등기하 개념에 기초한 구조부재의 형상 최적화

Shape Optimization of Structural Members Based on Isogeometry Concept

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요 지

본 연구는 구조 시스템에서 중요 부위, 즉 응력이 집중되는 영역에서의 형상 최적화를 다룬 것이다. 등기하 해석은 기하학적 모델링(CAD)과 수치적 해석(CAE)을 통합하는 효율적인 방법으로 잘 알려져 있다. 이는 NURBS에 의한 기하학적 모델링을 직접 이용함으로써 이루어 질 수 있다. 본 연구에서는 등기하 개념을 도입한 효율적인 구조해석 컴퓨터 코드를 개발하였다. 여기에서는 CAD에 대한 정보를 유한요소 모델링에 직접 이용할 수 있다. 본 연구에서 개발한 코드의 타당성을 보이기 위해, 본 연구에서 개발한 코드에 의한 구조해석 결과를 유한요소해석 상용 패키지인 MSC/NASTRAN에 의한 결과와 비교하였다. 구조역학적인 문제에서 최적화를 다룰 수 있도록 본 연구의 등기하 해석 과정을 최적화 과정과 통합하였다. 본 시스템을 브라켓이 있는 외팔 구조의 형상 최적화에 성공적으로 적용하였다. 본 연구를 통해 개발한 시스템의 타당성을 검증하였다. 이 논문의 끝 부분에서는 본 연구방법의 실용적 적용성과 추후 연구에 대해 언급하였다.

핵심용어 : 유한요소해석, 등기하 해석, 조정점, 형상 최적화, NURBS

Abstract

This study is concerned with the shape optimization of structural members frequently found in critical area in a structure system, that is, highly stressed zone. Isogeometry analysis is well known to be the very efficient way to integrate the geometric modeling(CAD) and computational analysis(CAE). This can be accomplished by directly using the geometric modeling by NURBS(Non-Uniform Rational Basis Spline). In this study, an efficient computer code adopting the isogeometry concept has been developed for the structural analysis, in which CAD information can be directly used in the finite element modeling. In order to show the validity of the present code, the present results are compared with those by using the commercial package, that is, MSC/NASTRAN. The present isogeometric analysis procedure has been integrated with the optimization procedure to deal with the optimization problem found in the context of structural mechanics. The present system has been successfully applied to the shape optimization of cantilever structure having bracket. From the present study, it can be seen the validity of the present approach and computer codes developed in this study. This paper ends with some discussions about the practical usefulness of the present approach which is based on isogeometry analysis, and extension of the present study.

Keywords : *finite element analysis, isogeometric analysis, control points, shape optimization, NURBS*

1. 서 론

The basic concept of isogeometric analysis is to use the same functions between the solution space and the geometric modeling. Concept of The isogeometric analysis was introduced by Cho and Roh(2003), and NURBS based isogeometric analysis was developed by Hughes et al.(2005). Through isogeometric analysis, the geometric modeling from computer aided design (CAD) can be directly adopted to the mesh for computer aided engineering(CAE) without the extra regeneration process. The skip of this extra job can save the time by about 80% of overall analysis time(Hughes et al., 2005). NURBS basis is able to generate the exact geometry when compared with other basis functions. The isogeometric analysis can be, therefore, applied

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to the optimization problem(Cho and Ha, 2008; Bazilevs et al., 2007) in which the geometric sensitivity is important and to fluid dynamic simulations which needs much efforts in mesh generation based on CAD modeling(Wall et al., 2008).

Objective of the present paper is to apply the isogeometric analysis to a structural member to examine the efficiency of isogeometric analysis by comparing results with those by MSC/NASTRAN, and to conduct the shape optimization of structural member based on isogeometry concept.

The computer codes for the isogeometric analysis have been developed through the present study. To illustrate the validity of the present approach and computer codes developed based on it, structural analysis has been carried out for the cantilever plate having bracket as the relatively simple structural model. The results by the present codes have been compared with those by using the commercial package, MSC/NASTRAN. The results by the present computer codes for the isogeometric analysis show a good agreement with those by MSC/NASTRAN from view points of both displacement and stress. The computer codes for the isogeometric analysis has been integrated with the optimization procedure to produce the optimum shape design result.

From the present study, it can be said that the present computer codes for the isogeometric analysis and its based optimization can be extended to the application to more complex and large structural system.

