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8 Antenna Polar Switching Up-Down Relay Networks

Jun Li[1] · Moon Ho Lee[2] · Yier Yan[3] · Bu Shi Peng[4] · Gun Joon Hwang[5]

Abstract

In this paper, we propose a reliable 8×8 up-down switching polar relay code based on 3GPP LTE standard, motivated by 3GPP LTE down link, which is 30 bps/Hz for 8×8 MIMO antennas, and by Arikan's channel polarization for the frequency selective fading (FSF) channels with the generator matrix Q_8. In this scheme, a polar encoder and OFDM modulator are implemented sequentially at both the source node and relay nodes, the time reversion and complex conjugation operations are separately implemented at each relay node, and the successive interference cancellation (SIC) decoder, together with the cyclic prefix (CP) removal, is performed at the destination node. Use of the scheme shows that decoding at the relay without any delay is not required, which results in a lower complexity. The numerical result shows that the system coded by polar codes has better performance than currently used designs.

Key words: Polar Code, Up-Down Relay Network, SIC Decoder.

Ⅰ. Introduction

Channel polarization shows a construction of provably capacity-achieving coding sequences with belief propagation (BP) decoders [1], [2]. It provides a potential method to meet the elusive goal of multi-fold binary-input discrete memoryless channels, where channel combining and splitting operations are applied to improve its symmetric capacity [1]. In actuality, the polarization of multiple channels is a commonplace phenomenon and thus almost impossible to avoid as long as several channels are synthesized in a proper density with certain arrangements. During the past decade, the space time coding (STC) communication system has been well studied as a promising way to increase spectral efficiency, channel capacity and link reliability [4]~[7]. It shows that the coding gain and diversity can be simultaneously achieved with suitable coding schemes. The problem with the relay schemes is the data rate loss that occurs as the number of relay nodes increases, since relay nodes are assumed to be assigned orthogonal channels. This leads to the use of polar coding sequences in relay channels, where relay nodes are allowed to transmit simultaneously over the same channels.

In this paper, we consider a simple design of the relay system that achieves the fascinating symmetric capacity of the relay channels based on polar coding with a belief propagation (BP) decoder at destination node. The paper is organized as follows. In Section Ⅱ, we describe the state switching Up-Down polar Relay System and the capacity at the source and the each relay. In Section Ⅲ, we systematically decode the design of a simple polar relay scheme for up-down polar relay channels by using a successive cancellation structure. Some simulation results are also depicted in order to show the BER performance behavior and robustness of this polar relay system. Finally, conclusions are drawn in Section Ⅳ.

Ⅱ. Conventional Polar Code

In Arikan's paper [1], a polar code is proposed that is a class of capacity-achieving codes. The main motivation of polar code was theoretical to show the existence of a family of codes that are provably capacity achieving and that have low complexity encoding and decoding algorithms.

Polar code is the first provably capacity achieving code for arbitrary B-DMC W with low encoding and decoding complexity. The polar codes are constructed to transform $Q_2^{\otimes 3}$ to a block of N=2^n bits and transmit the output through independent copies of a B-DMC where '⊗' denotes Kronecker product. As n grows large, the channels seen by individual bits start polarizing: they approach either a noiseless channel or a pure-noise cha-

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[1]Div. of Electronic & Information Engineering, Chonbuk National University, Jeonju, Korea.
[2]Div. of Electronic Engineering, Chonbuk National University, Jeonju, Korea.
[3]Mechanical and Electronic Engineering, Guangzhou University, Guangzhou, P. R. China.
[4]Div. of Electronic & Information Engineering, Chonbuk National University, Jeonju, Korea.
[5]Supporting Center of Information & Communication, Jeonju University, Jeonju, Korea.
Corresponding Author : Moon Ho Lee (e-mail : moonho@jbnu.ac.kr)

nnel, where the fraction of channels becoming noiseless is close to the symmetric mutual information I(W) that presents high rates in reliable wireless communication channel as inputs of W with equal frequencies.

Constructions of polar codes of order 2^n are derived from radix 2^n channel polarization, which is an operation whereby one manufacture out of N independent copies of a given BDMC W for $N=2^n$ yields a second set of N channels $W_N^i : 1 \le i \le N$ that show a polarization effect in the sense that, as N becomes large, the symmetric capacity terms $I(W_N^i) : 1 \le i \le N$ tend towards 0 or 1 for all but a vanishing fraction of indices i. This operation consists of a channel-combining phase and a channel-splitting phase. This phase combines copies of a given B-DMC in a recursive manner to produce a vector channel W_N for $N=2^n$. The recursion begins with only one copy of W and we set W_1=W The first level of the recursion combines two independent copies, as shown in Fig. 1, and obtains the channel W_2 with the transition probabilities:

$$W_2(y_1^2|u_1^2) = W(y_1|\otimes_{i=1}^2 u_i)W(y_2|u_2) \tag{1}$$

where $\otimes_{i=1}^2 u_i$. The mapping $W_2 : u_1^2 \to y_1^2$ from input of W_2 can be denoted by:

$$W_2(y_1^2|u_1^2) = W^2(y_1^2|u_1^2 G_2) \tag{2}$$

where

$$G_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \tag{3}$$

The mapping $W_4 : u_1^4 \to y_1^4$ from the input of W_4 to the output of W_4, and hence the next level of the recursion, is shown in Fig. 2, where four independent copies of W_2 are combined to create the channel W_{22} with transition probabilities denoted by:

$$\begin{aligned} W_4(y_1^4|u_1^4) &= W^4(y_1^4|u_i^4 G_4) \\ &= W_2(y_1^2|\otimes_{i=2}^2 u_i, \otimes_{i=3}^4 u_i)W_2(y_3^4|u_2, u_4) \end{aligned} \tag{4}$$

where

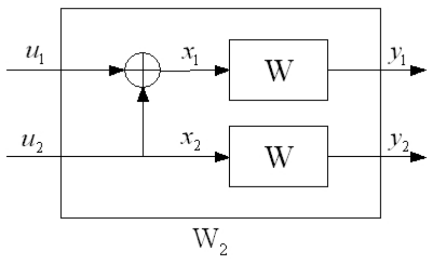

Fig. 1. The 2 by 2 Arikan polar code.

