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원통형 회전 분리기를 감시하기 위한 전기저항법의 이용

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An Application of Electrical Resistance Method for Monitoring of Rotating Cylindrical Separator

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요 약

본 연구에서는 방사성 폐기물을 처리하는 원통형 회전 분리기를 감시하기 위해 전기 저항법을 제안하고 수학적 모델을 연구하였다. 회전형 방사성 폐기물 분리기에서 관 벽에 설치된 한 쌍의 전극의 전기 저항은 표면을 따라 형성된 불용해성 입자의 환상 영역의 두께와 그 영역의 불용해성 입자의 농도와 관련이 있다. 본 연구는 전위 방정식에 대한 2차원 해와 실험적인 전도도-농도 관계를 기반으로 전술한 인자들 사이의 해석적 관계를 기술하였다. 또한, 원통형 회전 분리기를 감시하기 위한 전기 저항법의 적용 가능성을 논의 하였다.

주요어 : 방사성 폐기물 분리, 전기 저항법, 동심 환형 구조

Abstract — In order to monitor a rotating cylindrical separator for radioactive waste, an electrical resistance method is proposed and its mathematical model is investigated. In a rotating radioactive waste separator, the electrical resistance between a pair of electrodes mounted on the inner wall of the vessel is related to the thickness of annular region of insoluble particle formed around the periphery and the concentration of the insoluble particle in that region. This work presents an analytical relationship among the aforementioned parameters based on a two-dimensional solution to the electrical potential equation and an empirical conductivity-concentration relation. Also, the feasibility of electrical resistance method for monitoring rotating cylindrical separators is discussed.

Key words : Radioactive waste separation, Electrical resistance method, Concentric annular geometry

1. Introduction

This work considers an application of electrical resistance method to monitor waste separation process in a rotating cylindrical separator, where solid waste particles fed into solvent are separated according to their solubility. The electrical method has been widely used in monitoring the process of two immiscible liquids [1], gas-liquid mixture [2], and gas-solid flow [3]. Cattle and West [4] adopted a dual-modality tomography technique which is combined with the electrical impedance tomography(EIT) and the gamma ray emission tomography(GET) for monitoring the insoluble particle distribution in a radioactive waste processing. Recently, Park et al. [5] applied the particle

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swarm optimization(PSO) algorithm to estimate the conductivity, that is, the concentration distribution in the electrical resistance monitoring of the rotating cylindrical waste separator.

In a rotating cylindrical separator, there are two regions: the solvent region in the core and the sediment region in the periphery. The regions form a concentric annular geometry. It is assumed that the sediment region contains the insoluble particles while the solvent region does not. The resistance between two electrodes attached on the inner wall of the vessel is related to the conductivity of each region and the thickness of the sediment region. We will derive an analytical expression of the conductance, or the inverse of the resistance, in terms of the conductivity and the thickness of the sediment region. The expression can be used to analyze the waste separation process and the design of the sensor electrodes.

2. Mathematical Model

In a rotating separator, soluble and insoluble solid particles in the solvent are separated by centrifugal force and there are two distinct regions: the solvent region in the core and the sediment region in the periphery as shown in Fig. 1. It is assumed that the volume fraction distribution of the insoluble solid particle is axi-symmetric and each region has a constant electrical conductivity. Two identical electrodes flush mounted with the inner surface of the vessel of radius a are positioned each other in the opposite direction. The height of the electrode is H. The angle between the electrodes is 2δ(Fig. 1). Voltages are applied to the right and the left electrodes at $+V$ and $-V$, respectively. Now, we derive an analytical expression of the resistance in terms of the conductivity and the volume fraction of the sediment region.

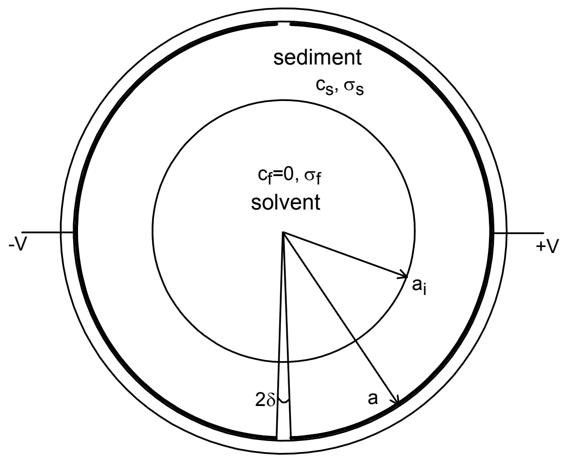


Fig. 1. Problem domain description.