2. Nurbs

NURBS(Non-Uniform Rational Basis Splines) are built from B-splines. A knot vector is a set of coordinates in parametric space of B-spline. In one dimension, the knot vector can be expressed as

$$\Xi = \{\xi_1, \xi_2, \cdots, \xi_{n+p+1}\} \tag{1}$$

where n is the number of basis function and p is the polynomial order. Basically, two types of knot vectors can be used, periodic and open, in two flavors, uniform and non-uniform(Rogers, 2001; Pigel and Tiller, 1997). Uniform knot vectors are incremented by equal interval but non-uniform knot vectors are incremented by nonequal interval. If a knot vector values at ends are repeated $p+1$ times for a given order p, this is said to be open.

B-spline basis functions are defined by the Cox-de Boor recursion formula(Rogers, 2001).

$$N_{i,0}(\xi) = \begin{cases} 1 \text{ if } \xi_i \le \xi < \xi_{i+1} \\ 0 \; otherwise \end{cases} , p = 0 \tag{2}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \; p = 1, 2, \ldots \tag{3}$$

The properties of B-spline basis functions are (Hughes et al., 2005),

(1) The sum of the B-spline basis functions for $\forall \xi$

$$\sum_{i=1}^{n} N_{i,p}(\xi) = 1 \tag{4}$$

(2) Each basis function is positive or zero for all parameter values. That is, $N_{i,p}(\xi) \ge 0$, $\forall \xi$.

(3) Each i-th knot vector with the order p basis function, $N_{i,p}(\xi)$ is contained in the interval $[\xi_i, \xi_{i+p+1}]$

NURBS is expressed as the sum of the product of rational basis function and the control point as

$$R_i^p(\xi) = \frac{N_{i,p}(\xi) w_i}{\sum_{\hat{i}=1}^{n} N_{\hat{i},p}(\xi) w_{\hat{i}}} \tag{5}$$

$$C(\xi) = \sum_{i=1}^{n} R_i^p(\xi) B_i \tag{6}$$

where R_i^p is the rational basis function, w_i is the i-th weight and B_i is the control point. Similarly the basis functions of NURBS surfaces and solids are

$$R_{i,j}^{p,q}(\xi,\eta)=\frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)w_{\hat{i},\hat{j}}} \quad (7)$$

$$R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta)=\frac{N_{i,p}(\xi)M_{j,q}(\eta)L_{k,r}(\zeta)w_{i,j,k}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}\sum_{\hat{k}=1}^{l}N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)L_{\hat{k},r}(\zeta)w_{\hat{i},\hat{j},\hat{k}}} \quad (8)$$

To increase the flexibility of B-spline curve, two methods(knot insertion and order elevation) can be considered. Inserting a single knot is referred to as knot insertion, whereas inserting multiple knots is called knot refinement(Pigel and Tiller, 1997). Knot may be inserted without changing the shape of the curve to increase the flexibility of the curve. For the given order of basis functions, p and number of basis functions, n, new knot insertion makes $n+1$ basis functions and $n+1$ control points formed from n control points. Order elevation is to increase the polynomial order of basis functions without changing the geometry. Each unique knot value is repeated to preserve discontinuities in the p-th derivative of the curve.

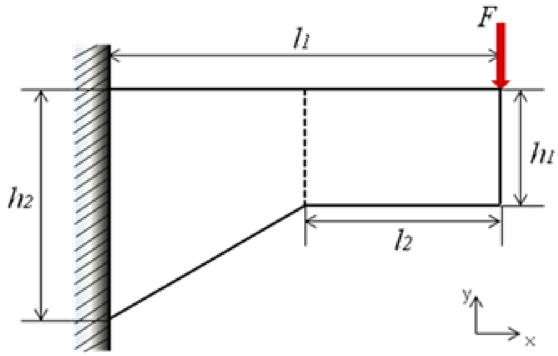
3. Structural Analysis using the Isogeometry Concept

To verify the present isogeometric analysis procedure, the cantilever plate having bracket as shown in Fig. 1 is selected as a test model, and the results are compared with those by the commercial package, MSC/NASTRAN. Fig. 1 shows the schematics of the geometry and the problem conditions. Total load of 10kN is uniformly distributed on the free edge of the model. This cantilever shape is modeled by C^0-NURBS basis function with knots vectors and contol points as follows and as shown in Table 1. Weights of all control points are given as 1.0. Modeling is shown in Fig. 2.

$$\Xi=\{0,0,0,5,1,1\}\in x-direction \quad (9)$$
$$H=\{0,0,1,1\} \quad \in y-direction$$

Based on the simple modeling of the given geometry, the mesh can be refined through knot insertion and order elevation. The number of control points in x and y direction are 9 and 5 with NURBS basis function of quadratic order, respectively. And the number of elements in MSC/NASTRAN is 439. Figs. 3 and 4 compare the results by MSC/NASTRAN and by the present isogeometric analysis.

