 | 8<br>6<br>1 | 8<br>1<br>2 | 6<br>5<br>4 | 6<br>1<br>5 |
| team<br>8 | 3<br>4<br>5 | 4<br>5<br>6 | 5<br>6<br>7 | 6<br>7<br>1 | 7<br>1<br>2 | 1<br>2<br>3 | 2<br>3<br>4 |

## Ⅲ. Determining the Ranks of Pairs

One may apply the BHR to the problem of determining the ranks of the pairs of nodes, over the

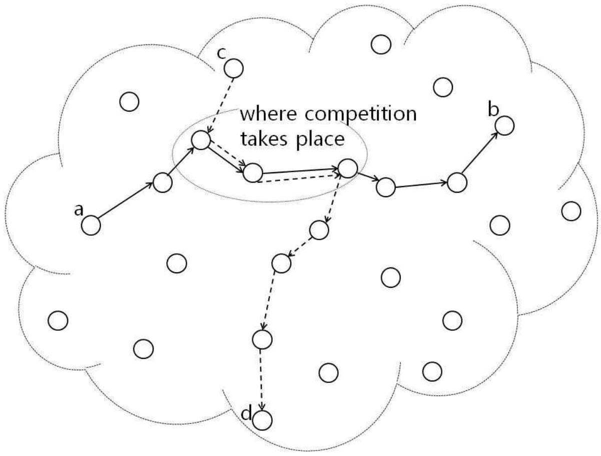

그림 2. 네트워크상의 경쟁중인 노드쌍
Fig. 2. Competing Pairs on the Network.

ad hoc network during the current time interval, according to the communication throughput between the two nodes of the pairs.

A team of a BHR corresponds to a pair of nodes in the network. The game of the two teams in BHR corresponds to the 'sending and receiving' an amount of signals concurrently and continuously, in a short given time interval, along each of the two paths of two pairs, which corresponds to the two teams, of nodes in the network. That is, the two nodes $a$ and $b$ in the pair $(a, b)$, which is regarded as a team in a BHR, send and receive signals with each other, while the two in $(c, d)$ do the same. While the tries, of the two pairs, of 'sending and receiving' signals are influencing each other, the pair with better conditions may produce higher communication throughput. As in a BHR, the throughput of $(a, b)$ is not compared with that of $(c, d)$, instead it is compared relatively with the throughputs of the other three pairs in the same board, and then evaluated to a score. In the example of 'Games on Four Tables,' the games from four different boards are performed concurrently on the four tables to reduce the overall time. However, over the network, the 'sending and receiving's from boards are performed just serially. The nodes, in the two pairs, are 'sending and receiving' signals in a given time interval, by the programs, to run the prescribed procedure, which are installed in each node. The results of 'sending and receiving' are saved in each node, and to be used as routing information after the procedure is completed.

As shown next by the mathematical proof, one can not apply BHR to arbitrarily many pairs of nodes in a network, because a BHR can be constructed when the number of teams meets some conditions. Actually, the conditions are hard to be met. Therefore, one should partition the network by locations so that the number of partitions meets the condition, by which the BHR can be constructed. One may properly assume that the nodes within a near-by area — inside a partition— practically have almost the same conditions of communications, as long as all the nodes have the same architecture. So, one may choose a node from a partition so that it should represent the nodes in there. As for the example of 8 teams in a BHR, one may divide given network into 8 partitions. Then, 'the pair of nodes' can be extended as 'the pair of partitions'. Now it should be considered how one may partition the network so that the above assumed matter can be properly accepted. To know which are located near to one node, there are many messaging protocols, the use of GPS, and other techniques. Though each technique may give out different partitions, geometric vicinity of nodes may generally help to compromise the differences. That is, even when two nodes are fall into two different partitions, the two partitions will behave more similarly if the two nodes are closer each other.

