Journal of Magnetics 16(4), 391-397 (2011) http://dx.doi.org/10.4283/JMAG.2011.16.4.391

Inductance Measurement of Interior Permanent Magnet Synchronous Motor in Stationary Frame of Reference

Geun-Ho Lee[1], Woongchul Choi[1], Byeong-Hwa Lee[2], Jae-Woo Jung[2], and Jung-Pyo Hong[2]*

[1]*Department of Automotive Engineering, Kookmin University, Seoul 136-702, Korea*
[2]*Department of Automotive Engineering, Hanyang University, Seoul 133-791, Korea*

(Received 29 July 2011, Received in final form 31 October 2011, Accepted 31 October 2011)

An inductance measurement method for interior permanent magnet synchronous machine (IPMSM) is proposed in this paper. In this method, the motor is measured at standstill condition, and only a 3-phase voltage source, an oscilloscope and a DC voltage source are required. Depending on the deductive dq-axis voltage equations in the stationary frame of reference, the dq-axis inductances at different current magnitude and vector angle can be calculated by the measured 3-phase voltages and currents. And hence, the saturation and cross-magnetizing effect of the inductances are measurable. This paper introduces the principle equations, experiment setup, data processing, and results comparison on the concentrated-winding and distributed-winding IPMSMs.

Keywords : cross-magnetizing effect, current vector, inductance, interior permanent magnet synchronous motors, standstill, stationary frame of reference

1. Introduction

Owing to the permanent magnet, salient structure, and flux barrier, it is difficult to calculate and test the d- and q-axis inductances of interior permanent magnet synchronous motor (IPMSM) [2]. A few numerical methods have been proposed to solve the calculation problem [2-5]. The saturation, cross-magnetizing and other effect can be considered and correctly calculated in these methods.

There are also several solutions to measure the inductances [1, 3-5]. In [1] a method called AC standstill is introduced. This method applies a single phase AC voltage source to one phase motor winding, and measures the currents and voltages of this phase and another phase in order to calculate the self- and mutual-inductances, and further calculate d- and q-axis inductances with them. It is called standstill because the rotor is locked at each test position. It is obvious that the effect of current vector angle cannot be reflected, and hence the cross-magnetizing effect is regardless in this method. Additionally, the flux path in two-phase exciting will be different with the one of three-phase exciting. The other standstill method with considering the both saturation and cross-magnetizing effect is introduced in [3]. It fixes the rotor position and uses a vector controller to generate a stepwise d- or q-axis voltage, while the other axis current is kept constant. According to the current response, the two-axis inductances can be calculated. The difficulty of this method is the generation of the stepwise d- or q-axis voltage. In the ordinary 3-phase inverter, it cannot be directly obtained from the pulse width modulation (PWM) voltage. A high precision low-pass filter must be used. According to phase shift between the flux linkages under the load condition and no-load condition in the steady state, authors of [4] measured the dq-axis inductances in the operation conditions. The d-axis current constant, and the load torque is adjusted, so that the total magnitude and vector angle of the load current can be covered. The errors in this method are the unregarded PM demagnetization, and much varying resistance. Based on [4], an improved method is proposed in [5]. In this method, a look-up table is used to correct the error due to the demagnetization of the PM. The complicated and relative expensive system is the shortage of this method. The dynamometer, power meter, vector-control motor drive, low-pass filter, etc. are necessary in order to measure many required variables.

*Corresponding author: Tel: +82-2-2220-4466
Fax: +82-2-2220-4465, e-mail: hongjp@hanyang.ac.kr

As the mentioned above, the methods in [3, 5] can measure the inductance with considering the cross-magnetizing and saturation effects. When the proper motor drive is absent, however, these inductance test methods become unavailable. Considering the practical requirements, this paper proposes a simple method to measure the d- and q-axis inductance of IPMSM. It is processed in standstill condition so that the dynamometer is not necessary. It uses a 3-phase AC voltage source so that the vector control drive is not required. It only measures the phase currents and phase voltages, so the power meter is eliminated. Hence, it is very suitable for normal laboratory experiment. The most meaningful point is that this method also can consider the saturation and cross-magnetizing effect. In this paper, first, the principle of this method will be introduced. And then, based on the deductive equations, the experiment scheme and the processing methods of measured data will be proposed. After briefly introducing the inductance calculation method used in this paper, both a concentrated-winding IPMSM and a distributed-winding IPMSM will be tested and compared with the corresponding calculated results.

