# Constant Envelope Enhanced FQPSK and Its Performance Analysis

Zhidong Xie, Gengxin Zhang, and Dongming Bian

Abstract: It's a challenging task to design a high performance modulation for satellite and space communications due to the limited power and bandwidth resource. Constant envelope modulation is an attractive scheme to be used in such cases for their needlessness of input power back-off about 2~3 dB for avoidance of nonlinear distortion induced by high power amplifier. The envelope of Feher quadrature phase shift keying (FQPSK) has a least fluctuation of 0.18 dB (quasi constant envelope) and can be further improved. This paper improves FQPSK by defining a set of new waveform functions, which changes FOPSK to be a strictly constant envelope modulation. The performance of the FQPSK adopting new waveform is justified by analysis and simulation. The study results show that the novel FQPSK is immune to the impact of HPA and outperforms conventional FQPSK on bit error rate (BER) performance. The BER performance of this novel modulation is better than that of FQPSK by more than 0.5 dB at least and 2 dB at most.

Index Terms: Constant envelope enhanced Feher quadrature phase shift keying (FQPSK), constant envelope modulation (CEM), FQPSK, high power amplifier (HPA), satellite and space communications, Viterbi algorithm.

#### I. INTRODUCTION

It's a challenging task to design a high performance modulation for satellite and space communications due to the limited power and bandwidth resource. During last few decades, there have been extensive research efforts for bit error rate (BER) and bandwidth trade-off. Quadrature phase-shift-keying (QPSK) family such as QPSK, offset QPSK (QQPSK), and  $\pi/4$ QPSK are certainly among the most popular BER-bandwidth optimized modulation techniques in relatively high noisy channel such as satellite and space channel. OOPSK is a modulation that has no 180 phase shifts, and therefore has a much higher spectral containment than QPSK. However, the signal amplitude of these is not constant, leading to poorer power efficiency compared with constant envelope modulation (CEM). It is well known that CEM signal is immune to nonlinear distortion and nonlinear amplification does not produce spectral "re-growth" in the transmitted waveform, resulting in higher power efficiency.

In order to achieve better BER performance under nonlinear amplification, yet with good spectrum utilization, Feherpatented QPSK (FQPSK) [1], [2] has been proposed. FQPSK is certainly one of the promising modulation methods satis-

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The authors are with the Institute of Communication Engineering, PLA University of Science and Technology, Nanjing 210007, China, email: xzd313@gmail.com, gengxin\_zhang@126.com, bian\_dm@163.com.

fying above requirements. Not only is it bandwidth-efficient, but has quasi-CEM characteristics with BER performance approaching OPSK limit, being suitable for power efficient system. It is worth noticing that these merits are important especially in telemetry system, where both power and spectrum efficiency is preferred over other requirements. Since then, much more attention has been paid to the detection of FQPSK [3]-[9], while little to FQPSK itself. However, the envelope of FQPSK still fluctuates about 0.18 dB at least and can be further improved. Reference [1] points out a new enhanced FOPSK (EFQPSK) whose spectral roll-off rate was improved. Unfortunately, the envelope fluctuation became worse than before, resulting that it is more sensitive to the nonlinear impact of high power amplifier (HPA). Reference [10] presents a new constant envelope FQPSK, but pays no attention to the power spectral density (PSD) and BER performance. Its spectral performance is close to that of FOPSK but worse than EFOPSK. We had proposed a preliminary version of constant envelope enhanced FQPSK in [11], whose BER and spectral performance are immutable. However, we will point out that its BER performance is flexible and can be improved further in this paper.

In this paper, we improve FQPSK by redefining the waveform functions, which changes FQPSK to be a strictly constant envelope modulation. The set of functions can provide an alterable tradeoff between power and bandwidth efficiency due to the specific application which been achieved by a figure-of-merit (parameter q in functions). The greater the value of q, the better BER performance is while the worse PSD is. Then, the PSD and BER performance of the new modulation are derived and analyzed. The FQPSK adopting novel waveforms not only improves the power efficiency, but also improves the bandwidth efficiency when q is small. Moreover, it is immune to the impact of nonlinear HPA. Whatever detector is adopted, the BER performance of this novel modulation is better than that of FQPSK by more than 0.5 dB at least. On the ideal case, the improvement of BER is up to 2 dB.