The comparison of the displacement in y-direction is shown in Fig. 3. The maximum displacement in y-direction is -0.273mm and -0.270mm by MSC/NASTRAN and isogeometric analysis, respectively. From von-Mises stress contours shown in Fig. 4, it can be seen the stress concentration zone around the junction of bracket and main body(lower part of mid-span, that is, the part of red color).



l_1 =300mm, l_2 =150mm, h_1 =100mm, h_2 =200mm, t=10mm, F=10kN
Young prime's modulus=200GPa, Poisson ration=0.3

Fig. 1 Schematics of cantilever problem

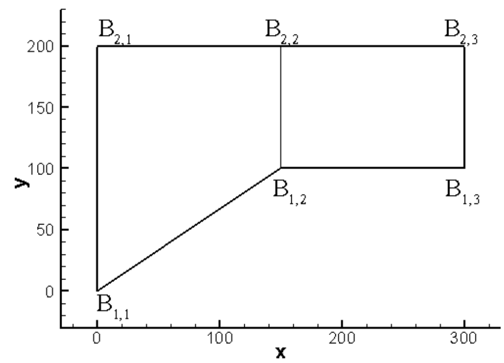
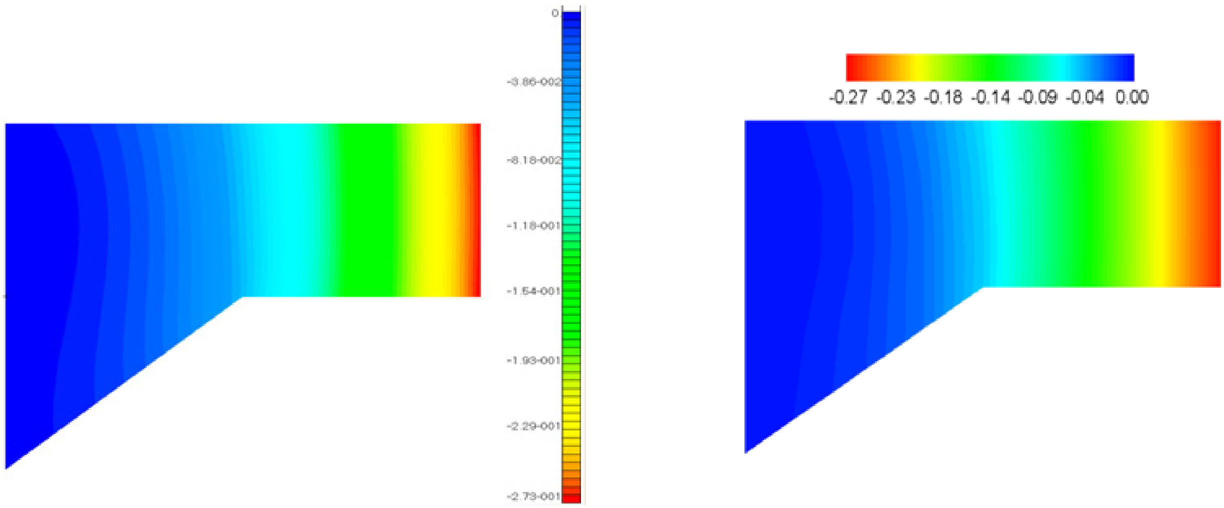