2. Principle of Inductance Measurement in Stationary Frame of Reference

2.1. Inductance in Stationary Frame of Reference

The voltage equation of the IPMSM in the stationary frame of reference is described in [6]

$$\begin{bmatrix} v_q^s \\ v_d^s \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}\begin{bmatrix} i_q^s \\ i_d^s \end{bmatrix} + \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}\begin{bmatrix} \lambda_q^s \\ \lambda_d^s \end{bmatrix}$$

$$\begin{bmatrix} \lambda_q^s \\ \lambda_d^s \end{bmatrix} = \begin{bmatrix} L_q^s & -L_{qd}^s \\ -L_{qd}^s & L_d^s \end{bmatrix}\begin{bmatrix} i_q^s \\ i_d^s \end{bmatrix} + \begin{bmatrix} \lambda_m \sin\theta_{er}^s \\ \lambda_m \cos\theta_{er}^s \end{bmatrix} \quad (1)$$

$$L_q^s = L + \Delta L\cos(2\theta_{er}^s)$$
$$L_d^s = L - \Delta L\cos(2\theta_{er}^s)$$
$$L_{qd}^s = \Delta L\sin(2\theta_{er}^s)$$

where r_s is the phase resistance, λ_m is the flux linkage of PM, p represents the *d/dt* operator, the subscript e represents the unit in electrical angle, θ_{er}^s is the rotor position in stationary frame of reference, and the L and ΔL are calculated by

$$L = \frac{L_q^r + L_d^r}{2}$$

$$\Delta L = \frac{L_q^r - L_d^r}{2} \quad (2)$$

where L_q^r and L_d^r are the desired q- and d-axis inductances in the rotation reference frame.

2.2. Fundamental Equations of Measurement Method in Stationary Frame of Reference

Expand (1), and It can be obtained that

$$v_q^s = r_s i_q^s + (L + \Delta L\cos(2\theta_{er}^s))\frac{d}{dt}i_q^s - 2\omega_{er}^s\Delta L\sin(2v)i_q^s$$
$$-\Delta L\sin(2\theta_{er}^s)\frac{d}{dt}i_d^s - 2\omega_{er}^s\Delta L\cos(2\theta_{er}^s)i_d^s + \omega_{er}^s\lambda_m\cos\theta_{er}^s$$
$$v_d^s = r_s i_d^s + (L - \Delta L\cos(2\theta_{er}^s))\frac{d}{dt}i_d^s + 2\omega_{er}^s\Delta L\sin(2\theta_{er}^s)i_d^s$$
$$-\Delta L\sin(2\theta_{er}^s)\frac{d}{dt}i_q^s - 2\omega_{er}^s\Delta L\cos(2\theta_{er}^s)i_q^s - \omega_{er}^s\lambda_m\sin\theta_{er}^s \quad (3)$$

It is obvious that the terms with ω_{er}^s can be eliminated in the standstill condition. And in order to eliminate the sine and cosine terms, the rotor position θ_{er}^s is set to 0° (or 90°). Thus the equations are simplified as

$$v_q^s = r_s i_q^s + L_q^r\frac{d}{dt}i_q^s$$
$$v_d^s = r_s i_d^s + L_d^r\frac{d}{dt}i_d^s \quad (4)$$

where the v_q^s, v_d^s, i_q^s and i_d^s are the q- and d-axis voltages and currents. According to the 3-phase to 2-phase transformation in the stationary frame of reference, they can be represented by 3-phase voltages and currents that are directly measurable variables. In practice, (4) is modified as (5) considering the sampling data.

$$[2v_a(k) - v_b(k) - v_c(k)] = r_s[2i_a(k) - i_b(k) - i_c(k)]$$
$$+L_q^r\frac{[2i_a(k) - i_b(k) - i_c(k)] - [2i_a(k\text{-}1) - i_b(k\text{-}1) - i_c(k\text{-}1)]}{T_s}$$
$$[v_c(k) - v_b(k)] = r_s[i_c(k) - i_b(k)] \quad (5)$$
$$+L_d^r\frac{[i_c(k) - i_b(k)] - [i_c(k\text{-}1) - i_b(k\text{-}1)]}{T_s}$$