To provide a tenfold increase in capacity over the current satellite communication systems, one of the methods is the use of more bandwidth-efficient modulations, especially those best suited for use with nonlinear power amplifiers. Gaussian minimum shift keying (GMSK) [12] is another popular waveform choice for bandwidth-constrained systems using nonlinear amplifiers. Since GMSK is a CEM, it has much better spectral containment than QPSK, and OQPSK which are used in the many of the current satellite communications systems. We will compare GMSK with our methods as well OQPSK in this paper.

The rest of the paper is arranged as follows. The mathematical model of conventional FQPSK is depicted in brief in Section II. The novel waveform for constant envelope FQPSK is proposed

out in Section III. The performance analysis and numerical results about this novel modulation are given in Section IV and Section V concludes the paper.

#### II. RELATED WORK

In this section, the mathematical model of conventional FQPSK is depicted. Then, several kinds of detector for FQPSK are depicted in brief.

### A. The Mathematical Model of FQPSK

The baseband signal of FQPSK is essentially constructed by sixteen waveforms  $s_i(t), 0 \le i \le 15$ . These waveforms are described in [1]. Each waveform occupies only one symbol interval. In every symbol interval, a particular waveform is chosen for the I channel and another waveform is chosen for the Q channel. The selection of an I or a Q waveform depends on the most recent data transition on that channel as well as two most recent successive transitions on the other channel.

The baseband I and Q channel waveforms  $x_{\rm I}(t)$  and  $x_{\rm Q}(t)$  during the *n*th signal interval are assigned wavelets  $s_i(t)$  and  $s_i(t)$ , respectively, where the indices i and j are given by (1).

$$i = I_3 \times 2^3 + I_2 \times 2^2 + I_1 \times 2^1 + I_0 \times 2^0$$
  

$$j = Q_3 \times 2^3 + Q_2 \times 2^2 + Q_1 \times 2^1 + Q_0 \times 2^0$$
 (1)

where

$$\begin{split} I_0 &= D_{Q,n} \oplus D_{Q,n-1}, & Q_0 = D_{1,n+1} \oplus D_{I,n}, \\ I_1 &= D_{Q,n-1} \oplus D_{Q,n-2}, & Q_1 = D_{1,n} \oplus D_{I,n-1}, \\ I_2 &= D_{I,n} \oplus D_{I,n-1}, & Q_2 = D_{Q,n} \oplus D_{Q,n-1}, \\ I_3 &= D_{I,n}, & Q_3 = D_{Q,n} \end{split}$$

where  $D_{I,n}$  and  $D_{Q,n}$  are the nth I and Q channel inputs (0 or 1) to the modulator, respectively. We have  $x_I(t) = s_i(t-nTs)$  and  $x_Q(t) = s_j(t-nTs+Ts/2)$ . The overall complex envelope of the transmitted signal x(t) is

$$x(t) = \sum_{n} s_i(t - nTs) + j \sum_{n} s_j \left( t - nTs + \frac{Ts}{2} \right)$$
 (3)

where  $s_i(t)$  and  $s_j(t)$  have support only over [-Ts/2, Ts/2]. The envelope M(t) is

$$M(t) = \sqrt{x_{\rm I}(t)^2 + x_{\rm Q}(t)^2}.$$
 (4)

The received complex envelope is y(t) = x(t) + n(t), where n(t) is additive white Gaussian noise (AWGN). When a non-linear HPA model which was presented in [13] is adopted, the received signal will be

$$y(t) = A[M(t)] \cos{\{\Phi[M(t)]\}} + n(t) \tag{5}$$

where A[M(t)] is an odd function of M(t) representing amplitude modulation (AM)/AM effects of HPA, and  $\Phi[M(t)]$  is an even function of M(t) representing AM/phase modulation (PM) conversion. The following equations are used in our study

in which  $\rho$  denotes amplitude of signal.

