등기하 해석법을 이용한 구조해석

On the Structural Analysis Using the Isogeometry Analysis Approach

이 주 성† 장 경 식* 노 명 일**

Lee, Joo-Sung Chang, Kyoungsik Roh, Myoung Il

요 지

본 논문에서는 NURBS의 기저함수를 이용하는 등기하 해석을 선형 탄성 문제에 적용하였다. 등기하 해석의 목적은 기하학적 모델링 (CAD)와 수치적 해석 (CAE)를 통합하는 것인데, 이는 계산 망으로써 NURBS에 의한 기하학적 모델링 결과를 직접 이용해서 이룰 수 있다. NURBS 곡면은 조정점과 노트 벡터들을 이용하여 정확한 기하학적 형상을 표현할 수 있으며, 또한 요소의 정밀화 과정이 상대적으로 용이하다는 장점이 있다. 본 연구를 통해 개발된 컴퓨터 코드의 정당성을 보이기 위해 비교적 단순한 형태의 두 가지 구조모델에 적용하였다 ; 1) 균일 내압을 받는 실린더, 2) 균일 인장력이 작용하는 중앙에 구멍이 있는 정사각형 판. 이 두 모델은 정해가 있는 경우로서 절점을 추가하는 h-정밀화와 기저함수의 차수를 증가하는 p-정밀화에 의한 등기하 해석법을 적용한 근사해의 수렴성을 분석하였다.

핵심용어 : 유한요소해석, 등기하 해석, NURBS, h-정밀화, p-정밀화

Abstract

In the present work, isogeometric analysis in linear elasticity problem is conducted using the basis functions from NURBS. The objectives of isogeometric analysis introduced is to integrate both geometric modeling(CAD) and computational analysis(CAE), and this can be accomplished from direct usage of geometric modeling by NURBS as the computational mesh. The merit of the isogeometry analysis is that NURBS surface are able to represent exact geometry from the control points and knot vectors, and also subsequent refinement is relatively simple relatively. In order to verify the computer codes developed in this study, it has been applied to two structural models of which geometry are simple ; 1) circular cylinder subjected to the constant internal pressure loading, 2) square plate with circular hole at center subjected to uniform tension. The exact solutions of these two models are available. Convergence of the approximate solutions by the present code for the isogeometry analysis are investigated by mesh refinement with inserting knots (h-refinement) and by mesh refinement with order elevation of the basis functions (p-refinement).

***Keywords** : finite element analysis, isogeometric analysis, NURBS, h-refinement, p-refinement*

1. 서 론

The basic concept of the isogeometric analysis is to use the same functions between the solution space and the geometric modeling. This isogeometric analysis was introduced conceptually by Cho and Roh(2003), and NURBS based isogeometric analysis was developed by Hughes et al.(2005). Through isogeometric analysis, the geometric modeling from Computer Aided Design (CAD) is directly adopted to the mesh for Computer Aided Engineering(CAE) without the extra regeneration process. The omission of this extra job can save the time by about 80% of overall analysis time(Hughes et al.,2005). NURBS basis is able to generate the exact geometry when compared with other basis functions. Also the refinement procedure for more accurate solutions in computational analysis is simple and efficient. Therefore isogeometric analysis can be applied

† 책임저자, 정회원 · 울산대학교 조선해양공학부 교수
Tel: 052-259-2162 ; Fax: 052-259-2836
E-mail: jslee2@ulsan.ac.kr
* 울산대학교 조선해양공학부 교수
** 정회원 · 울산대학교 조선해양공학부 교수

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to the optimization problem(Cho and Ha, 2007) in which the geometric sensitivity is important and to fluid dynamic simulations which needs most of times for mesh generation based on CAD modeling(Bazilevs et al., 2007).

The objectives of the present paper are to use isogeometric analysis for the linear elasticity problem to : (i) examine the parametric study of the basis order and the knot vectors ; (ii) study the features of h-refinement and p-refinement.

2. Nurbs

2.1 Knot vectors

A knot vector is a set of coordinates in parametric space of B-spline. In one dimension, the knot vector can be expressed as $\Xi=\{\xi_1, \xi_2, \cdot\cdot, \xi_{n+p+1}\}$, where n is the number of basis functions and p is the polynomial order. Basically, two types of knot vectors can be used, periodic and open, in two flavors, uniform and non-uniform(Rogers, 2001; Pigel and Tiller, 1997). Uniform knot vectors is incremented by equal interval but nonuniform knot vectors by nonequal that. If a knot vector values at the ends is repeated $p+1$ times for a given order p, this is said to be open.