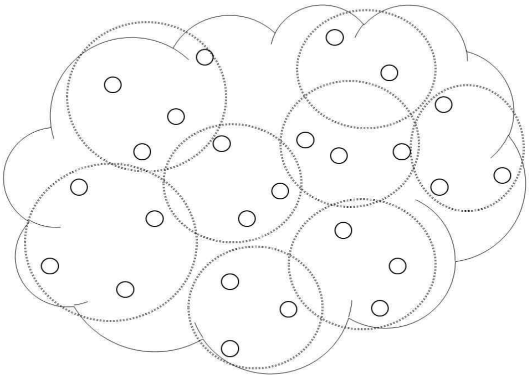

그림 3. 노드의 8 파티션
Fig. 3. Eight Partitions of Nodes.

Now regarding a pair of partitions as a team in a BHR, one may apply the BHR to determine the ranks among the communicating pairs. Once the upper ranked pairs of partitions are chosen, any pair of nodes from the partition-pairs will be taken into the backbone of the route over the current network. To determine the nodes of partitions, one may proceed as shown in Fig. 4. One node, say $n_{P_1}$, which is in the backbone of current route, is chosen to run the partitioning program. Then the program calls all other nodes for their GPS's. After all nodes have sent their current GPS's to $n_{P_1}$, the program runs as follows. With the number of total nodes and the

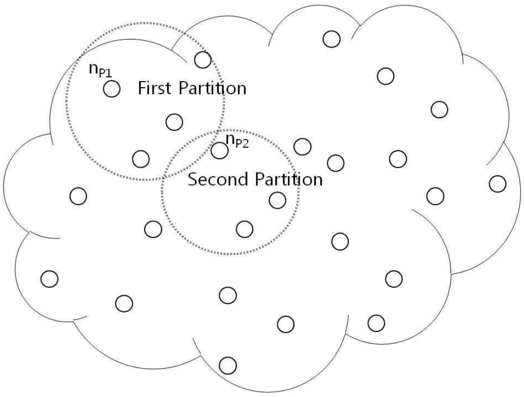

그림 4. 파티션 단계
Fig. 4. Stage of Partitioning.

number $n$ of BHR, the program may know the size $p_s$ of a partition. Then, by the distances from the GPS's of other nodes, $n_{P_1}$can determine to include its near-by neighbor nodes one by one into its partition until the number of nodes reaches $p_s$. This group of nodes becomes the first partition, where $n_{P_1}$ becomes its representative. By determining the second representative $n_{P_2}$ one may proceed the same procedure to know the second partition, etc. The second representative $n_{P_2}$ can reasonably be determined by choosing the next closer node to the last chosen node of the first partition.

Once a node is notified to be a representative, it may get into the tournament when the time comes. The tournament can be performed periodically or by demands from nodes in a troubled area: the area of poor communication conditions.

## Ⅳ. The Existence of BHR

**Theorem 1** Let $n-1$ be a product of twin prime powers, i.e., $n = PQ+1$ where $P = p^\alpha$ and $Q = q^\beta$ with $P+2 = Q$ for some primes $p$ and $q$. Then $BHR(n)$ exists unless $\alpha$ is an odd number at least 3, $\beta$ is an even number at least 2 and $q = 3$.

Proof. To prove this theorem, it suffices to prove the existence of $BHR(PQ+1)$ in the following three cases.

Case 1. $q \neq 3$.
Case 2. $q = 3$ and $\beta$ is odd.
Case 3. $q = 3$, $\beta$ is even and $\alpha = 1$.
Case 4. $q = 3$, $\beta$ is even and $\alpha$ is even.

Case 1 has been proved by Du and Hwang[5] by using Galois domain[8]. The following lemma can be found in their proof.