Fig. 2 Coarse mesh and control point

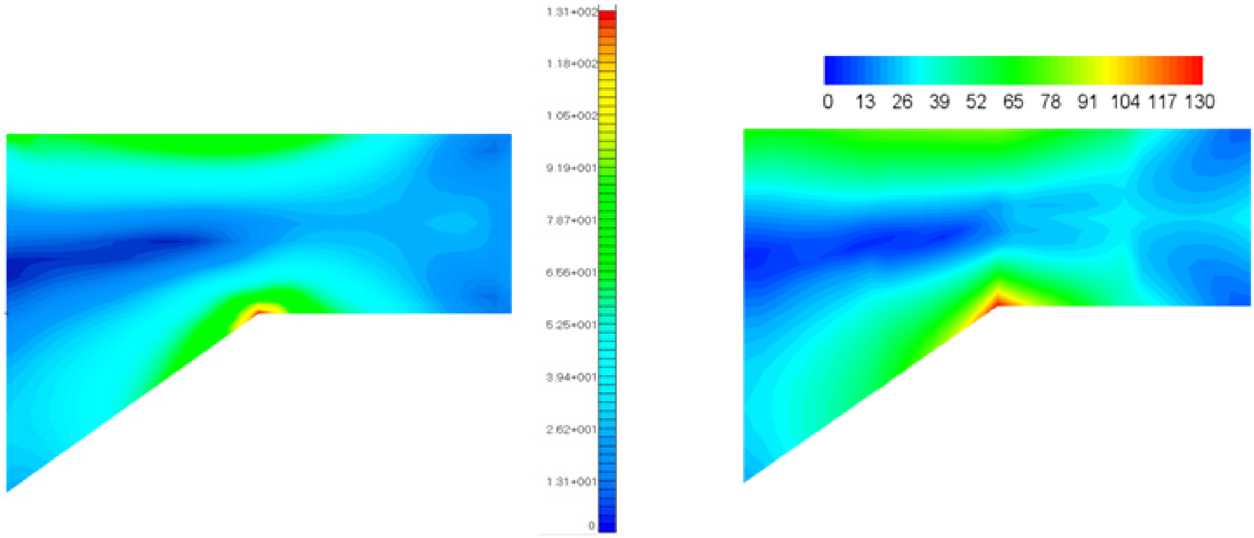
Table 1 Coordinates of control points

i	$B_{i,1}$	$B_{i,2}$	$B_{i,3}$
1	(0, 0)	(150, 100)	(300, 100)
2	(0, 200)	(150, 200)	(300, 200)



(a) by MSC/NASTRAN (b) by isogeometry analysis

Fig. 3 Contours of displacement y-direction(unit : mm)



(a) by MSC/NASTRAN (b) by isogeometry analysis

Fig. 4 von Mises stress contours(unit : MPa)

4. Shape Optimization

This section deals with the shape optimization problem. In this section, illustrated is the shape optimization based on the isogeometry analysis concept. The same structure model as in the previous section, that is, the cantilever plate having bracket shown in Fig. 1 is selected for the present study.

The present optimization problem is to find the coordinates of control points which minimize the weight of the structure, that is, area of the structural member. Hence, the coordinates of control points become the design variables. The statement of the problem is as follows.

- Minimize : area of the structural member
- Subject to the following constraints with three control points in both x- and y-directions as shown in Fig. 5.

4.1 Geometric constraints

In this paper, three geometric constraint cases are to be dealt with to illustrate the change in the

Table 2 Geometric constraints

Case	Constraints
Case 1	- points A, C, D, I : fixed - points B, G, H: movement in y-direction only is allowed - point F: movement in x-direction only is allowed - point E: movement in x- and y-directions is allowed
Case 2	- points A, C, I : fixed - points B, G, H: movement in y-direction only is allowed - point F: movement in x-direction only is allowed - points D, E: movement in x- and y-directions is allowed
Case 3	- points C, I : fixed - points A, B, G, H: movement in y-direction only is allowed - point F: movement in x-direction only is allowed - points D, E: movement in x- and y-directions is allowed

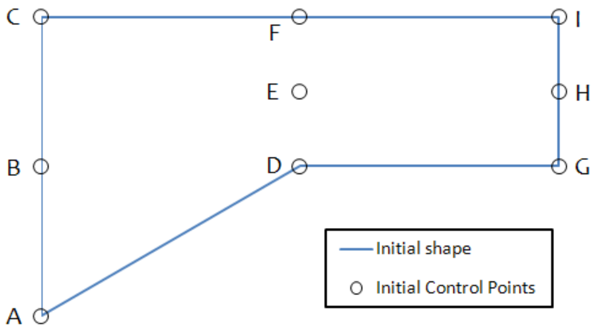


Fig. 5 Initial shape and initial locations of control points

optimized shape to geometric constraints.