2.2 B-spline basis functions

B-spline basis functions are defined by the Cox-de Boor recursion formula(Rogers, 2001).

$$N_{i,0}(\xi)=\begin{cases}1 \text{ if } \xi_i \le \xi < \xi_{i+1} \\ 0 \ otherwise\end{cases} \tag{1}$$

For $p=1, 2, \ldots$

$$N_{i,p}(\xi)=\frac{\xi-\xi_i}{\xi_{i+p}-\xi_i}N_{i,p-1}(\xi)+\frac{\xi_{i+p+1}-\xi}{\xi_{i+p+1}-\xi_{i+1}}N_{i+1,p-1}(\xi) \tag{2}$$

The properties of B-spline basis functions are, (Hughes et al., 2005)

(1) The sum of the B-spline basis functions for $\forall\xi$,

$$\sum_{i=1}^{n} N_{i,p}(\xi)=1 \tag{3}$$

(2) Each basis function is positive or zero for all parameter values.

(3) Each ith with the order p basis function is contained in the interval $[\xi_i, \xi_{i+p+1}]$.

NURBS(Non-Uniform Rational B-Splines) is expressed as the sum of the product of rational basis function and the control points as

$$R_i^p(\xi)=\frac{N_{i,p}(\xi)w_i}{\sum_{\hat{i}=1}^{n} N_{\hat{i},p}(\xi)w_{\hat{i}}} \tag{4}$$

$$C(\xi)=\sum_{i=1}^{n} R_i^p(\xi)B_i \tag{5}$$

where R_i^p is the rational basis function, w_i is the ith weight and B_i is the control points. Similarly the basis functions of NURBS surfaces and solids are

$$R_{i,j}^{p,q}(\xi,\eta)=\frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m} N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)w_{\hat{i},\hat{j}}} \tag{6}$$

$$R_{i,j,k}^{p,q,r}(\xi,\eta,\zeta)=\frac{N_{i,p}(\xi)M_{j,q}(\eta)L_{k,r}(\zeta)w_{i,j,k}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}\sum_{\hat{k}=1}^{l} N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)L_{\hat{k},r}(\zeta)w_{\hat{i},\hat{j},\hat{k}}} \tag{7}$$

2.3 Mesh refinement

To increase the flexibility of B-spline curve, two methods, say knot insertion and order elevation, are basically considered.

Inserting a single knot is referred to as knot insertion, whereas inserting multiple knots is called knot refinement(Pigel, 1997). Knot may be inserted without changing the shape of the curve to increase the flexibility of the curve(h-refinement). At the given basis order (p) and basis function (n), new knot insertion makes $n+1$ basis functions and $n+1$

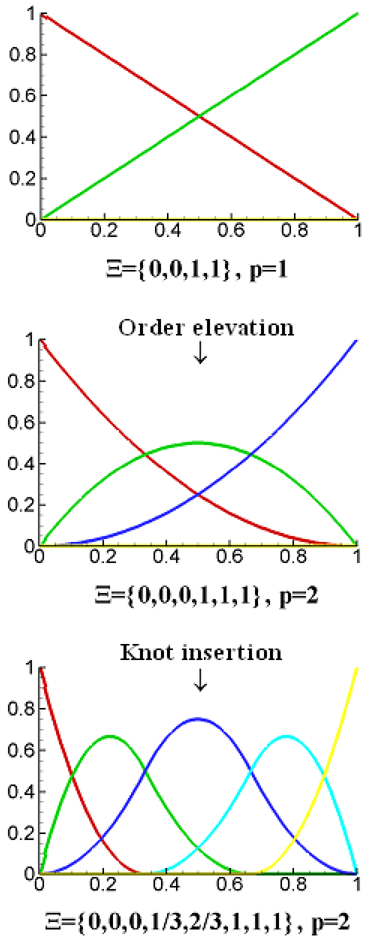

Fig. 1 k-refinement procedure

control points formed from n control points.

Order elevation(p-refinement) is to increase the polynomial order of basis functions without changing the geometry. Each unique knot value is repeated to preserve discontinuities in the p-th derivative of the curve.