$$A(\rho) = \begin{cases} 1.24\rho + 0.33\rho^2 - 0.73\rho^3 + 0.17\rho^4 - 0.047\rho^5, \, \rho < 1.0 \\ 1.0, \, \rho \ge 1.0, \end{cases}$$

$$\Phi(\rho) = \begin{cases} 13\rho + 0.8\rho^2 - \rho^3 - 4.2\rho^4, & \rho < 1.0 \\ 8.6 + 80(\rho - 1) - 48(\rho - 1)^2 + 9.3(\rho - 1)^3, & \rho \ge 1.0. \end{cases}$$

The signal baseband model of output has a form given by

$$y(t) = (a+jb)x(t) \tag{7}$$

where

$$a = A[M(t)] \cos{\{\Phi[M(t)]\}},$$
  
 $b = A[M(t)] \sin{\{\Phi[M(t)]\}}.$ 

From (6) and (7), we can get the conclusion that the fluctuation of modulated signal would induce nonlinear distortion when it passes through HPA. Therefore, a constant envelope modulation is a preferred scheme to be used in some cases for their needlessness of input back-off about  $2\sim3$  dB for avoidance nonlinear distortion induced by HPA, such as in satellite and deep space communications.

### B. Receiver Structures

There had been several methods proposed for detecting FQPSK during past few decades [3]–[9]. Here, we categorize the receiver structures into optimal and symbol-by-symbol detector. The received signal vector can be expressed as  $\mathbf{y}_{\mathrm{I},k} = \mathbf{s}_i + \mathbf{n}_{\mathrm{I}}$ , where the vector  $\mathbf{s}_i$  consists of samples of  $s_i(t)$  and  $\mathbf{n}_{\mathrm{I}}$  is the in-phase noise sample vector.

### **B.1** Symbol-by-Symbol Detection

Let  ${\bf R}$  be a linear filter of r taps. The mean squared error (MSE) at the output of the filter is  $\varepsilon=E\{\left|{\bf R}^T{\bf y}_{{\bf I},k}-a_{{\bf I},k}\right|^2\}$ . Setting  $\varepsilon$  to zero, the optimal minimum MSE filter is obtained as  ${\bf R}_{opt}={\bf V}^{-1}\bar{\bf s}$ , where  ${\bf V}=(1/8)\sum_{i=0}^7{\bf s}_i{\bf s}_i^T+\sigma_n^2{\bf I}$ , and  $\bar{\bf s}=(1/8)\sum_{i=0}^7{\bf s}_i$ .

When  $\mathbf{R}=\overline{\mathbf{s}}$ , it is an averaged matched filter (AMF) receiver. It is to be noted that, at very low signal noise ratio (SNR),  $\mathbf{V}\approx\sigma_n^2\mathbf{I}$ , and then,  $\mathbf{R}_{opt}=\sigma_n^2\overline{\mathbf{s}}$ , which is the averaged matched filter receiver.  $\mathbf{R}=[1\ 1\ \cdots\ 1]^T$  denotes an integrate and dump (I&D) receiver.

### **B.2** Optimal Detection

The optimal receiver structure uses the Viterbi algorithm (VA) with 16 states. Each state represents

$$(a_{I, k-1}, a_{I,k}, a_{Q,k-2}, a_{Q,k-1})$$

four bits. The transition from state

$$\delta_s = (a_{I,k-2}, a_{I,k-1}, a_{O,k-3}, a_{O,k-2})$$

to state

$$\delta_e = (a_{I,k-1}, a_{I,k}, a_{Q,k-2}, a_{Q,k-1})$$

is associated with the branch metric

$$\lambda(\delta_s, \delta_e, k) = \|\mathbf{y}_{I,k} - \mathbf{s}_i\|^2 + \|\mathbf{y}_{Q,k} - \mathbf{s}_j\|^2$$
(8)

where the waveform identification number i and j are obtained from (1), and  $||\cdot||$  denotes Euclidean norm.