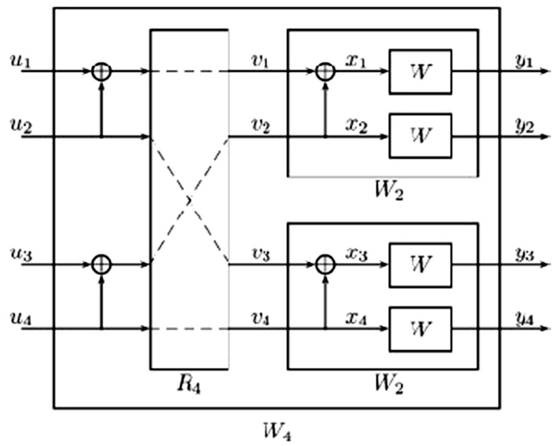

Fig. 2. The 4 by 4 Arikan polar code.

$$G_4 = R_4 G_2^{\otimes 2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \tag{5}$$

Note that $R_4(u_1^4) = (u_1, u_2, u_3, u_4)$.

The general form of the recursion is specified where four independent copies of $W_{N/2}$ are combined to produce the channel W_N for N=2n. The mapping $u_1^N \to y_1^N$ from the input of the synthesized channel to the input of the underlying raw channels is clearly linear over GF (2).

Thus, it is represented by a matrix $G(N)$, such that

$$W_N\left(y_1^N|u_1^N\right) = W^N\left(y_1^N|u_1^N G_N\right) \tag{6}$$

where $y_1^N \in \eta_1^N, u_1^N \in \chi^N$, and $G^N = G_2^{\otimes n}$.

The 2×2 and 4×4 polar codes are shown in Fig. 1 and Fig. 2, respectively. As shown in [1], asymptotically, the polar codes as a function of code block length N, can be encoded in complexity $O(N\log N)$, and decoded using a successive cancellation decoder in complexity $O(N\log N)$. The polar code can be used in a MIMO network system.

Ⅲ. Proposed Channel Polarization: Up-Down-Polar Relay System

In [3], the authors presented a MIMO system that employed reconfigurable antennas that were used as both transmitter and receiver parts. This MIMO system has a good diversity gain and multiplexing gain, as shown in Table 1. As the number of antennas increase, the diversity gain and multiplexing gain also increase proportionally. This is why we consider 8 antennas in our proposed scheme.

As Fig. 3 shows, we can easily see that the multiple

Table 1. Error probability and channel capacity with different number of antennas.

Number of antennas	Error probability(P_e)	Capacity(C), bps/Hz
N_s=N_r=1 (SISO)	$P \propto SNR^{-1}$	C=log(SNR)
N_s=1, N_r>1 (SIMO)	$P \propto SNR^{-N_r}$	C=log(SNR)
N_s>1, N_r>1 (MIMO)	$P \propto SNR^{-N_t N_r}$ $N_t N_r$=Diversity gain	C=min(N_t, N_r) log(SNR) gain min(N_t, N_r)= spectral multiplexing

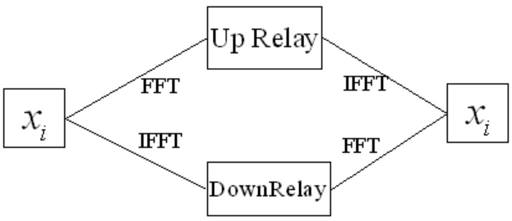

Fig. 3. Up-Down relay network model.

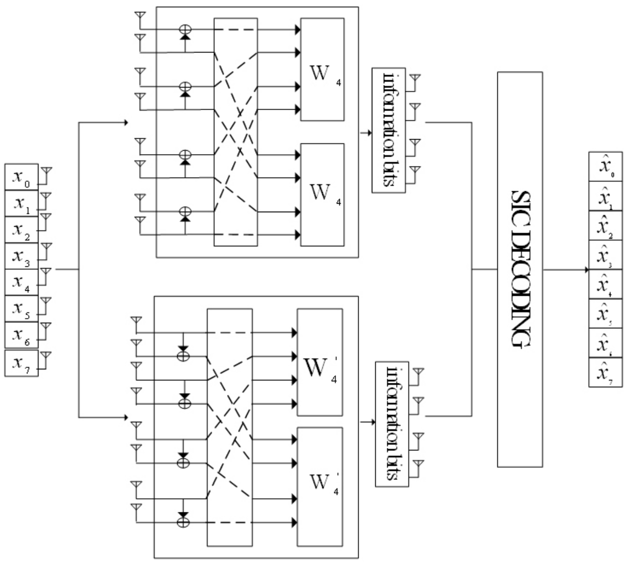

Fig. 4. Proposed 8 antenna polar switching up-down relay networks.

antenna system has a higher diversity gain and multiplexing gain. We therefore consider a distributed wireless system based on OFDM modulation with N subcarriers. The system has one source node S, one destination node D, and two relay nodes $R \triangleq \{R_1, R_2\}$, as shown in Fig. 4. The numbers of antennas at S, R, and D are N_s, N_r, and N_d, respectively. We consider a scenario where N_s OFDM symbols are transmitted for $N_s=2^n$. We consider a design of the polar relay scheme that can mitigate relay synchronization errors. Each relay node, $R_k, \forall k \in \{1,2\}$, is assumed to be capable of processing the OFDM symbols independently and correctly. The average transmission power at source node S is p_t. In this paper, we let N_s=N_r=N_d=1. This assumption is applicable for any nodes equipped with multiple antennas. The relay scheme is half-duplex, meaning that S and R do not transmit and receive simultaneously. The N_s independent OFDM symbols are transmitted simultaneously from source node S to destination node D in two stages. The constraint on the total network power is $p=p_t+2p_r$. We also adopt the power allocation strategy suggested in [3], and thus have

$$p_t = 2p_r = p/2 \tag{7}$$

This channel model, denoted by $\mathrm{H} \in C^{2\times 2}$, is created between source node S and relay nodes R, and between $\mathrm{K} \in C^{2\times 2}$ and D. Here, entries of H and K are assumed independent and identically distributed (i.i.d.) with distribution CN(0, 1).