Bazilevs et al.(2007) proposed new refinement strategy called k-refinement which has advantages of efficiency and robustness than previous p-refinement. In k-refinement, order elevation is followed by knot insertion, as it is shown in Fig. 1.

3. Isogeometry Analysis

3.1 Circular cylinder subjected to internal pressure

To validate isogeometric analysis we selected the circular cylinder subjected to the constant internal pressure loading as the test problem. Fig. 2 shows the schematics of the geometry and the problem conditions. The value of outer(r_{out}) and inner(r_{in}) radii are 2.0 and 1.0, respectively. The cylinder length is L=5 and the loading pressure is set to be 100. The simulated results are compared with the exact solution of Gould(2005) given as,

$$u_r(r)=\frac{1}{E}\frac{PR_i^2}{R_o^2-R_i^2}\left((1-\nu)r+\frac{R_o^2(1+\nu)}{r}\right) \tag{8}$$

$$\alpha_{rr}(r)=\frac{PR_i^2}{R_o^2-R_i^2}-\frac{PR_i^2R_o^2}{r^2(R_o^2-R_i^2)} \tag{9}$$

$$\alpha_{\theta\theta}(r)=\frac{PR_i^2}{R_o^2-R_i^2}+\frac{PR_i^2R_o^2}{r^2(R_o^2-R_i^2)} \tag{10}$$

where u_r is the displacement in the radial direction and σ_{rr} and $\sigma_{\theta\theta}$ are the stress components in radial and circumferential direction, respectively.

In the present works, the thick cylinder meshes are generated using NURBS and the modelled geometry is refined by knot insertion(h-refinement). Also isogeometric analysis is conducted with the same NURBS basis function to solve linear elasticity equation. The convergence is checked with respect to the order of the basis functions and the number of knot vectors. Finally the effects of k-refinement by Hughes et al.(2005) are studied. Fig. 3 represents the coarsest mesh and the subsequent meshes generated by h-refinement. In the case of the coarsest mesh, quadratic basis functions are used in the circumferential, radial and longitudinal directions. The knot vectors are shown below and the control points including weight are presented detail in Hughes et al.(2005).

$$\Xi=\{0,0,0,1,1,2,2,3,3,4,4,4\} \tag{11}$$

$$H=\{0,0,0,0.5,1,1,1\}$$

$$Z=\{0,0,0,1,1,1\}$$

Mesh 2 is produced by inserting the knot vector in the knot vectors of mesh 1. For example, the additionally inserted knot vector is 0.5, 1.5, 2.5

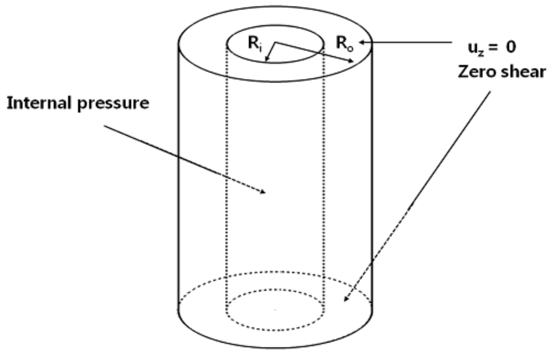

Fig. 2 Schematics of thick cylinder problem

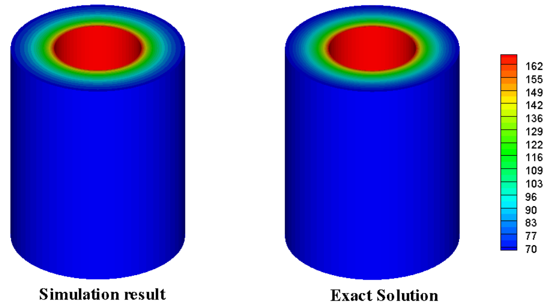

Fig. 4 Comparison of stress contours in circumferential direction. left) simulated results, right) exact solution (Gould, 1999).