# III. THE NOVEL WAVEFORM FOR ENVELOPE ENHANCED FQPSK

The FQPSK is regarded as quasi-constant envelope modulation and it means that the modulated signal is not of strictly constant envelope. In order not to affect the distribution of the power spectrum of the modulated signal and to enhance the power efficiency while keeping the bandwidth efficiency, the signal should have no slope discontinuity anywhere, similar to EFQPSK [1]. It is hoped that we can modify the waveform to obtain constant envelope signal. Therefore, we redefine the signal as (9), where  $A = 1/\sqrt{2}$  and  $q = 2, 4, 6, 8 \cdots$ . The modulated signal envelope will keep strictly constant if only q is an even number. Looking into the waveform functions, the value of q decides a symbol energy which determines the BER performance of modulation. Furthermore, larger q will introduce the higher slope in waveform which would deteriorate the spectral performance of modulation. Therefore, the parameter q is a figure-of-merit for this modulation which can be used to achieve compromise between BER and spectral performance to fit different requirements. Both PSD and BER performance are related to the value of q. The bigger q is, the better BER performance is while the worse PSD is. In our study later, we set q = 6 for a tradeoff.

We call the novel FQPSK which adopts new waveforms as constant envelope enhanced FQPSK (CEEFQPSK) for convenience. Fig. 1 indicates that the envelope fluctuation comparison between EFQPSK and CEEFQPSK waveforms and it clearly shows that CEEFQPSK is a strictly constant modulation and the FQPSK has envelope fluctuations. The CEEFQPSK is immune to nonlinear distortion of HPA and can improve the power transition efficiency of the radio frequency amplifiers wonderfully. The order of waveform function in CEEFQPSK would be higher as q increases, and the complexity of implementation would increase. Especially, it is more complex than FQPSK when q increases. Luckily, the complexity is almost the same as FQPSK

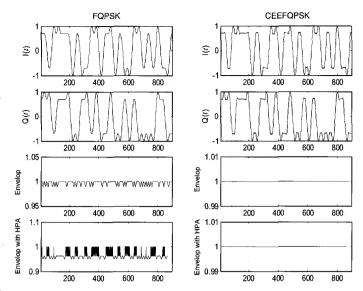


Fig. 1. Waveform and envelope of FQPSK and CEEFQPSK.

when q is equal to 2, and the limited increase when q is equal to 6 (which is better choice for q).

# IV. PERFORMANCE ANALYSIS AND NUMERICAL RESULTS

There had been several methods proposed for detecting FQPSK, including symbol by symbol detector and optimal detector. The CEEFQPSK is derived from the FQPSK. Therefore, it also can be demodulated by VA as an optimal receiver. In order to verify the performance of the proposed CEEFQPSK, several comparisons between CEEFQPSK and the FQPSK are made. At the same time, OQPSK performance is included which usually used in satellite communication. All following simula-

$$s_{0}(t) = A, \qquad -\frac{T_{s}}{2} \le t \le \frac{T_{s}}{2}, \quad s_{8}(t) = -s_{0}(t)$$

$$s_{1}(t) = \begin{cases} A, & -\frac{T_{s}}{2} \le t \le 0 \\ \sqrt{1 - \left[\sin\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}} - (1 - A)\sin^{q}\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}}\right]^{2}}, & 0 \le t \le \frac{T_{s}}{2}, \\ s_{2}(t) = \begin{cases} \sqrt{1 - \left[\sin\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}} - (1 - A)\sin^{q}\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}}\right]^{2}}, & -\frac{T_{s}}{2} \le t \le 0 \\ A, & 0 \le t \le \frac{T_{s}}{2}, \end{cases}$$

$$s_{3}(t) = \sqrt{1 - \left[\sin\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}} - (1 - A)\sin^{q}\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}}\right]^{2}}, & -\frac{T_{s}}{2} \le t \le \frac{T_{s}}{2}, \quad s_{11}(t) = -s_{2}(t) \end{cases}$$

$$s_{3}(t) = \sqrt{1 - \left[\sin\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}} - (1 - A)\sin^{q}\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}}\right]^{2}}, & -\frac{T_{s}}{2} \le t \le \frac{T_{s}}{2}, \quad s_{11}(t) = -s_{3}(t) \end{cases}$$

$$s_{4}(t) = \begin{cases} \sin\frac{\pi(t + \frac{T_{s}}{2})}{T_{s}} - (1 - A)\sin^{q}\frac{\pi t}{T_{s}}, & 0 \le t \le \frac{T_{s}}{2}, \quad s_{12}(t) = -s_{4}(t) \end{cases}$$