where k means the kth value of data, and T_s is the sampling time of the measurement equipment. Finally, in order to express the relationship between the inductances and current vector, the 3-phase current should be converted to the magnitude and angle of the vector in the rotation frame of reference with Park's transformation

$$\begin{bmatrix} I_a\cos\beta \\ -I_a\sin\beta \end{bmatrix} = \begin{bmatrix} i_q^r \\ i_d^r \end{bmatrix}$$
$$= \frac{2}{3}\begin{bmatrix} \cos\theta_{er}^s & \cos(\theta_{er}^s - 120°) & \cos(\theta_{er}^s + 120°) \\ \sin\theta_{er}^s & \sin(\theta_{er}^s - 120°) & \sin(\theta_{er}^s + 120°) \end{bmatrix}\begin{bmatrix} i_a^s \\ i_b^s \\ i_c^s \end{bmatrix} \quad (6)$$

where θ_{er}^{s} is 0° as assumed before, I_a is the magnitude of current vector, and β is the angle of current vector referred to q-axis. Due to the zero θ_{er}^{s}, the q- and d-axis currents are varying with time. Thus, the measured q- and d-axis inductances cover the various current vector states. Their saturation phenomena can be reflected by different current magnitude, and their cross-magnetizing effect can be measured by the variation of current vector angle.

3. Experiment Setup

3.1. Experiment configuration

According to the derivative equations, the ideal 3-phase AC voltage source (or current source) is required. In this paper, the voltage source will be applied. In the standstill, there is no back electromotive force (Back-EMF) in each phase. The rated phase current usually can be reached at very low voltage exciting. Therefore, the low voltage range has priority when selecting the voltage source, in order to increase the precision. In addition, there are current components in the equivalent iron-loss resistances rising as the source frequency increases, which are not the torque-producing component. Thus relatively low frequency of the AC source also is preferred.

As described in (5), totally there are six variables that should be measured. Due to the asymmetric phase inductance distribution, there is voltage component in the motor neutral line, i.e. the sum of the 3-phase voltages is no longer zero. Meanwhile, the sum of 3-phase currents always equals to zero. Thus, 3-phase voltage and 2-phase current should be measured. In addition, a DC voltage generator will be helpful to find the rotor zero position. The experiment setup applied in this paper is shown in Fig. 1(a). The total experiment devices include a 50-Hz 3-phase AC source, a 4-channel oscilloscope, a vice grid pliers, and a DC voltage generator.

3.2. Experiment IPMSM model

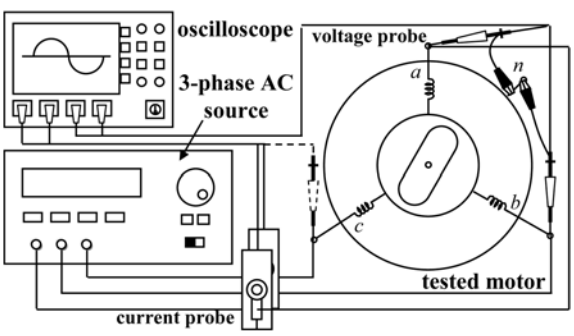

Fig. 1. Schemes of inductance measurement system with 3-phase AC source.

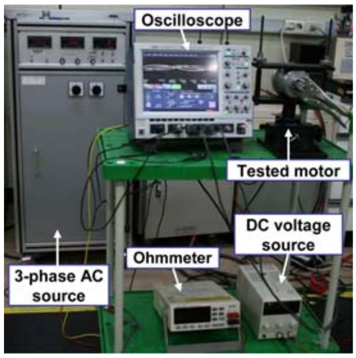

Fig. 2. (Color online) Experiment setting of inductance measurement in this paper.

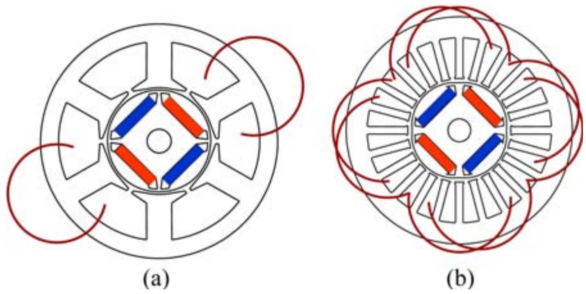

Fig. 3. (Color online) Cross section and winding configuration of test motors: (a) Concentrated winding IPMSM; (b) Distributed winding IPMSM.