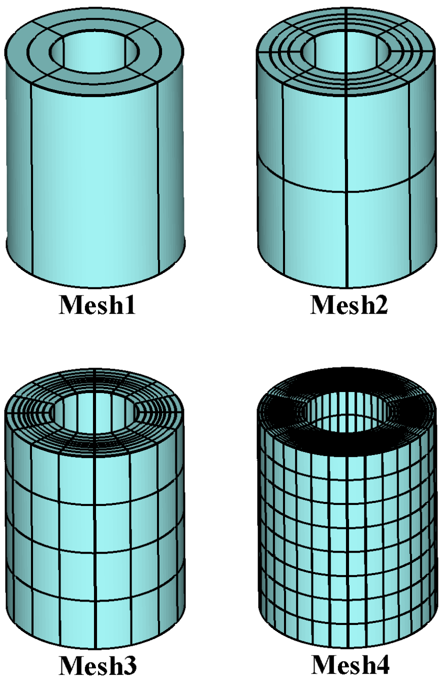

Fig. 3 Meshes produced by h-refinement

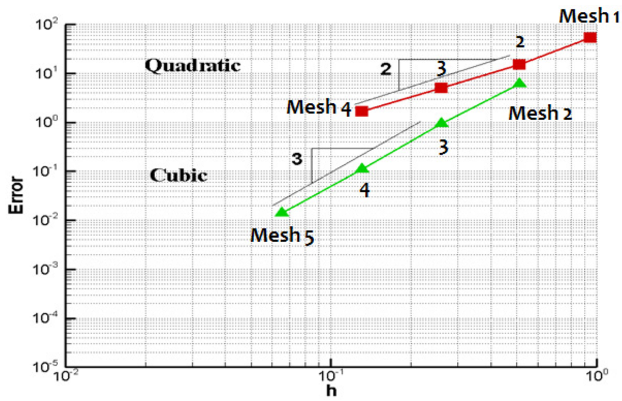

Fig. 5 Error in L_2-norm of stress and mesh parameter for cylinder model

and 3.5 between 0, 1, 2, 3, 4 in the circumferential direction. Other meshes are generated in the same manner.

The simulated stress component in circumferential direction of mesh 4 case is compared with the exact solution by Gould(1999) as it is shown in Fig. 4 which is corresponding to the quadratic basis function. Apparently nearly same results have been obtained. The convergence rates of the error between the simulated and analytic stress components are checked by L_2-norm. The mesh of the cubic basis can be obtained by k-refinement(Hughes et al., 2005) and the continuity of this is increased to C2. When compared the knot vectors between the quadratic basis adopting one k-refinement process and the coarsest cubic basis, the repeated value at the end of the knot vector is 3 and 4, respectively. The convergence rates for quadratic and cubic basis functions are order of 2 and 3, which has been confirmed as it is shown in Fig. 5.

3.2 Square plate with hole subjected to uniform tension

Square plate with circular hole at center is modeled using knot vectors and control points of NURBS of which information is given in the paper of Hughes et al.(2005). At this time this geometry is under the constant in-plane tension in the x-direction. Details are shown in Fig. 6. T_x is the magnitude of the applied stress, R is the radius of the hole, L is the length of the finite quarter plate, E is Young's modulus and ν is Poisson's ratio.

The solutions from isogeometric analysis are also compared with the exact solutions given as Eq.(12)

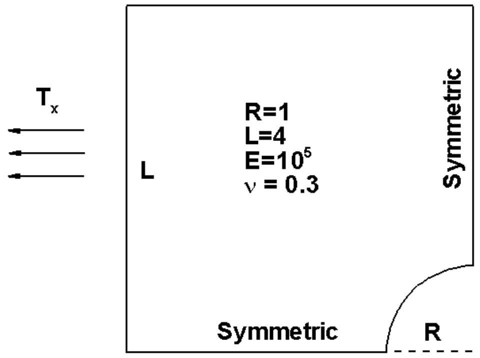

Fig. 6 Quarter model of square plate with circular hole at center

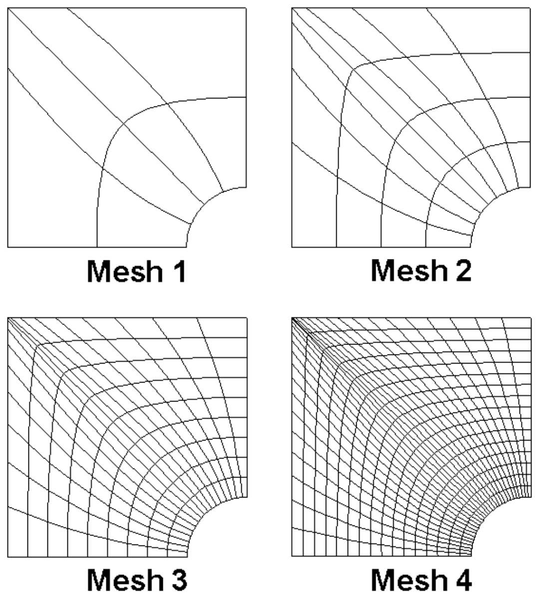

Fig. 7 Meshes generated by h-refinement

(Gould, 1999).