**Lemma 1** Let $x$ be a generator of Galois field $GF(p^\alpha)$ of order $p^\alpha$ and $y$ a generator of Galois field $GF(q^\beta)$ of order $q^\beta$. Suppose that $k$, $m$ and $z$ satisfy

$$x^{2k}+1 = x^m,\ 0 \le m \le (P-2)/2,\ 2 = y^z$$

where $z-m$ is either 0 or 1. Then $BHR(PQ+1)$ exists.

To prove the existence of $BHR(PQ+1)$ in Case 2, we first prove the following lemma.

**Lemma 2** Let $p^\alpha \equiv 1(\mathrm{mod}8)$ and $x$ a generator of $GF(p^\alpha)$. Then $-2 = x^k$ for some even number $k$.

Proof. $GF(p^\alpha)$ has a unique subfield of order $p$. Denote $u = p^{\alpha-1}+p^{\alpha-2}+\cdots+1$. Then $x^u$ is a generator of the subfield of order $p$. If $p \equiv 1(\mathrm{mod}8)$, then 2 is a quadratic residue modulo $p$ (see[9]). Since $x^u$ is a generator of $GF(p)$, $2 = x^{2k'u}$ for some $k'$. Hence, $-2 = x^{2k'u+\frac{p^\alpha-1}{2}}$ where $2k'u+\frac{p^\alpha-1}{2}$ is an even number.

Next, we assume $p \not\equiv 1(\mathrm{mod}8)$. Note that $p$ is odd and $3^2 \equiv 1\,(\mathrm{mod}8)$, $5^2 \equiv 1\,(\mathrm{mod}8)$ and $7^2 \equiv 1\,(\mathrm{mod}8)$. Thus,

$$p^\alpha = \left\{ \begin{matrix} p\,(\mathrm{mod}8), & \mathrm{if}\,\alpha\,is\,odd, \\ 1\,(\mathrm{mod}8), & \mathrm{if}\,\alpha\,is\,even \end{matrix} \right\}$$

Since $p^\alpha \equiv 1(\mathrm{mod}8)$, $\alpha$ must be even. Therefore, $u$ is an even number. Since $-2$ is an element in the subfield $GF(p)$, we have $-2 = x^{k'u}$ for some $k'$ where $k'u$ is an even number. □

In Case 2, since $\beta$ is odd,

$$p^\alpha = 3^\beta - 2 = 3 \times 9^{\frac{\beta-1}{2}} - 2 \equiv 1(\mathrm{mod}8)\cdot$$

By Lemma 2, there exists $k$ such that $x^{2k} = -2$. Note that $-1 \equiv 2 \,(\mathrm{mod}\, 3)$. Thus, in $GF(3^{\beta})$, $2 = y^{(P+1)/2}$. Choose $z = (P+1)/2 = (Q-1)/2$, $m = (P-1)/2$. Then $x^{2k} + 1 = -1 = x^{m}$, $0 \le m \le \frac{P-1}{2}$, $2 = y^{z}$ and $z - m = 1$. By Lemma 1, $BHR(PQ+1)$ exists.

In Case 3, $PQ+1 = p(p+2)+1 = (p+1)^{2}$. To prove the existence of $BHR(PQ+1)$ in this case, we need a multiplication theorem given by Du and Hwang[10]. To describe this theorem, let us introduce the concept of partitionable starter.

Let $G$ be an additive Abelian group of order $g(g \equiv 3 \,(\mathrm{mod}\, 4))$ and $G^{*}$ the set of nonzero elements. A partitionable starter of $G$ is a collection of $(g-1)/2$ disjoint ordered pairs $(x_i, y_i)$, $i = 1,2,\dots,(g-1)/2$, of elements in $G^{*}$ satisfying the following conditions.

(a) Every element of $G^{*}$ appears in $\{\pm(x_i - y_i) | i = 1,2,\dots,(g-1)/2\}$ exactly once.