Table 1. Specification of the test motors.

Terms	Concentrated winding	Distributed winding
Stator/rotor outer radii [mm]	80 / 34.5	
Airgap/stack length [mm]	0.8 / 35	
Volume of PM [mm^3]	16 × 3.5 × 34	
Material of core	Cogent	
No. of series connected turns	58	52
No. of parallel circuits	2	
Phase resistance (@20°C) [Ω]	0.159	0.145
Rated current [Arms]	8.8	8.3
DC link voltage [V]	300	

Two IPMSMs with concentrated winding and distributed winding are analyzed and tested in this paper to verify the applicability of the proposed method. The cross-sections of these two motors are shown in Figs. 3 (a) and (b), respectively. And their specifications are shown in Table 1.

4. Experiment Data Processing

4.1. FFT filter for smoothing measured data

The waveforms in Fig. 4 are measured and saved with a

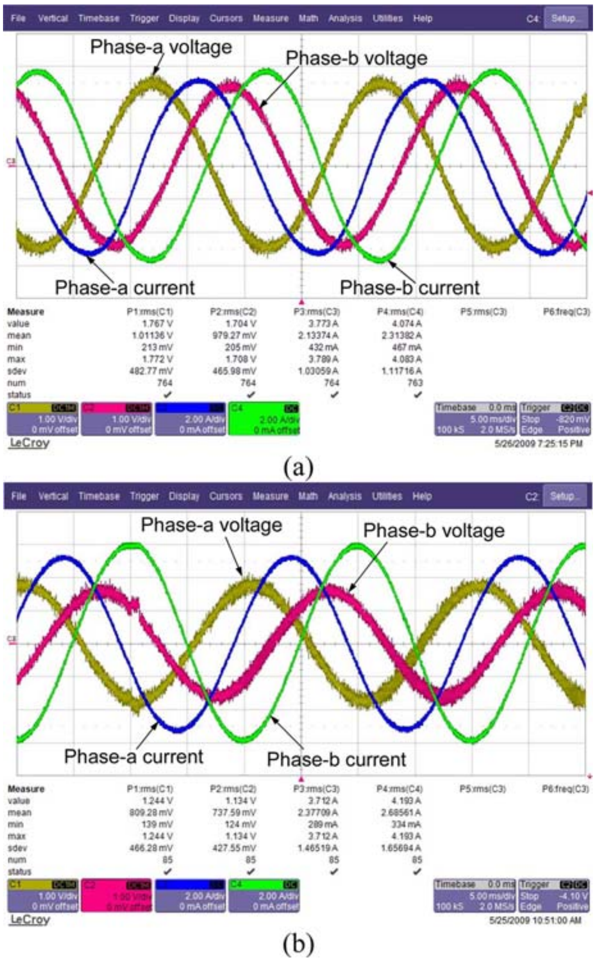

Fig. 4. (Color online) Measured phase voltages and currents in one period at 3 Arms: (a) concentrated winding IPMSM; (b) distributed winding IPMSM.

digital oscilloscope. The measured voltages and currents hence are discrete-time data. It is obvious that there is much noise in the measured wave forms so that the data cannot be used directly. By means of the Fast Fourier Transform (FFT) filter, the Fourier components whose frequencies are higher than the frequency in (7) can be removed from the original experiment data.

$$f_{threshold} = \frac{1}{n\Delta T} \tag{7}$$

where n is the number of data points considered at one time, and ΔT is the abscissa spacing between two adjacent data points. Fig. 5 shows the comparison between the original data and filtered wave form of phase a voltage.

4.2. Ripple elimination

Fig. 6 shows the raw calculated inductances by using the measured phase voltages and currents. It can be seen that the raw inductance results have some ripples. The

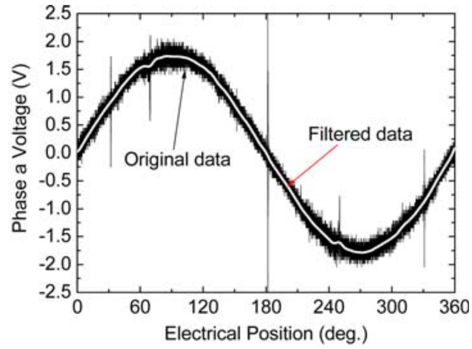

Fig. 5. (Color online) Comparison of the original data and the filtered data of phase-a voltage waveforms.