4.2 Strength constraints

The strength constraint can be given as following inequality;

$$\sigma_{\max} \leq \sigma_{aw} \tag{10}$$

where $\sigma_{\max}$ is maximum von Mises stress and σ_{aw} is the allowable stress level. Material of the present member is assumed to be mild steel of which yield stress is σ_{yp}= 240MPa. In this study, the allowable stress σ_{aw} is assumed to be 50% of yield stress.

From the above statements of the present optimization problem, design variables involved in the present optimization problem can be defined as in Table 3.

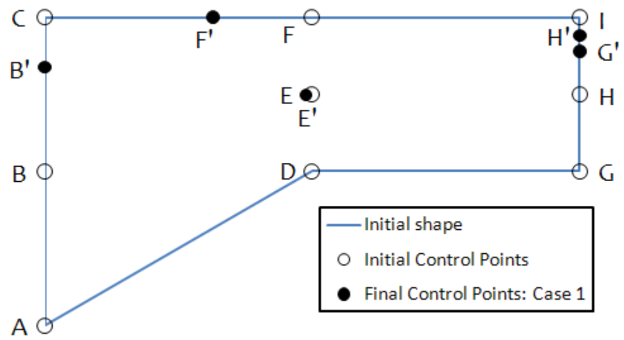
Table 3 Definition of design variables

Case	Design variables	No. of design variables
Case 1	- y-coordinates of points B, G and H - x-coordinate of point F - x- and y-coordinates of point E	6
Case 2	- y-coordinates of points B, G and H - x-coordinate of point F - x- and y-coordinates of points D and E	8
Case 3	- y-coordinates of points A, B, G and H - x-coordinate of point F - x- and y-coordinates of points D and E	9

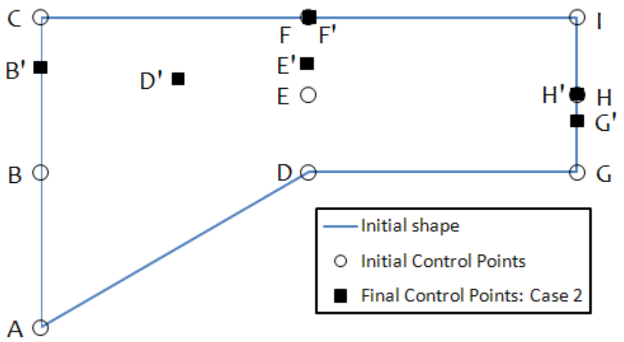
Since the problem is constrained optimization problem, any relevant optimization algorithm can be used for the present purpose. In this study, Rosenhill algorithm is used(Kuester and Mize, 1973). Since this study is intended to illustrated the development and application of the integrated computer codes for the isogeometric analysis based optimization, the relatively simple optimization algorithm has been used. And so, the optimization algorithm can be replaced by any other one which is expected to be more suitable to get the present goal, if needed.

Used are 3 control points in both x- and y-direction, and cubic basis functions as for illustration. Initial shape of the structural member and locations of control points are shown in Fig. 5. As the results of the present shape optimization based on the isogeometric analysis, Fig. 6 illustrates the movement of control points at the final stage of optimization procedure for three cases in Table 2. In Fig. 6, hollow circles, rectangles and triangles denote locations of control points at the final stage of the optimization procedure for three cases. It can be seen that movement of control points are apparently different to the geometric constraint conditions.

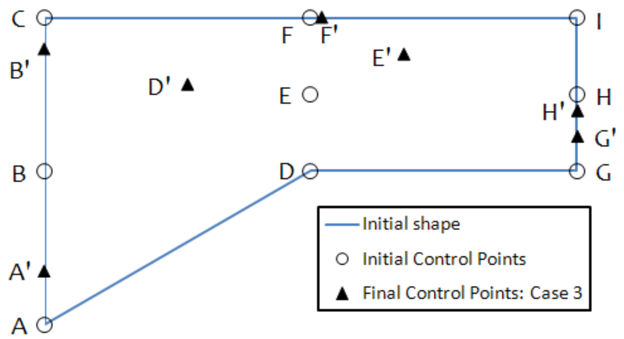
Geometric shape and stress contour at the initial stages are shown in Fig. 7, and those at the final stages are illustrated in Fig. 8 for the present three geometric constraint cases. It can be also seen that the stress concentration around the knuckled part (lower middle part) has been much mitigated due to