The relay wireless system with source-relay-destination triplet structure is equivalent to two partial wireless systems. The polar channel model is shown as:

$$up\ polar\ channel: \mathrm{H} = diag(\phi_1, \phi_2), \tag{8}$$

$$down\ polar\ channel: K = diag(\kappa_1, \kappa_2). \tag{9}$$

where the entries ϕ_k and κ_k are independent complex Gaussian random variables with zero-mean and unit-variance.

Based on the MIMO relay channels H and K in (8) and (9), respectively, we designed the polar system for the transmission of the signal vector x, in which we switch to the polar system for four time slots; i.e., up-down-polarizing MIMO relay communication models in a turning switch.

The system has two phases, as follows. In phase 1, the source node broadcasts the information OFDM symbols that are polarized at source node S to each relay node R_k. In phase 2, S stops the transmission and each relay node R_k that has polarized the received OFDM symbols now processes and simultaneously retransmits the resulting symbols to destination node D.

Ⅳ. Up-Down State Switching Polarizing Cooperative Relay System

At source node S, the transmitted information is modulated into complex symbols x_{ij} and then each N modulated symbol as a block is poured into an OFDM modulator of N subcarriers. We denote four consecutive OFDM blocks by $x_i = (x_{i,0}, x_{i,1}, \cdots, x_{i,N-1})^{\mathrm{T}}, \forall i \in Z_8$. We define $(x_i + x_j) = (x_{i,0} + x_{j,0}, x_{i,1} + x_{j,1}, \cdots, x_{i,N-1} + x_{j,N-1})^T, \forall i, j \in Z_8$, for calculation.

In the first time slot, the eight consecutive OFDM blocks are processed with the up-polarizing 8×8 matrix Q_8 at source node S, i.e.,

$$U = Q_8 X, \tag{10}$$

where $U=(u_0,u_1,u_2,u_3,u_4,u_5,u_6,u_7)$ denotes the polarized matrix of size $N\times 8$, $X=(x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7)$ denotes the signal matrix of size $N\times 8$ corresponding to eight OFDM blocks, $Q_8 = Q_2\otimes Q_2\otimes Q_2 = Q_2^{\otimes 3}$, which is given as:

$$up: Q_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},\ down: Q_2^T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{11}$$

The following indicates the up switch:

$$Q_8 = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0 \\ 1&1&1&1&0&0&0&0 \\ 1&0&0&0&1&0&0&0 \\ 1&1&0&0&1&1&0&0 \\ 1&0&1&0&1&0&1&0 \\ 1&1&1&1&1&1&1&1 \end{pmatrix}. \tag{12}$$

and the down of the transpose of the above matrix is

$$Q_8^T = \begin{pmatrix} 1&1&1&1&1&1&1&1 \\ 0&1&0&1&0&1&0&1 \\ 0&0&1&1&0&0&1&1 \\ 0&0&0&1&0&0&0&1 \\ 0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}. \tag{13}$$

In the OFDM modulator, the eight consecutive blocks are modulated by N-point FFT. Each block is then precoded by a cyclic prefix (CP) with length l_{cp}. Thus, each OFDM symbol consists of $L_s = N + l_{cp}$ samples. Finally, the four OFDM symbols are broadcasted to two relay nodes denoted by τ_{sd2}, the overall relative delay from source node S to relay node R_2, and then to destination node D, where the delay is relative to relay node R1. In order to combat against both frequency selective fading channels and timing errors, we assume that $l_{cp} \ge \max_{l,k}\{\tau_{l,sk}+\tau_{l,rk}+\tau_{sd2}\}$. We denote eight consecutive OFDM symbols by $\breve{u}_i, \forall i\in Z_8$, where $\breve{u}_i$ consists of $FFT(u_i)$ and the corresponding CP.

At each relay R_k, the received noisy signals will be simply processed and forwarded to destination node D. We assume that the channel coefficients are constant during eight OFDM symbol intervals. We define two processed vectors $\mathrm{u}_1 = (\breve{u}_0^T,\breve{u}_1^T,\breve{u}_2^T,\breve{u}_3^T)^T$ and $\mathrm{u}_2 = (\breve{u}_4^T,\breve{u}_5^T,\breve{u}_6^T,\breve{u}_7^T)^T$, which are polarized at R_1 and R_2, respectively.

Therefore, the received signals at $R_k, \forall k\in\{1,2\}$, for eight successive OFDM symbol durations can be generated as:

$$\begin{aligned}
\breve{r}_{k0} &= \sqrt{p_t}\breve{u}_0 * \phi_k + \breve{n}_{k0},\\
\breve{r}_{k1} &= \sqrt{p_t}\breve{u}_1 * \phi_k + \breve{n}_{k1},\\
\breve{r}_{k2} &= \sqrt{p_t}\breve{u}_2 * \phi_k + \breve{n}_{k2},\\
\breve{r}_{k3} &= \sqrt{p_t}\breve{u}_3 * \phi_k + \breve{n}_{k3},\\
\breve{r}_{k4} &= \sqrt{p_t}\breve{u}_4 * \phi_k + \breve{n}_{k4},\\
\breve{r}_{k5} &= \sqrt{p_t}\breve{u}_5 * \phi_k + \breve{n}_{k5},\\
\breve{r}_{k6} &= \sqrt{p_t}\breve{u}_6 * \phi_k + \breve{n}_{k6},\\
\breve{r}_{k7} &= \sqrt{p_t}\breve{u}_7 * \phi_k + \breve{n}_{k7}.
\end{aligned} \tag{14}$$

where $\sqrt{p_t}$ is the transmission power at source node S, ϕ_k is an $L\times 1$ vector defined as $\phi_k = (\alpha_{sk}(0),\alpha_{sk}(1),\cdots,\alpha_{sk}(L-1))$, $*$ denotes the linear convolution, and $\breve{n}_{ki}, \forall i \in Z_8$, denotes the corresponding additive white Gaussian noise(AWGN) at relay node R_k with zero-mean and unit-variance, in four successive OFDM symbol durations.