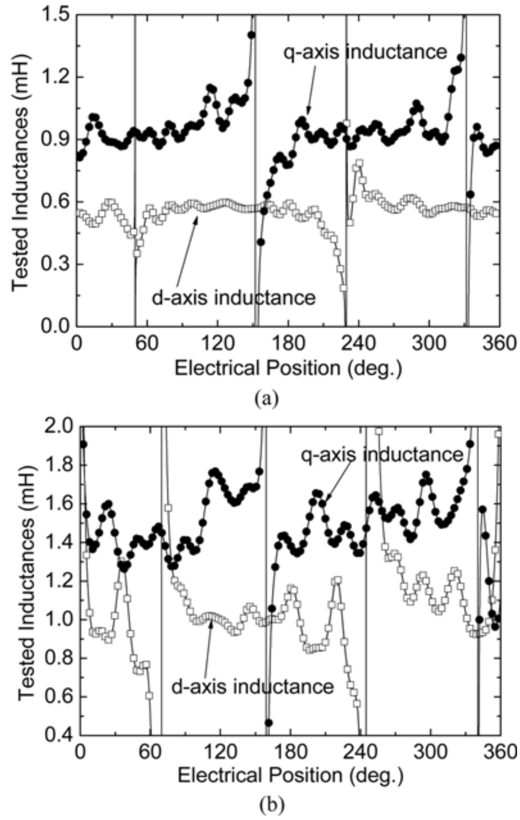

Fig. 6. Raw measured d- and q-axis inductances at certain voltage: (a) concentrated winding IPMSM; (b) distributed winding IPMSM.

dominate reason is that the slots and teeth of stator produces the different permeability in the spatial distribution. That is the reason why the inductance curves of the distributed winding IPMSM are smoother than those of the concentrated winding IPMSM. Additionally, due to the asymmetric circuit, the variation of current magnitude and vector angle also may generate different saturation and cross-magnetizing effect. In order to eliminate the ripple of calculated inductances, two methods are pro-

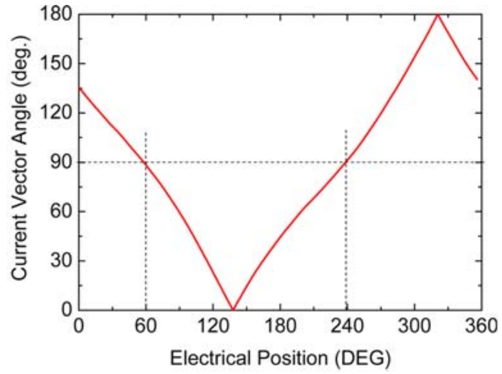

Fig. 7. (Color online) Relationship between the current vector angle and electrical position.

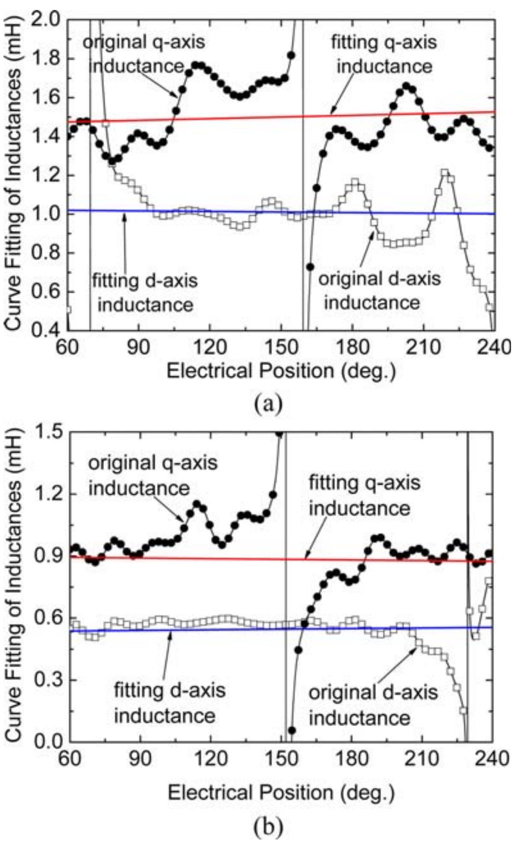

Fig. 8. (Color online) Curve fitting of raw inductance results: (a) concentrated winding IPMSM; (b) distributed winding IPMSM.

posed in this paper. One is that each current point is tested twice. One time the rotor is aligned to pole, the other time the rotor is rotated to align with the central of slot. The wave form is smoothed solving the mean value of the calculated inductances of both times.