$$s_{5}(t) = \begin{cases} \sin\frac{\pi t}{T_{s}} + (1 - A)\sin^{q}\frac{\pi t}{T_{s}}, & -\frac{T_{s}}{2} \le t \le 0 \\ \sin\frac{\pi t}{T_{s}}, & 0 \le t \le \frac{T_{s}}{2}, \quad s_{13}(t) = -s_{5}(t) \end{cases}$$

$$s_{6}(t) = \begin{cases} \sin\frac{\pi t}{T_{s}}, & -\frac{T_{s}}{2} \le t \le 0 \\ \sin\frac{\pi t}{T_{s}}, & 0 \le t \le \frac{T_{s}}{2}, \quad s_{14}(t) = -s_{6}(t) \\ -\frac{T_{s}}{2} \le t \le \frac{T_{s}}{2}, \quad s_{15}(t) = -s_{7}(t) \end{cases}$$

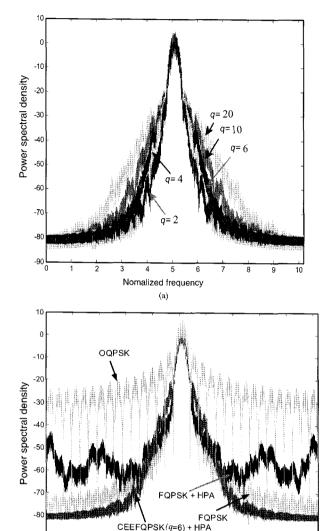


Fig. 2. PSD of CEEFQPSK: (a) For different q and (b) with HPA.

Normalized frequency

(b)

tions and analysis are under assumptions of ideal timing recovery and ideal phase recovery.

# A. Power Spectral Density

The modulated signal envelope will keep strictly constant if only q is an even number. However, the bigger q is, the worse frequency spectrum becomes. Fortunately, we will point out that the BER performance is better when q is bigger. The PSD of CEEFQPSK for different q are shown in (a) of Fig. 2. The PSD of CEEFQPSK is better than that of FQPSK when q is smaller than 6. When the HPA mentioned above is used, the PSD of FQPSK and CEEFQPSK are compared in (b) of Fig. 2. We can see clearly that the nonlinearity of HPA induces the spread of PSD of FQPSK as well OQPSK and the PSD of CEEFQPSK is immune to the nonlinearity of HPA.

### B. BER Performance with Symbol-by-Symbol Detector

In order to get the BER performance of CEEFQPSK in AWGN channel, we can adopt an analogous manner used in

[1] for FOPSK. We induced BER performance with I&D and AMF receiver as follow. Firstly, we should compute out the waveform for the interval  $0 \le t \le Ts$ . Focusing our attention on the I channel, each of these new waveforms is composed of the latter half (i.e., that which occurs in the interval  $0 \le t \le Ts/2$  ) of the I channel waveform transmitted in the interval  $-Ts/2 \le t \le Ts/2$ , followed by the first half (i.e., that which occurs in the interval  $Ts/2 \le t \le Ts$  ) of the I channel waveform transmitted in the interval  $Ts/2 \le t \le 3Ts/2$ . For  $a_{\rm I,0}=1$  and  $x_{\rm I}(t)=s_0(t)$  in the interval  $-Ts/2 \le t \le Ts/2$ , the transmitted signal,  $S_i(t)$ , during the interval  $0 \le t \le Ts$  is composed of the latter half of  $s_0(t)$  followed by the first half of either  $s_0(t)$ ,  $s_1(t)$ ,  $s_{12}(t)$ , or  $s_{13}(t)$ . Therefore, we can get  $S_0(t)$ and  $S_1(t)$  because there are only two distinct waveforms. Following a similar procedure (still for  $a_{I,0} = 1$ ), we can get other possible distinct waveforms in  $0 \le t \le Ts$ , defined as (10). Then, the average symbol error probability is given by

$$P(E) = \frac{1}{8} \sum_{i=0}^{7} P_{si}(E)$$
 (11)

where

$$P_{si}(E) = \frac{1}{2} \operatorname{erfc} \left\{ \sqrt{\frac{\left[ \int_0^{T_s} S_i(t) \overline{S}(t) dt \right]^2}{N_0 E_{\overline{s}}}} \right\}. \tag{12}$$

For I&D receiver, we can set  $\overline{S}=1$  and  $E_{\overline{s}}=Ts$ , while for AMF receiver  $\overline{S}=\frac{1}{8}\sum_{i=0}^{7}S_{i}(t)$  and  $E_{\overline{s}}=\int_{0}^{Ts}\overline{S}^{2}dt$ .