Each relay node R_k then polarizes, processes, and forwards the received noisy signals $u_i, \forall i\in\{1,2,\cdots,8\}$. After performing the processing operations, each relay node R_k amplifies the yielded symbols with a scalar $\lambda = \sqrt{p_r/p_t+1}$ while retaining the average transmission power p_r. In order to make the PF scheme available for frequency selective fading channels, each relay R_k can only implement the time reversal operations or the complex conjugation operations on the received OFDM symbols.

At destination node D, the CP is removed for each OFDM symbol. We note that relay node R1 implements the time reversions of the noisy signals including both information symbols and CP. What we need is to obtain the time reversal version of only information symbols, i.e., $\zeta(FFT(u_i)), \forall i\in Z_8$, after the CP removal. Then, by using some properties of FFT/IFFT, we achieve the feasibility of the definition as follows.

Definition 2.1 Consider the processed OFDM symbols at relay node R_1 we can obtain

$$\zeta(FFT(u_i)) * \zeta(\phi_1')$$

at destination node if we remove the CP as in a conventional OFDM system, to get an N-point vector and shift the last $\tau_1' = l_{cp} - \tau_1 + 1$ samples of the N-point vector as the first τ_1' samples. Here ϕ_1' is an $N\times 1$ vector defined as $\phi_1' = (\alpha_{s1}(0),\cdots,\alpha_{s1}(L-1),0,\cdots,0)$, and τ_1 denotes the maximum path delay of channel ϕ_1 from source node S to relay node R1, i.e., $\tau_1 = \max_l\{\tau_{l,s1}\}$. In a similar way, we define another $N\times 1$ vector $\kappa_1^{'}$ as $\kappa_1^{'}=\left(\alpha_{r1}(0),\cdots,\alpha_{r1}(L-1),0,\cdots,0\right)$.

The received OFDM symbols are then transformed by the N-point FFT [6], [8]. As mentioned before, because of the timing errors, the OFDM symbols from relay node R2 arrive at destination node τ_{sd2} samples later than the symbols from relay node R1. Since l_{cp} is long enough, we can still maintain the orthogonality between subcarriers. The delay τ_{sd2} in the time domain corresponds to a phase change in the frequency domain; i.e.:

$$f^{\tau_{sd2}}=\left(1,e^{-\imath 2\pi\tau_{sd2}/N},\cdots,e^{-\imath 2\pi\tau_{sd2}(N-1)/N}\right)^T, \tag{15}$$

where $f=\left(1,e^{-\imath 2\pi\tau_{sd2}/N},\cdots,e^{-\imath 2\pi(N-1)/N}\right)^T$ and $\imath=\sqrt{-1}$. Similarly, the shift of $\tau_1^{'}$ samples in the time domain also corresponds to a phase change $f^{\tau_1^{'}}$, and hence the total phase change is f^{τ_2}. We denote $\breve{y}_i=\left(\breve{y}_{i0},\breve{y}_{i1},\cdots,\breve{y}_{i(N-1)}\right)$, $\forall i\in Z_8$, the received signals for four consecutive OFDM blocks at destination node D after the CP removal and the FFT transformations. Consequently, we have

$$\begin{aligned}
\breve{y}_0&=\lambda\left[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_0)\right)\right)\circ\breve{\phi}_1\circ\breve{\kappa}_1+\breve{\bar{n}}_{10}\circ\breve{\kappa}_1\right]+\breve{n}_0\\
\breve{y}_1&=\lambda\left[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_1)\right)\right)\circ\breve{\phi}_1\circ\breve{\kappa}_1+\left(\breve{\bar{n}}_{10}+\breve{\bar{n}}_{20}\right)\circ\breve{\kappa}_1\right]+\breve{n}_1\\
\breve{y}_2&=\lambda\left[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_0+u_2)\right)\right)\circ f^{\tau_2}\circ\breve{\phi}_2\circ\breve{\kappa}_2+\breve{\bar{n}}_{21}\circ\breve{\kappa}_2\right]+\breve{n}_2\\
\breve{y}_3&=\lambda[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_1+u_3)\right)\right)\circ f^{\tau_2}\circ\breve{\phi}_2\circ\breve{\kappa}_2\\
&\quad+\left(\breve{\bar{n}}_{21}^*+\breve{\bar{n}}_{23}^*\right)\circ\breve{\kappa}_2]+\breve{n}_3\\
\breve{y}_4&=\lambda\left[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_4)^*\right)\right)\circ\breve{\phi}_5\circ\breve{\kappa}_5+\breve{\bar{n}}_{50}\circ\breve{\kappa}_5\right]+\breve{n}_4\\
\breve{y}_5&=\lambda\left[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_4+u_5)^*\right)\right)\circ\breve{\phi}_5\circ\breve{\kappa}_5+\left(\breve{\bar{n}}_{50}+\breve{\bar{n}}_{60}\right)\circ\breve{\kappa}_5\right]+\breve{n}_5\\
\breve{y}_6&=\lambda\left[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_4+u_6)^*\right)\right)\circ f^{\tau_6}\circ\breve{\phi}_6\circ\breve{\kappa}_6+\breve{\bar{n}}_{61}\circ\breve{\kappa}_6\right]+\breve{n}_6\\
\breve{y}_7&=\lambda[\sqrt{p_t}FFT\left(\zeta\left(FFT(u_4+u_5+u_6+u_7)^*\right)\right)\circ f^{\tau_6}\circ\breve{\phi}_6\circ\breve{\kappa}_6\\
&\quad+\left(\breve{\bar{n}}_{61}^*+\breve{\bar{n}}_{63}^*\right)\circ\breve{\kappa}_6]+\breve{n}_7
\end{aligned}\tag{16}$$

where $\breve{\phi}_1=FFT\left(\zeta\left(\phi_1^{'}\right)\right)$, $\breve{\kappa}_1=FFT\left(\kappa_1^{'}\right)$, $\breve{\phi}_2=FFT\left(\left(\phi_1^{'}\right)^*\right)$, $\breve{\kappa}_2=FFT\left(\kappa_2^{'}\right)$, $\breve{\bar{n}}_{ki}=FFT\left(\bar{n}_{ki}\right)$, and $\breve{\bar{n}}_{ki}=FFT\left(\bar{n}_{ki}\right)$, $\forall k\in$ and $\{1,\cdots,8\}$ $\forall i\in Z_8$.