The other method is to use the polynomial least-square function to fit the curve. The general k orders polynomial

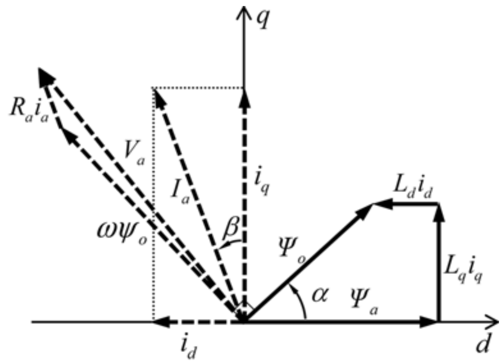

Fig. 9. Phasor diagram of IPMSM.

least-square function is described in

$$f(x) = a_0 + a_1x + a_2x^2 + ... + a_kx^k \tag{8}$$

where a_0, a_1, $a_2...a_k$ are chosen to minimize the least-square loss function as described in (9) [7].

$$x^2 = \sum_{i=1}^{N}\left[\frac{y_i - \sum_{k=1}^{M} a_kX_k(x_i)}{\sigma_i}\right]^2 \tag{9}$$

where σ_i is the measurement error of the ith data.

According to the relationship between the current vector angle and the electrical position as shown in Fig. 7, the data from 60° to 240° electrical position can cover the current vector angle from the −90° to 90°. Thus, the data in this section is selected and processed by curve fitting. The fitting results of the concentrated winding IPMSM and the distributed winding IPMSM are shown in Figs. 8 (a) and (b), respectively.

5. Calculation Method

The inductance calculation method used in this paper is described in [1]. A phasor diagram of IPMSM is shown in Fig. 9. In the solid-line part, it can be seen that there are the relationships as

$$L_d = \frac{\psi_0 \cos\alpha - \psi_a}{i_d}$$
$$L_q = \frac{\psi_0 \sin\alpha}{i_q} \tag{10}$$

where ψ_a is the flux linkage generated by permanent magnet in no-load condition, ψ_0 is the flux linkage generated by permanent magnet and excited armature current, and α is the phase shift between these two flux linkages. According to these equations, the d- and q-axis inductances are calculated in the procedure described in [1].

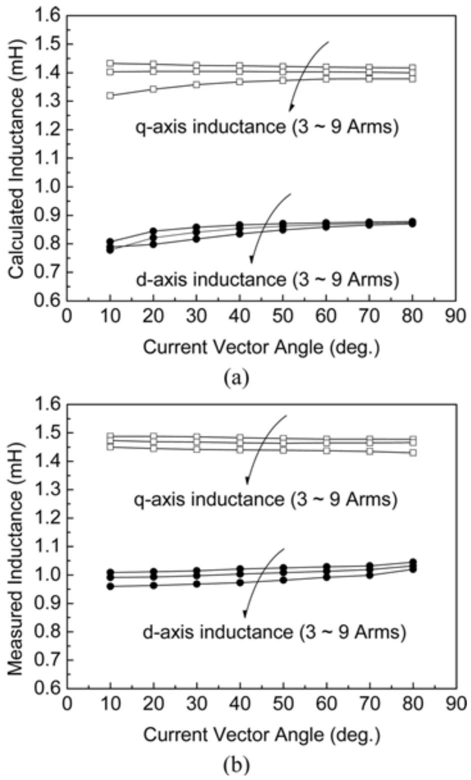

Fig. 10. Inductances of the concentrated winding IPMSM: (a) calculated inductances; (b) measured inductances.