$$\sigma_{rr}=\frac{T_x}{2}\left(1-\frac{R^2}{r^2}\right)+\frac{T_x}{2}\left(1-4\frac{R^2}{r^2}+3\frac{R^4}{r^4}\right)\cos 2\theta \qquad (12)$$

$$\sigma_{\theta\theta}=\frac{T_x}{2}\left(1+\frac{R^2}{r^2}\right)-\frac{T_x}{2}\left(1+3\frac{R^4}{r^4}\right)\cos 2\theta$$

$$\sigma_{r\theta}=-\frac{T_x}{2}\left(1+2\frac{R^2}{r^2}-3\frac{R^4}{r^4}\right)\sin 2\theta$$

Basic mesh with 5 points in ξ direction and 3 points in η-direction are generated using 2nd order NURBS basis function like mesh1 of Fig. 7. Knot insertion process(h-refinement) is conducted in each direction based on the basic mesh to produce more fine meshes(meshes 2, 3 and 4 in Fig. 7).

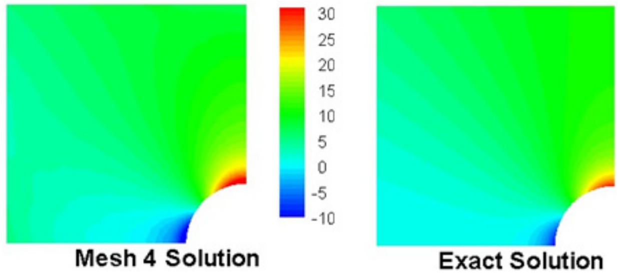

Fig. 8 Comparison of σxx contour plots. left) mesh 4 case, right) exact solution of Gould(1999)

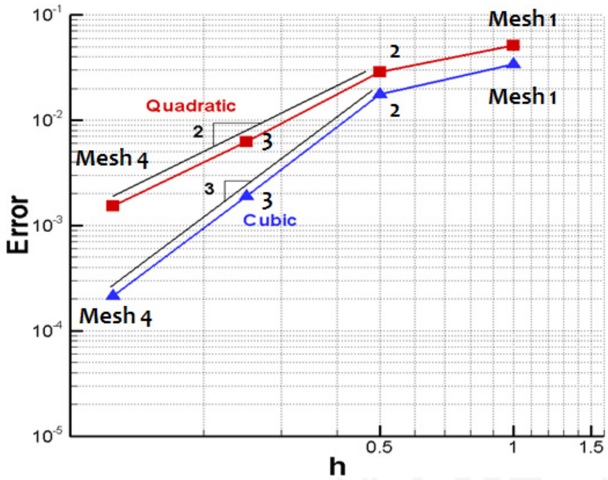

Fig. 9 Error in L_2-norm of stress and mesh parameter for square plate model

Fig. 8 shows the contour plots of σxx in mesh 4 case. When the applied stress T_x is 10, the stress concentration is $\sigma_{xx}=30$ at $r=R$, $\theta=3/2\pi$. Fig. 8 shows good agreement with the exact solution of Gould(1999).

In the same way as the previous circular cylinder model, the convergence rates of the error between the simulated and analytic stress components are checked by L_2-norm as shown in Fig. 9. The convergence rates for quadratic and cubic basis functions are approximately obtained with the same order of 2 and 3 in quadratic and cubic NURBS cases, respectively.

4. Conclusions

In the present study, isogeometric analysis based on the NURBS basis function is conducted to the linear elasticity problem. Circular cylinder having thick thickness and infinite plate with the hole is produced by quadratic NURBS and is refined by knot insertion(h-refinement) and order elevation (p-refinement). Accuracy of the approximate solutions for displacement and stress components has been verified through comparison with the exact solution. It has been confirmed that order elevation and knot insertion based on the basic geometry are simple and efficient for more accurate solution. Also as the number of the knot vectors increases, the error decrease by the same order of the basis function.

The extension of the present study to the structures having more complex geometry, and the results will be presented in the judicial proceedings and journals.

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