4-1 The Up Polar Relay Case

For the up-polar relay case, we define the new mapping method at the relay from the 2 by 2 up-polar code:

$$\begin{aligned}
u_0&=x_0,\\
u_1&=x_0+x_1,\\
u_2&=x_2,\\
u_3&=x_2+x_3,\\
u_4&=x_0+x_4,\\
u_5&=x_1+x_5,\\
u_6&=x_2+x_6,\\
u_7&=x_3+x_7.
\end{aligned}\tag{17}$$

According to the properties of the well-known FFT transforms for an $N\times 1$ point vector x, we have $\left(FFT(x)\right)^*=IFFT\left(x^*\right)$ and $FFT\left(\zeta\left(FFT(x)\right)\right)=IFFT\left(FFT(x)\right)=x$. Therefore, the formulas in (16) can be written in the polar code form on each subcarrier $u,\forall u\in Z_N$, as follows:

$$\begin{pmatrix}y_{0u}\\y_{1u}\\y_{2u}\\y_{3u}\\y_{4u}\\y_{5u}\\y_{6u}\\y_{7u}\end{pmatrix}=\begin{pmatrix}
\breve{\phi}_1\breve{\kappa}_1&0&0&0&0&0&0&0\\
0&\breve{\phi}_1\breve{\kappa}_1&0&0&0&0&0&0\\
0&0&\breve{\phi}_1\breve{\kappa}_1&0&0&0&0&0\\
0&0&0&\breve{\phi}_1\breve{\kappa}_1&0&0&0&0\\
0&0&0&0&\Phi_2&0&0&0\\
0&0&0&0&0&\Phi_2&0&0\\
0&0&0&0&0&0&\Phi_2&0\\
0&0&0&0&0&0&0&\Phi_2
\end{pmatrix}\begin{pmatrix}u_0\\u_1\\u_0+u_2\\u_1+u_3\\u_4^*\\u_4^*+u_5^*\\u_4^*+u_6^*\\u_4^*+u_5^*+u_6^*+u_7^*\end{pmatrix}+e_0 \tag{18}$$

$$\begin{pmatrix}y_{0u}\\y_{1u}\\y_{2u}\\y_{3u}\\y_{4u}^*\\y_{5u}^*\\y_{6u}^*\\y_{7u}^*\end{pmatrix}=\begin{pmatrix}
\breve{\phi}_1\breve{\kappa}_1&0&0&0&0&0&0&0\\
\breve{\phi}_1\breve{\kappa}_1&\breve{\phi}_1\breve{\kappa}_1&0&0&0&0&0&0\\
\breve{\phi}_1\breve{\kappa}_1&0&\breve{\phi}_1\breve{\kappa}_1&0&0&0&0&0\\
\breve{\phi}_1\breve{\kappa}_1&\breve{\phi}_1\breve{\kappa}_1&\breve{\phi}_1\breve{\kappa}_1&\breve{\phi}_1\breve{\kappa}_1&0&0&0&0\\
\Phi_2^*&0&0&0&\Phi_2^*&0&0&0\\
\Phi_2^*&\Phi_2^*&0&0&\Phi_2^*&\Phi_2^*&0&0\\
\Phi_2^*&0&\Phi_2^*&0&\Phi_2^*&0&\Phi_2^*&0\\
\Phi_2^*&\Phi_2^*&\Phi_2^*&\Phi_2^*&\Phi_2^*&\Phi_2^*&\Phi_2^*&\Phi_2^*
\end{pmatrix}\begin{pmatrix}x_{0u}\\x_{1u}\\x_{2u}\\x_{3u}\\x_{4u}\\x_{5u}\\x_{6u}\\x_{7u}\end{pmatrix}+e \tag{19}$$

where $\Phi_2=f_\in^{\tau_2}\breve{\phi}_2\breve{\kappa}_2,\Phi_2^*=\left(f_\in^{\tau_2}\breve{\phi}_2\breve{\kappa}_2\right)^*$, e_0 and e are the polarized noises given by