The region interface is located at $r=a_i$, which separates the sediment region and the solvent region. It is assumed that sediment region has a constant insoluble particle volume fraction c_s while the solvent region does not contain insoluble particles and the particle volume fraction c_f is set to zero as illustrated in Fig. 1. From the conservation of mass, the global volume fraction of insoluble solid particles c_g can be expressed in terms of the volume fraction of the solvent region, $\alpha=(a_i/a)^2$ and the concentration of the insoluble solid particles in the sediment region, c_s:

$$c_g=(1-\alpha)c_s. \tag{1}$$

The conductance, the inverse of the resistance, is related to the electrical conductivity distribution, which can be converted into the concentration distribution by using the relationship between the concentration and the electrical conductivity. Many experimental and theoretical studies have been conducted to express the effective conductivity in terms of the volume fraction of a suspension in a liquid [6,7.8]. Recently, Feitosa et al. presented a new correlation based on nine sets of experimental data for gas bubbles in liquid, oil-in-water emulsions, and solid particles in aqueous solution [9]:

$$\kappa=\frac{\sigma_{eff}}{\sigma_{solvent}}=\frac{2(1-c)(13-12c)}{26-11c-9c^2}, \tag{2}$$

$$c=\frac{(1-\kappa)(1+23\kappa)}{1+25\kappa+10\kappa^2}. \tag{3}$$

where σ_{eff} is the effective conductivity of the mixture, $\sigma_{solvent}$ the conductivity of the solvent, κ is the ratio of $\sigma_{eff}/\sigma_{solvent}$, and c the insoluble particle concentration.

In each region, the electrical potential distribution satisfies the following equations;

$$\nabla \cdot \sigma_s \nabla u_s = 0 \quad a_i \le r \le a \quad \text{sediment region} \quad (4)$$

$$\nabla \cdot \sigma_f \nabla u_f = 0 \quad 0 \le r \le a_i \quad \text{solvent region} \quad (5)$$

where u is the potential, σ is the conductivity and the subscripts 's' and 'f' denote the sediment and the solvent region, respectively. The governing equations are subjected to the boundary and interfacial conditions;

$$u_s(a,\theta) = \tilde{V}(\theta) \quad (6)$$

$$u_s(a_i,\theta) = u_f(a_i,\theta) \quad (7)$$

$$\sigma_s \left.\frac{\partial u_s}{\partial r}\right|_{r=a_i} = \sigma_f \left.\frac{\partial u_f}{\partial r}\right|_{r=a_i} \quad (8)$$

where $\tilde{V}(\theta)$ is the applied voltage on inner wall of the vessel. The applied voltages on the electrodes centered at $\theta_1 = 0$ and $\theta_2 = \pi$ are set to V and $-V$. As for the gaps between the electrodes, the proper boundary condition is the insulation condition. With the above boundary conditions, however, the analytic solution does not seem to be feasible. Instead, we assume that the voltage along the gap is linear function of position:

$$\tilde{V}(\theta) = \tilde{V}(-\theta) = \begin{cases} V & \text{for } 0 \le \theta \le \pi/2-\delta \\ -\dfrac{V}{\delta}(\theta - \pi/2 + \delta) + V & \text{for } \pi/2-\delta \le \theta \le \pi/2+\delta . \\ -V & \text{for } \pi/2+\delta \le \theta \le \pi \end{cases} \quad (9)$$

3. Results

With the conditions given in the mathematical model, the following analytic solution can be obtained by using the method of separation of variables:

$$u_s(r,\theta) = \sum_{n=odd}^{\infty} K_n \left[\left(\frac{r}{a}\right)^n + \frac{(1-\kappa_s)\alpha^n}{(1+\kappa_s)-(1-\kappa_s)\alpha^n} \left[\left(\frac{r}{a}\right)^n - \left(\frac{r}{a}\right)^{-n} \right] \right] \cos n\theta, \quad (10)$$

where the conductivity ratio is defined as $\kappa_s = \sigma_s/\sigma_f$ and the coefficient is

$$K_n = \frac{4V \sin(n\pi/2)\sin n\delta}{\pi \delta n^2}, \qquad n = odd. \quad (11)$$