Therefore, we can compute the BER performance by (11) and (12), but it is difficult to compute integral of some terms and to get the result in closed form. Fortunately, we can get the numerical value of integral assisted by computer.

Because there is a mistake in the formula of BER for FQPSK in [1], we compute it again herein. The result is as follows.

$$P_{s}(E) = \frac{1}{16} \operatorname{erfc}\left(\sqrt{\frac{A^{2}}{E}} \frac{E_{b}}{N_{0}}\right) + \frac{1}{16} \operatorname{erfc}\left(\sqrt{\frac{(\frac{A}{2} + \frac{A}{\pi})^{2}}{E}} \frac{E_{b}}{N_{0}}\right) + \frac{1}{16} \operatorname{erfc}\left(\sqrt{\frac{(\frac{1}{2} + \frac{A}{\pi})^{2}}{E}} \frac{E_{b}}{N_{0}}\right) + \frac{1}{16} \operatorname{erfc}\left(\sqrt{\frac{(\frac{1}{4} + \frac{A}{4} + \frac{1}{\pi})^{2}}{E}} \frac{E_{b}}{N_{0}}\right) + \frac{1}{16} \operatorname{erfc}\left(\sqrt{\frac{(\frac{A}{\pi})^{2}}{E}} \frac{E_{b}}$$

where  $\bar{E}=(7+2A+15A^2)/32$ . With an I&D receiver, the BER for different values of are illustrated in Fig. 3. What is also included in this figure is the performances of OQPSK and FQPSK corresponding to the I&D receiver. We observe that the bigger value of q is, the better performance is. When q is equal to 20, CEEFQPSK is better than the FQPSK by more than 2 dB at BER lower than  $10^{-5}$ . Given that Euclidean distance of OQPSK is equal to 2, under the same situation, the Euclidean distance of GMSK is 1.69 and 1.78 when BT equals 0.25 and 0.3, respectively, then that of CEEFQPSK is close to 2 as q becomes infinite, such as 1.8322 when q equals 20, 1.9378 when q equals 200, 1.9786 when q equals 2000, 1.9930 when q up to 20000, and so on. Therefore, as shown in Fig. 3, there is a tendency that the BER performance of CEEFQPSK is near to the performance

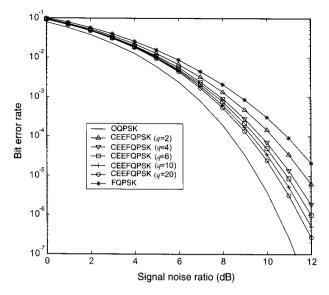


Fig. 3. BER performances of CEEFQPSK for different q.

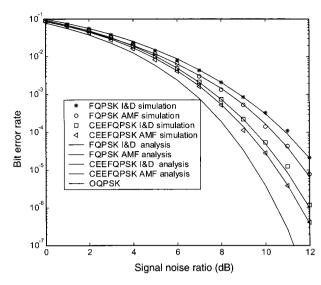


Fig. 4. A comparison of BER performances between FQPSK and CEEFQPSK with I&D and AMF receiver.

of OQPSK when q is infinite. Certainly, the PSD of CEEFQPSK is deteriorated seriously when q equals 20, but it is still much better than that of OQPSK which can be seen from Fig. 2. Excitedly, we can obtain a conclusion that the CEEFQPSK performs better on BER than FQPSK no matter what q is. However, the gain improvement resulting from increase 2 for value of q will decrease when q is bigger, such as the Euclidean distance gap is about 0.04 when q changes from 4 to 6 while it is only 0.026 when q changes from 6 to 8. Taking both the PSD and BER performance into the consideration, q equaling 6 is an appropriate tradeoff. In this paper later, we set q equals six. Fig. 4 shows comparison between FQPSK and CEEFQPSK detected by I&D receiver and AMF receiver, including both BER of analysis and simulation respectively. The results of simulation are in accordance with the analysis. From the results mentioned above, the excellent BER performance of CEEFQPSK is verified.