(a) case 1



(b) case 2



(c) case 3

Fig. 6 Movement of control points at the final stage

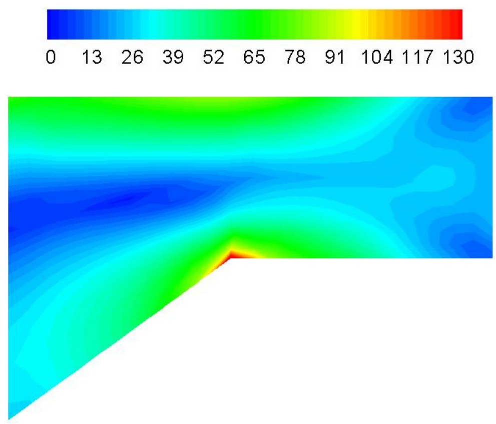
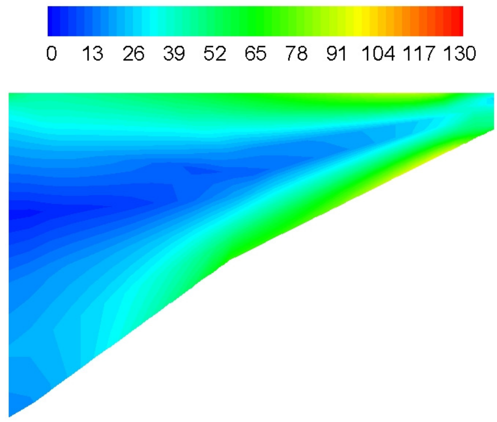
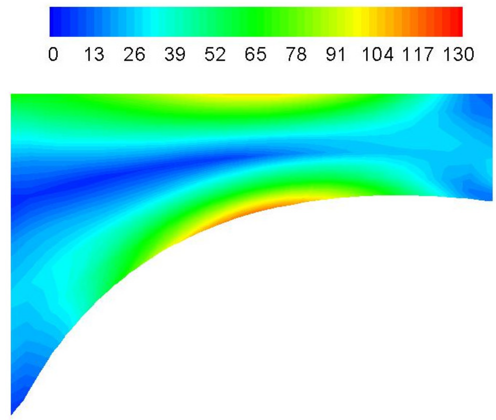


Fig. 7 von Mises stress contour before optimization

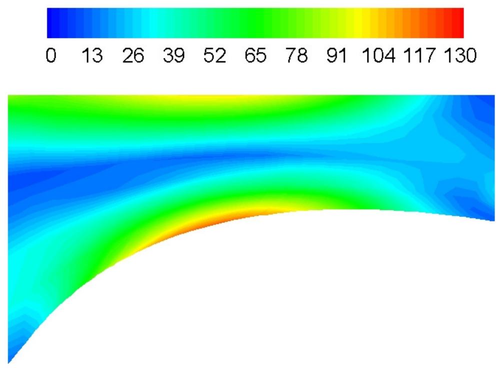
the smoothness of geometric shape. From the present shape optimization procedure, the ratio of area at the final stage to the area of the original shape is 0.825,



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 8 von Mises stress contours after optimization

0.740 and 0.709 for Case 1, 2 and 3, respectively. That is 17.5, 26.0 and 29.1% weight saving by specifying the three cases of geometric constrain, respectively can be achieved as far as present results are concerned.

As far as the present results are concerned, it can be said that the shape optimization procedure developed in the present study can give the reasonable optimized results.

5. Conclusions

In the present study, isogeometric analysis based on the NURBS basis function has been carried out for the structural mechanics problem. Cantilever geometry is generated by quadratic NURBS and MSC/NASTRAN. Accuracy and efficiency of isogeometric analysis have been verified through comparing the results by the present isogeometric analysis with those by MSC/NASTRAN. Results by isogeometric analysis show very good agreement with those by MSC/NASTRAN in terms of both displacement and stress.

Shape optimization based on isogeometry concept has been also carried out to find the optimal shape of cantilever with changing the geometric constraints. It can be seen that stress concentration has been much mitigated although area of the structural member was remarkably reduced for the optimized geometries, and in addition weight saving can be achieved at the same time. From the present findings, as far as the present results are concerned, it can be said that the present isogoemetric analysis procedure and computer codes developed based on it are reasonable.

The present computer codes developed based on the isogeometric analysis concept will be extended to apply to structural analysis and shape optimization of more complex structures, and the results will be presented ate the judicial proceedings and journal.

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