It is noted that $\kappa_s \le 1$ since $\sigma_s \le \sigma_f$. The equality is valid only when there is no insoluble particle. From the definition of the conductance G, we have

$$G = \frac{current}{voltage\ difference} = \frac{I}{2V} = \frac{H}{2V}\int_{-\pi/2+\delta}^{\pi/2-\delta} \sigma_s \left.\frac{\partial u_s}{\partial r}\right|_{r=a} a d\theta$$
$$= \frac{2\sigma_s H}{\pi\delta}\left[\sum_{n=odd}^{\infty} \frac{\sin 2n\delta}{n^2} + \sum_{n=odd}^{\infty} \frac{2(1-\kappa_s)\alpha^n}{(1+\kappa_s)-(1-\kappa_s)\alpha^n}\frac{\sin 2n\delta}{n^2} \right]. \quad (12)$$

If the vessel is filled with solvent only, the conductance will be

$$G_f = \frac{2\sigma_f H}{\pi\delta} \sum_{n=odd}^{\infty} \frac{\sin 2n\delta}{n^2}. \quad (13)$$

From Eqs. (12) and (13) the normalized conductance G^* becomes

$$G^* = \frac{G}{G_f} = \kappa_s + \frac{\displaystyle\sum_{n=odd}^{\infty} \frac{2(1-\kappa_s)\alpha^n}{(1+\kappa_s)-(1-\kappa_s)\alpha^n}\frac{\sin 2n\delta}{n^2}}{\displaystyle\sum_{n=odd}^{\infty}\frac{\sin 2n\delta}{n^2}}. \quad (14)$$

Here note that the volume fraction of the solvent region should not be greater than $1-c_g$, namely $0 \le \alpha \le 1-c_g$.

Now, a mathematical model to describe the electrical conductance as a function of the volume fraction of the solvent region and the conductivity value of each region is available. Initially, the conductance of the solvent without particles, G_f, is measured. Then, the particles are well mixed with the solvent. After centrifugal separation, the conductance of the solvent with particles, G, is measured. From Eq. (14), we can obtain the ratio of the conductance $G^* = G/G_f$.

The normalized conductance as a function of the volume fraction of the solvent region can be obtained for various global insoluble solid particle concentrations. As an example, Fig. 2 shows the normalized conductance for the half-gap angle δ=0.025. For a given c_g and a fixed half-gap angle δ, from the normalized conductance reading we can measure the volume fraction of the solvent region, or the thickness of the sediment region. In Fig. 3, the dependency of the normalized conductance on the volume fraction of the solvent region is shown for 4 different half-gap angles, δ=0.01, 0.025, 0.05, 0.1. Based on Fig. 3, it is observed that as the gap angle decreases the normalized conductance becomes more sensitive to the volume fraction of the solvent region. When δ=0.025(Fig. 2), for large c_g, e.g. $c_g > 0.5$, the normalized conductance G^* tends to decreases somewhat linearly as the volume fraction of the solvent region. On the other hand, for small c_g, e.g. $c_g < 0.1$, G^* tends to be insensitive to α over a wide range of α. If $c_g > 0.13$ with δ=0.025, the sensitivity of G^* with respect to α, $|dG^*/d\alpha|$, is greater than 0.1 for all possible α as can be seen in Fig. 4. It means that for small c_g the electrical resistance method becomes less useful to analyze

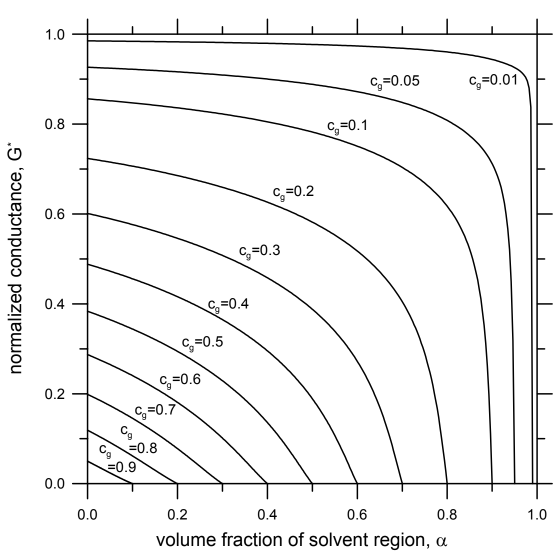
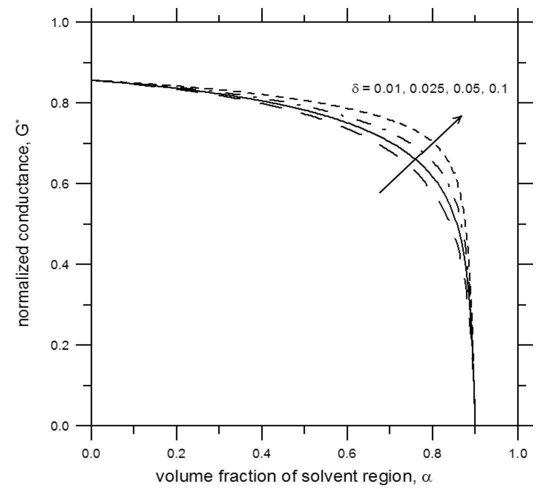
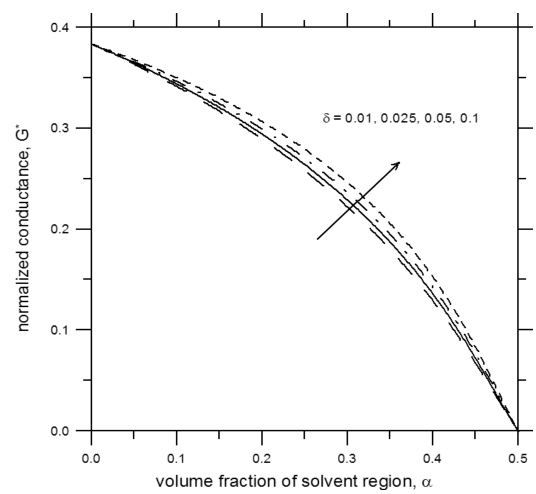