## C. BER Performance with Optimal Detector

The CEEFQPSK is derived from the FQPSK. Therefore, it also can be demodulated by VA as an optimal receiver. Adopting VA depicted in Section II, the BER performance in AWGN is shown in Fig. 5. In order to compare the performance of different kinds of receiver, their performances are also included in Fig.5. From the figure, we can see that the performance gap between OQPSK and CEEFQPSK with VA detector is about 0.3 dB. We can also know that VA performs best while I&D detector performs worst, whose performance gap is about 0.8 dB. The AMF receiver and the Wiener Filter receiver perform closely, which fall into the midst between VA and I&D receiver. However, their complexity is reverse. Commonly, there is a tradeoff between performance and complexity according to specific application environment.

$$S_{0}(t) = A, \qquad 0 \le t \le Ts,$$

$$S_{1}(t) = \begin{cases} A, & 0 \le t \le \frac{Ts}{2} \\ \sin\frac{\pi t}{Ts} - (1 - A)\sin^{q}\frac{\pi t}{Ts}, & \frac{Ts}{2} \le t \le Ts, \end{cases}$$

$$S_{2}(t) = \sqrt{1 - \left[\sin\frac{\pi(t + \frac{Ts}{2})}{Ts} - (1 - A)\sin^{q}\frac{\pi(t + \frac{Ts}{2})}{Ts}\right]^{2}}, \qquad 0 \le t \le Ts,$$

$$S_{3}(t) = \begin{cases} \sqrt{1 - \left[\sin\frac{\pi(t + \frac{Ts}{2})}{Ts} - (1 - A)\sin^{q}\frac{\pi(t + \frac{Ts}{2})}{Ts}\right]^{2}}, & 0 \le t \le \frac{Ts}{2} \\ \sin\frac{\pi t}{Ts}, & \frac{Ts}{2} \le t \le Ts, \end{cases}$$

$$S_{4}(t) = \begin{cases} \sin\frac{\pi t}{Ts} - (1 - A)\sin^{q}\frac{\pi t}{Ts}, & 0 \le t \le \frac{Ts}{2} \\ A, & \frac{Ts}{2} \le t \le Ts, \end{cases}$$

$$S_{5}(t) = \sin\frac{\pi t}{Ts} - (1 - A)\sin^{q}\frac{\pi t}{Ts}, & 0 \le t \le Ts,$$

$$S_{6}(t) = \begin{cases} \sin\frac{\pi t}{Ts}, & 0 \le t \le \frac{Ts}{2} \\ \sqrt{1 - \left[\sin\frac{\pi(t + \frac{Ts}{2})}{Ts} - (1 - A)\sin^{q}\frac{\pi(t + \frac{Ts}{2})}{Ts}\right]^{2}}, & \frac{Ts}{2} \le t \le Ts, \end{cases}$$

$$S_{7}(t) = \sin\frac{\pi t}{Ts}, & 0 \le t \le Ts.$$

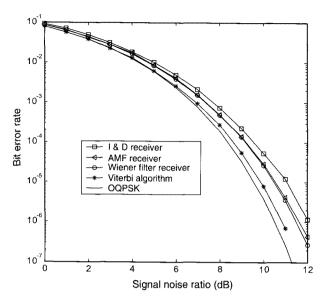


Fig. 5. BER performance of CEEFQPSK for different detector.