$$e_0=\lambda\begin{pmatrix}
\breve{\bar{n}}_{10,\in}\breve{\kappa}_{1,\in}+\breve{n}_{0,\in}\\
\left(\breve{\bar{n}}_{10,\in}+\breve{\bar{n}}_{12,\in}\right)\breve{\kappa}_{1,\in}+\breve{n}_{1,\in}\\
\breve{\bar{n}}_{21,\in}\breve{\kappa}_{2,\in}+\breve{n}_{2,\in}\\
\left(\breve{\bar{n}}_{21,\in}+\breve{\bar{n}}_{23,\in}\right)\breve{\kappa}_{2,\in}+\breve{n}_{3,\in}\\
\breve{\bar{n}}_{50,\in}\breve{\kappa}_{5,\in}+\breve{n}_{4,\in}\\
\left(\breve{\bar{n}}_{50,\in}+\breve{\bar{n}}_{62,\in}\right)\breve{\kappa}_{5,\in}+\breve{n}_{5,\in}\\
\breve{\bar{n}}_{61,\in}\breve{\kappa}_{6,\in}+\breve{n}_{6,\in}\\
\left(\breve{\bar{n}}_{61,\in}+\breve{\bar{n}}_{63,\in}\right)\breve{\kappa}_{6,\in}+\breve{n}_{7,\in}
\end{pmatrix},\quad e=\lambda\begin{pmatrix}
\breve{\bar{n}}_{10,\in}\breve{\kappa}_{1,\in}+\breve{n}_{0,\in}\\
\left(\breve{\bar{n}}_{10,\in}+\breve{\bar{n}}_{12,\in}\right)\breve{\kappa}_{1,\in}+\breve{n}_{1,\in}\\
\breve{\bar{n}}_{21,\in}\breve{\kappa}_{2,\in}+\breve{n}_{2,\in}\\
\left(\breve{\bar{n}}_{21,\in}+\breve{\bar{n}}_{23,\in}\right)\breve{\kappa}_{2,\in}+\breve{n}_{3,\in}\\
\breve{\bar{n}}_{50,\in}\breve{\kappa}_{5,\in}+\breve{n}_{4,\in}\\
\left(\breve{\bar{n}}_{50,\in}+\breve{\bar{n}}_{62,\in}\right)\breve{\kappa}_{5,\in}+\breve{n}_{5,\in}\\
\breve{\bar{n}}_{61,\in}\breve{\kappa}_{6,\in}+\breve{n}_{6,\in}\\
\left(\breve{\bar{n}}_{61,\in}+\breve{\bar{n}}_{63,\in}\right)\breve{\kappa}_{6,\in}+\breve{n}_{7,\in}
\end{pmatrix}^* \tag{20}$$

$f_\in^{\tau_2}=\exp\left(-\imath 2\pi\in\tau/N\right)$, $x_{i\in}$ is the $\in^{th}$ element of x_i, $\breve{\kappa}_{k,\in}$ is the $\in^{th}$ element of $\breve{\kappa}_k$, $\breve{\bar{n}}_{ki}$ is the $\in^{th}$ element of $\breve{\bar{n}}_i$, $\forall k\in\{1,\cdots,8\}$ and $\forall i\in Z_8$.

From equation (19), we can easily see that the channel coefficient is changed to 8 by 8 polar coding. By getting this signal, we can directly decode with a SIC recursive decoding method at the destination.

4-2 The Down Polar Relay Case

As we mentioned in the previous up-polar relay section, we can finally get an 8 by 8 up-polar signal at the destination. In order to get the 8 by 8 down-polar code, another mapping is specified from 2 by 2 down-polar code.

Therefore, we also use the same method for down-polar relay. For the down-polar relay case, we also define another mapping:

$$
\begin{aligned}
&u_0 = x_7,\\
&u_1 = x_7 + x_6,\\
&u_2 = x_5,\\
&u_3 = x_5 + x_4,\\
&u_4 = x_7 + x_3,\\
&u_5 = x_6 + x_2,\\
&u_6 = x_5 + x_1,\\
&u_7 = x_4 + x_0.
\end{aligned} \tag{21}
$$

Therefore, the formulas in (16) can be written in the polar code form on each subcarrier $d, \forall d \in Z_N$, as follows:

$$
\begin{pmatrix} y_{0d}\\ y_{1d}\\ y_{2d}\\ y_{3d}\\ y_{4d}\\ y_{5d}\\ y_{6d}\\ y_{7d} \end{pmatrix}
=
\begin{pmatrix}
\breve{\phi}_1\breve{\kappa}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \breve{\phi}_1\breve{\kappa}_1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \breve{\phi}_1\breve{\kappa}_1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \breve{\phi}_1\breve{\kappa}_1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \Phi_2 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \Phi_2 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \Phi_2 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_2
\end{pmatrix}
\begin{pmatrix} u_4^* + u_5^* + u_6^* + u_7^*\\ u_4^* + u_6^*\\ u_4^* + u_5^*\\ u_4^*\\ u_1 + u_3\\ u_0 + u_2\\ u_1\\ u_0 \end{pmatrix}
+ e_0 \tag{22}
$$

which can be rewritten as:

$$
\begin{pmatrix} y_{0d}^*\\ y_{1d}^*\\ y_{2d}^*\\ y_{3d}^*\\ y_{4d}\\ y_{5d}\\ y_{6d}\\ y_{7d} \end{pmatrix}
=
\begin{pmatrix}
\breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1 & 0\breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1\\
0 & \breve{\phi}_1\breve{\kappa}_1 & 0 & \breve{\phi}_1\breve{\kappa}_1 & 0 & \breve{\phi}_1\breve{\kappa}_1 & 0 & \breve{\phi}_1\breve{\kappa}_1\\
0 & 0 & \breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1 & 0 & 0 & \breve{\phi}_1\breve{\kappa}_1 & \breve{\phi}_1\breve{\kappa}_1\\
0 & 0 & 0 & \breve{\phi}_1\breve{\kappa}_1 & 0 & 0 & 0 & \breve{\phi}_1\breve{\kappa}_1\\
0 & 0 & 0 & 0 & \Phi_2^* & \Phi_2^* & \Phi_2^* & \Phi_2^*\\
0 & 0 & 0 & 0 & 0 & \Phi_2^* & 0 & \Phi_2^*\\
0 & 0 & 0 & 0 & 0 & 0 & \Phi_2^* & \Phi_2^*\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_2^*
\end{pmatrix}
\begin{pmatrix} x_{0u}\\ x_{1u}\\ x_{2u}\\ x_{3u}\\ x_{4u}\\ x_{5u}\\ x_{6u}\\ x_{7u} \end{pmatrix}
+ e \tag{23}
$$

where $\Phi_2 = f_{\in}^{\tau_2}\breve{\phi}_2\breve{\kappa}_2, \Phi_2^* = \left(f_{\in}^{\tau_2}\breve{\phi}_2\breve{\kappa}_2\right)^*$, e_0 and e are the polarized noises given by:

$$
e_0 = \lambda \begin{pmatrix}
\breve{\bar{n}}_{10,\in}\breve{\kappa}_{1,\in} + \breve{n}_{0,\in}\\
\left(\breve{\bar{n}}_{10,\in} + \breve{\bar{n}}_{12,\in}\right)\breve{\kappa}_{1,\in} + \breve{n}_{1,\in}\\
\breve{\bar{n}}_{21,\in}\breve{\kappa}_{2,\in} + \breve{n}_{2,\in}\\
\left(\breve{\bar{n}}_{21,\in} + \breve{\bar{n}}_{23,\in}\right)\breve{\kappa}_{2,\in} + \breve{n}_{3,\in}\\
\breve{\bar{n}}_{50,\in}\breve{\kappa}_{5,\in} + \breve{n}_{4,\in}\\
\left(\breve{\bar{n}}_{50,\in} + \breve{\bar{n}}_{62,\in}\right)\breve{\kappa}_{5,\in} + \breve{n}_{5,\in}\\
\breve{\bar{n}}_{61,\in}\breve{\kappa}_{6,\in} + \breve{n}_{6,\in}\\
\left(\breve{\bar{n}}_{61,\in} + \breve{\bar{n}}_{63,\in}\right)\breve{\kappa}_{6,\in} + \breve{n}_{7,\in}
\end{pmatrix}
\quad
e = \lambda \begin{pmatrix}
\breve{\bar{n}}_{10,\in}\breve{\kappa}_{1,\in} + \breve{n}_{0,\in}\\
\left(\breve{\bar{n}}_{10,\in} + \breve{\bar{n}}_{12,\in}\right)\breve{\kappa}_{1,\in} + \breve{n}_{1,\in}\\
\breve{\bar{n}}_{21,\in}\breve{\kappa}_{2,\in} + \breve{n}_{2,\in}\\
\left(\breve{\bar{n}}_{21,\in} + \breve{\bar{n}}_{23,\in}\right)\breve{\kappa}_{2,\in} + \breve{n}_{3,\in}\\
\breve{\bar{n}}_{50,\in}\breve{\kappa}_{5,\in} + \breve{n}_{4,\in}\\
\left(\breve{\bar{n}}_{50,\in} + \breve{\bar{n}}_{62,\in}\right)\breve{\kappa}_{5,\in} + \breve{n}_{5,\in}\\
\breve{\bar{n}}_{61,\in}\breve{\kappa}_{6,\in} + \breve{n}_{6,\in}\\
\left(\breve{\bar{n}}_{61,\in} + \breve{\bar{n}}_{63,\in}\right)\breve{\kappa}_{6,\in} + \breve{n}_{7,\in}
\end{pmatrix} \tag{24}
$$

$f_{\in}^{\tau_2} = \exp(-\iota 2\pi \in \tau / N)$, $x_{i\in}$ is the $\in^{th}$ element of x_i, $\breve{\kappa}_{k,\in}$ is the $\in^{th}$ element of $\breve{\kappa}_k$, $\breve{\bar{n}}_{ki}$ is the $\in^{th}$ element of $\breve{\bar{n}}_i$, $\forall k \in \{1,\cdots,8\}$ and $\forall i \in Z_8$. From (19) and (23), we can easily see the up and down 8 polar switching forms, and also, the up-down switch analysis is proved in Appendix A.

Ⅴ. Decoding of the Polar Relay System

In this subsection, we consider the successive interference cancellation (SIC) decoding for the proposed polar MIMO relay system with standard complex constellations, such as binary phase shift keying (BPSK) modulation constellation. Recall that each signal $x_{i,\varepsilon}, \forall\varepsilon \in Z_N$, in the ε^{th} subcarrier of OFDM block $x_i, i \in Z_8$, is independently transmitted across R_k and a channel output $y_{i,\varepsilon}$ is obtained with transition probability $W\left(y_{i,\varepsilon} \middle| x_{i,\varepsilon}\right)$. For each subcarrier in eight OFDM symbols we misuse the simplified notations $\mathrm{x} = \mathrm{x}_\varepsilon = \left(\mathrm{x}_{I,\varepsilon}^T, \mathrm{x}_{F,\varepsilon}^T\right)^T$, $\mathrm{y} = \mathrm{y}_\varepsilon = \left(\mathrm{y}_{1,\varepsilon}^T, \mathrm{y}_{2,\varepsilon}^T\right)^T$ for the up-down polarizing system, where $\mathrm{y}_1 = \mathrm{y}_{1,\varepsilon} = \left(y_{0,\varepsilon}, y_{1,\varepsilon}\right)^T$ and $\mathrm{y}_2 = \mathrm{y}_{2,\varepsilon} = \left(y_{2,\varepsilon}, y_{3,\varepsilon}\right)^T$.

We consider all single links of the relay channel H from each pair of transmit antenna of source node S and receive antenna of relay node R_k, and K from relay node R_k to destination node D, which are independent complex Gaussian random variables with zero-mean and unit-variance. Each single-link channel, denoted by W, has the transition probability $W(y|x)$, where $x, y \in \mathrm{A}$. As a useful measurement of the reliability of the wireless network, a conventional channel parameter, the symmetric capacity I(W), can be used with some modulations. Note that parameter I(W) is the highest rate at which the reliable communication is possible using inputs with equal probabilities.

The successive iterative cancellation (SIC) decoder of the down-polarizing system observes y and generates an estimate $\hat{\mathrm{x}}$ of x. We may visualize the decoder as consisting of four decision elements, each element $\hat{\mathrm{x}}_i$ for source element $x_i, \forall \in Z_8$.