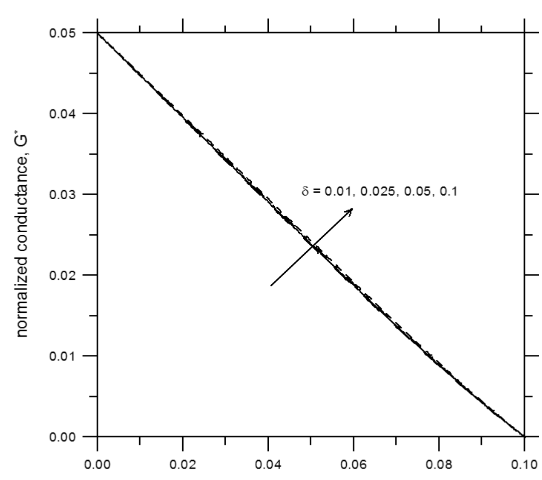
Fig. 2. Normalized conductance as a function of solvent volume fraction for various global insoluble solid particle concentrations($2\delta = 0.05$).



(a) $c_g = 0.1$



(b) $c_g = 0.5$



(c) $c_g = 0.9$

Fig. 3. Effect of gap angle on the relationship between the normalized conductance G^* and the volume fraction of the solvent region α.

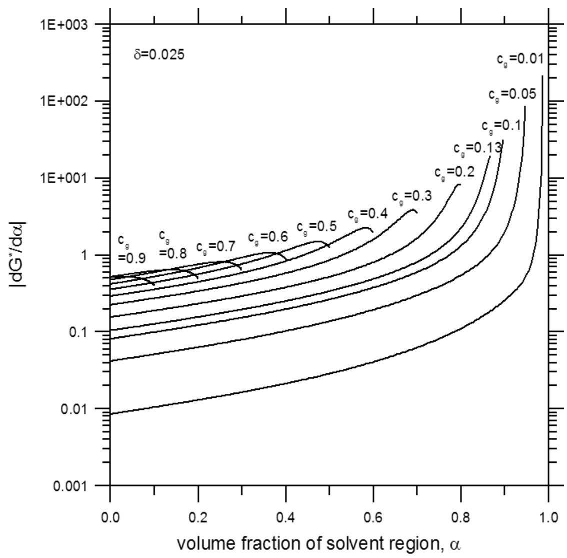


Fig. 4. Sensitivity of the normalized conductance G^* with respect to the change of the volume fraction of the solvent region α.

the separation process. For instance, for c_g = 0.1, the sensitivity $|dG^*/d\alpha|$ is less than 0.1 when α is smaller than about 0.8.

4. Conclusions

In this work, an electrical resistance method is applied to monitor the particle concentration distribution in a rotating cylindrical separator. In the separator, soluble and insoluble solid particles fed into solvent are separated into solvent and sediment regions after centrifugal separation in the vessel. An analytical expression of the electrical conductance is derived in terms of the insoluble solid particle concentration in the sediment region and the volume fraction of the solvent region. It has been observed that as the gap between the electrodes decreases and as the global solid concentration increases, the sensitivity of the normalized conductance becomes more sensitive to the volume fraction of the solvent region. The derived mathematical model shows that the electrical resistance method is applicable to monitoring of the separation process and can be used to design the electrical sensors.

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