(b) These $(g-1)/2$ pairs can be divided into two sub-collections $S_1$ and $S_2$ sets such that $|S_2| = |S_1| + 1$, and $\{0\} \cup \{x | x$ is an element in a pair of $S_1\}$ and $\{x | x$ is an element in a pair of $S_2\}$ are supplementary difference set (i.e., every nonzero element appears in the differences $x - y$ where $x$ and $y$ are in the same set among the two sets equally often.).

Du and Hwang[10] proved the following.

**Lemma 3** If $p$ is a prime with $p \equiv -1 \,(\mathrm{mod}\, 8)$ and -2 is a generator of $GF(p)$, then the partitionable starter of $Z_p$ exists.

**Lemma 4** If $BHR(m)$, $BHR(n)$ and the partitionable starter of a Abelian group $G$ of order $n-1$ exist, then $BHR(mn)$ exists.

In Case 3, since $\beta$ is even, $p = 3^{\beta} - 2 \equiv -1 \,(\mathrm{mod}\, 8)$. Note that $p + 2 = 3^{\beta}$. This implies that -2 is a generator of $GF(p)$ (see[9]). Therefore, by Lemma 3, the partitionable starter of order $p$ exists. From Berlekamp and Hwang[2], $BHR(p+1)$ exists. By Lemma 4, $BHR((p+1)^{2})$ exists.

표 4. $9^{\beta' - 2}$의 인수분해

Table 4. Factorization of $9^{\beta' - 2}$.

| $\beta'$ | $9^{\beta' - 2}$ | factorization |
|---|---|---|
| 1 | 7 | 7 |
| 2 | 79 | 79 |
| 3 | 727 | 727 |
| 4 | 6559 | 7 · 937 |
| 5 | 59047 | 137 · 431 |
| 6 | 531439 | 113 · 4703 |
| 7 | 4782967 | 7 · 17 · 40193 |
| 8 | 43046719 | 89 · 483671 |
| 9 | 387420487 | 23 · 3617 · 4567 |
| 10 | 3486784399 | 7 · 498112057 |
| 11 | 31381059607 | 31381059607 |

Finally, we indicate that Case 4 actually does not exist. In fact, since $\beta$ is even, we have

$$p^{\alpha} = 3^{\beta} - 2 \equiv -1 \ (\mathrm{mod}\, 8).$$

By the same argument in the proof of Lemma 2, we know that

$$p^{\alpha} = \left\{ \begin{array}{ll} p \,(\mathrm{mod}\, 8), & \mathrm{if}\, \alpha\, is\, odd, \\ 1 \,(\mathrm{mod}\, 8), & \mathrm{if}\, \alpha\, is\, even \end{array} \right\}$$

Therefore, $\alpha$ is odd. □

We conjecture that there does not exist a prime $p$ such that $p^{2\alpha' + 1} + 2 = 9^{\beta'}$ for some positive integers $\alpha'$ and $\beta'$. In fact, we have verified it for $\beta' \le 10$. The following Table 4 shows the factorizations of $9^{\beta' - 2}$ for $1 \le \beta' \le 11$. This fact implies the following corollary.

**Corollary 1** If $n-1$ is a product of twin prime power and $n < 10^{20}$, then $BHR(n)$ exists.

## V. Conclusion

It is concluded that $BHR(n)$ exists when $n-1$ is a product of twin prime number and $n < 10^{20}$. Unfortunately, it is implied that the conclusion can not be a practically satisfying condition in using the BHR for ranking the pairs of nodes according to their communication throughputs because the density of twin primes for the number of nodes in a normal mobile network shall normally be very low. Therefore, the use of $BHR(n)$ is extended by introducing the partitions, which are the gatherings of near-by nodes on the network. That is, $n$ of $BHR(n)$ becomes the number of the partitions on the network, where the number can be varied flexibly depending upon the variable size of the partitions. Besides, since the BHR itself is a reasonable way of making a tournament to quickly rank the pairs in dynamic communication environments, it would be a challenge to discover other properties or methods, which can widen the practical use of the BHR in similar communication environments.

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