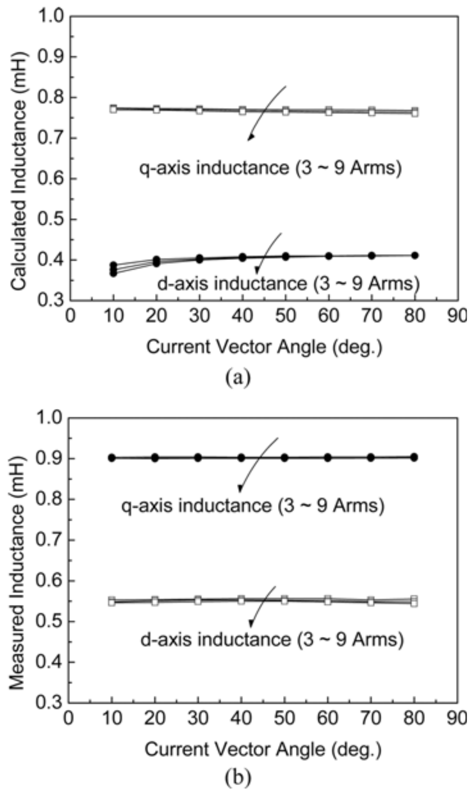

Fig. 11. Inductances of the distributed winding IPMSM: (a) calculated inductances; (b) measured inductances.

6. Comparison of Results and Discussion

The d- and q-axis inductances of the concentrated winding IPMSM and the distributed winding IPMSM calculated by the above method are shown in Fig. 10(a) and Fig. 11(a), respectively. It can be seen that the inductances of distributed winding IPMSM have no great differences as the current magnitude and vector angle vary, which means the significant cross-magnetizing and saturation effects can not be reflected well. Due to the air cooling method, the current can not reach high. Therefore, the motor always operates under the unsaturated condition.

The d- and q-axis inductances of these two motors measured with the proposed method are shown in Fig. 10(b) and Fig. 11(b), respectively. Compared with the calculated results, the experimental are very similar to them. It can be seen that the measured d-axis inductances are larger than those of calculation. This is because the rotor d-axis is aligned with the stator tooth at the 0^o electrical position. As mentioned before, the deductive equations are based on the sinusoidal winding distribution. The concentrated winding generates more space harmonics which strongly influence the accuracy of the principle equations. Additionally, the analysis process does not consider the current components in the iron-loss equivalent resistances. Therefore, larger current is used to produce the flux linkage in the numerical calculation process.

Due to the sinusoidal current wave form, the (5) may generate the singularity points in the entire electrical period. The measured inductances around these singularity points are strongly distorted, which restricts the measurable inductance range. The current or voltage limitation in the AC source is another drawback of this method, which implies the small power motor is preferred.

Approximately 20 percentage of error occurs between experimental value and analytical value. The large error occurs according to angle of current vector which can influence the degree of precision (error) by inducing 1.5 peak voltage applying AC Voltage source.

More accurate and lower rated voltage of AC voltage source need to be verified because inequality in phases affect on measurement of inductance.

7. Conclusion

The inherent drawback and complicated system configuration make the existed IPMSM inductance test methods not always available. Based on these problems, this paper proposed a relatively simple experiment method to measure the d- and q-axis inductance of IPMSM. In the stationary frame of reference, the relationships between the d- and q-axis inductances, and the 3-phase voltages and currents are derived. Thus, by using the measured 3-phase AC voltages and currents, FFT smoothing and least-square curve fitting, the d- and q-axis inductances reflecting cross-magnetizing and saturation effects can be obtained. Compared with the calculated results, the validity of this inductances measurement method is verified in the concentrated winding IPMSM and distributed winding IPMSM. This method is preferred to use in the simple laboratory experiment and quick parameter verification.

Acknowledgments

This work was supported (in part) by the new faculty research program 2011 of Kookmin University in Korea.

References

[1] IEEE Standard Procedure for Obtaining Synchronous Machine Parameters by Standstill Frequency Response Testing, IEEE Standard 115A-1987 (1987).

[2] Tao Sun, S. O. Kwon, S. H. Lee, and J. P. Hong, in Electrical Machines and Systems, ICEMS 2008, International Conference, pp. 3131-3136 (2008).

[3] B. Stumberger, G. Stumberger, D. Dolinar, A. Hamler, and M. Trlep, IEEE Trans. Ind. Appl. **39**, 1264 (2003).

[4] Edward C. Lovelace, Thomas M. Jahns, *et al.*, in Industry Applications Conference **4**, 2486 (2002).

[5] Khwaja A. Rahman and Silva Hiti, IEEE Trans. Ind. Appl. **41**, 557 (2005).

[6] G. D. Andreescu, I. Boldea, *et al.*, IEEE Trans. Energy Conversion **23**, 393 (2008).

[7] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, Numerical Recipes in C: The Art of Scientific Computing, Cambridge University Press, Cambridge (1992).