### D. BER Performance with Nonlinear HPA

Here, we adopt the HPA model given in [13], the AM/AM and AM/PM effects of HPA can be expressed as follows.

$$A(\rho) = \begin{cases} 1.24\rho + 0.33\rho^2 - 0.73\rho^3 + 0.17\rho^4 - 0.047\rho^5, \, \rho < 1.0 \\ 1.0, \, \rho \geq 1.0, \end{cases}$$

$$\Phi(\rho) = \begin{cases} 13\rho + 0.8\rho^2 - \rho^3 - 4.2\rho^4, & \rho < 1.0\\ 8.6 + 80(\rho - 1) - 48(\rho - 1)^2 + 9.3(\rho - 1)^3, & \rho \ge 1.0. \end{cases}$$
(14)

We can obtain the received signal as

$$y(t) = (a+jb)x(t)$$

$$= (a+jb)(S_i(t)+jS_q(t))$$

$$= (aS_i(t)-bS_q(t))+j(aS_q(t)+bS_i(t))$$
(15)

where

$$\begin{split} S_i(t) &= \sum_k s_i(t-kTs),\\ S_q(t) &= j \sum_k s_j \left(t-kTs + \frac{Ts}{2}\right). \end{split}$$

We know that the HPA induces the interference between I channel and Q channel, which will deteriorate the BER performance. The impact of HPA on the BER performance is illuminated in Fig. 6. It shows the BER curves of CEEFQPSK detected by I&D receiver and VA with and without HPA, respectively. Along with that for a purpose of comparison are the curves of FQPSK and OQPSK. First, we can see clearly that the CEEFQPSK improves the BER performance obviously whatever detector is adopted. When an I&D receiver is adopted, the CEEFQPSK is better than that of FQPSK by more than 1 dB. Then, the impact of HPA on the performance of FQPSK is serious while that on the CEEFQPSK is neglectable. Under nonlinear amplification, the CEEFQPSK outperforms FQPSK by more than 1 dB and 1.5 dB when VA receiver and I&D receiver are used, respectively.

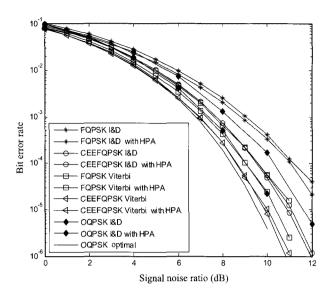


Fig. 6. BER performance of FQPSK and CEEFQPSK with and without HPA.

### V. CONCLUSION

FQPSK is the most promising modulation techniques in satellite and deep space communications, and has been recommended by consultative committee for space data system (CCSDS) [14]. Unfortunately, the envelope of the FQPSK modulated signal is non-constant and the power efficiency still can be improved. The CEEFQPSK presented in this paper with the absolutely constant envelope which enables the HPA work at saturated state, has improved power efficiency while maintains the same bandwidth efficiency. It is verified that CEEFQPSK has better BER performance than FQPSK both in AWGN and in nonlinear channel. The BER performance of this novel modulation is better than that of FQPSK by more than 0.5 dB at least and up to 2 dB at most.

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Gengxin Zhang was born in Pinghu County, Zhejiang Province, China, on January 1, 1967. He received the B.S., M.S., and Ph.D. degree in 1987, 1990, and 1994, respectively, all from Department of Radio Communication Engineering, Nanjing Institute of Communication Engineering, Nanjing, China. He is a Professor at the Key Lab of Satellite Communication, PLA University of Science and Technology, Nanjing, China, since 1999. From April 1990 to July 1999, he has worked at the Department of Radio Communication Engineering, Nanjing Institute of Communication Engineering,

deep space communications, and is heading several projects in these areas.



Zhidong Xie was born in Gansu Province, China, on March 13, 1984. He received the B.S. degree and Ph.D. degree in Communication Engineer from the PLA University of Science and Technology, China, in 2005 and 2010, respectively. Presently, he is a Lecturer at the Key Lab of Satellite Communication, PLA University of Science and Technology, China. His major research interests are in deep space communications, satellite communications, and networks.



Dongming Bian was born in Yuncheng County, Shandong Province, China, on November 22, 1975. He received his M.S. degree from the PLA University of Science and Technology in 2000, and Ph.D. degree from the PLA University of Information and Engineering in 2003, all in Communication Engineering. He is currently an Associate Professor at the Key Lab of Satellite Communication, PLA University of Science and Technology, Nanjing, China. His research interests include satellite communications and deep space communications.