The depolarizing algorithm of the polar relay system begins with the i[th] decision element $\hat{x}_i$ for the down-polarizing system. It waits until it has received all previous decisions $\hat{x}_{i-1}$, and upon receiving them, it calculates the likelihood ratio (LR) $L_8^{(i)}$ as follows:

$$
L_8^{(i)}\left(\mathrm{y}, \hat{x}_1 \cdots \hat{x}_{i-1}\right) = \frac{W_8^{(i)}\left(\mathrm{y}, \hat{x}_1 \cdots \hat{x}_{i-1} \middle| 0\right)}{W_8^{(i)}\left(\mathrm{y}, \hat{x}_1 \cdots \hat{x}_{i-1} \middle| 1\right)}, \tag{25}
$$

and generates its decision as:

$$\hat{x}_i = \begin{cases} 0, & \text{if } L_8^{(i)}\left(\mathrm{y}, \hat{x}_1 \cdots \hat{x}_{i-1}\right) \ge 1, \\ 1, & \text{otherwise,} \end{cases} \tag{26}$$

which is then sent to succeeding decision element $\hat{x}_{i+1}$.The proof of (25) is found in Appendix B. The complexity of the decoding algorithm is determined essentially by the complexity of calculating LRs, which is N(1+log2 N)=32 for computing one round.

However, for the up-polarizing system the low level of LRs are given in [1] by

$$\begin{aligned} &L_2^{'(0)}(\mathrm{y}_k) = \frac{L(y_{2k-2})L(y_{2k-1})+1}{L(y_{2k-2})+L(y_{2k-1})} \\ &L_2^{'(1)}(y_k, \hat{x}_{2k-2}) = (L(y_{2k-2}))^{1-2\hat{x}_{2k-2}} L(y_{2k-1}), \end{aligned} \tag{27}$$

Furthermore, the high level of LRs can be calculated as

$$\begin{aligned} &L_8^{(0)}(\mathrm{y}) = \frac{L_4^{'(0)}(y_1)L_4^{'(0)}(y_2)+1}{L_4^{'(0)}(y_1)+L_4^{'(0)}(y_2)} \\ &L_8^{(1)}(y_0, \hat{\mathrm{x}}_0) = (L_4^{'(0)}(y_1))^{1-2\hat{x}_0} L_4^{'(0)}(y_2), \\ &L_8^{(2)}(\mathrm{y}, \hat{\mathrm{x}}_F) = \frac{L_4^{'(1)}(y_1, \hat{x}_0)L_4^{'(1)}(y_2, \hat{x}_0+\hat{x}_1)+1}{L_4^{'(1)}(y_1, \hat{x}_0)+L_4^{'(1)}(y_2, \hat{x}_0+\hat{x}_1)} \\ &L_8^{'(3)}(y_0, \hat{\mathrm{x}}_0) = L_4^{'(1)}(y_1, \hat{x}_0)^{1-2\hat{x}_2} L_4^{'(1)}(y_2, \hat{x}_0+\hat{x}_1), \\ &L_8^{(4)}(\mathrm{y}) = \frac{L_4^{'(2)}(y_1)L_4^{'(2)}(y_2)+1}{L_4^{'(2)}(y_1)+L_4^{'(2)}(y_2)} \\ &L_8^{(5)}(y_0, \hat{\mathrm{x}}_0) = (L_4^{'(2)}(y_1))^{1-2\hat{x}_0} L_4^{'(2)}(y_2), \\ &L_8^{(6)}(\mathrm{y}, \hat{\mathrm{x}}_F) = \frac{L_4^{'(3)}(y_1, \hat{x}_0)L_4^{'(3)}(y_2, \hat{x}_0+\hat{x}_1)+1}{L_4^{'(3)}(y_1, \hat{x}_0)+L_4^{'(3)}(y_2, \hat{x}_0+\hat{x}_1)} \\ &L_8^{'(7)}(y_0, \hat{\mathrm{x}}_0) = L_4^{'(3)}(y_1, \hat{x}_0)^{1-2\hat{x}_2} L_4^{'(3)}(y_2, \hat{x}_0+\hat{x}_1), \end{aligned} \tag{28}$$

Thus far, we have calculated LRs of the polar MIMO OFDM relay system with two polar systems. The advantage of this depolarizing algorithm is the relationship of two level LRs in coordination. For example, two LRs, $L_4^{(0)}(\mathrm{y}_t)$ and $L_4^{(1)}(\mathrm{y}_t, \hat{x}_0)$, are assembled from the same pair of LRs, $L_2^{(0)}(y_1)$ and $L_2^{(0)}(y_2)$, while the other two LRs, $L_4^{(2)}(\mathrm{y}_t, \hat{\mathrm{x}}_1)$ and $L_4^{(3)}(\mathrm{y}_t, \hat{\mathrm{x}}_1\hat{\mathrm{x}}_2)$, are from LRs $L_2^{(1)}\left(y_1, \hat{x}_0\right)$ and $L_2^{(1)}\left(y_2, \hat{x}_0+\hat{x}_1\right)$ due to the symmetry properties of MIMO relay channels. In addition, two down level LRs $L_2^{(0)}(y_k)$ and $L_2^{(1)}(y_k, \hat{x}_{2k-2})$ are assembled from the lowest (initial) level LRs $L^{(0)}(y_{2k-2})$ and $L^{(0)}(y_{2k-1})$ $\forall k \in \{1, 2, \cdots 8\}$. This process proposes an elegant approach for an accurate count of the total number of LRs that are required for a full description of the decoding algorithm.

Next, we design an implementation of the SC decoder for switching the polar MIMO-OFDM relay system. There are 12 nodes corresponding to LRs for decision elements $\hat{x}$.

Note that, in the down-polarizing system, it is not necessary to generate $\hat{x}_2$ and $\hat{x}_3$ since they are frozen bits that have low-reliability while transmitting in the down-polarizing channels. Similarly, in the up-polarizing system, it is not necessary to generate $\hat{x}_0$ and $\hat{x}_1$ since they are frozen bits with the low-reliability while transmitting in the up-polarizing channels. In this way, the proposed depolarizing algorithm can be made while directly setting $\hat{x}_2 = \hat{x}_3 = 0$ in the down-polarizing system and setting $\hat{x}_0 = \hat{x}_1 = 0$ in the up-polarizing system. This decoding process continues until all information bits x are jointly decoded in the end. Next, we will show the BER performance behaviors of the polar MIMO-OFDM relay system with simulation results. Thus, we can obtain x from $\hat{\mathrm{x}}$ in the polar MIMO-OFDM relay system with the high-reliability.

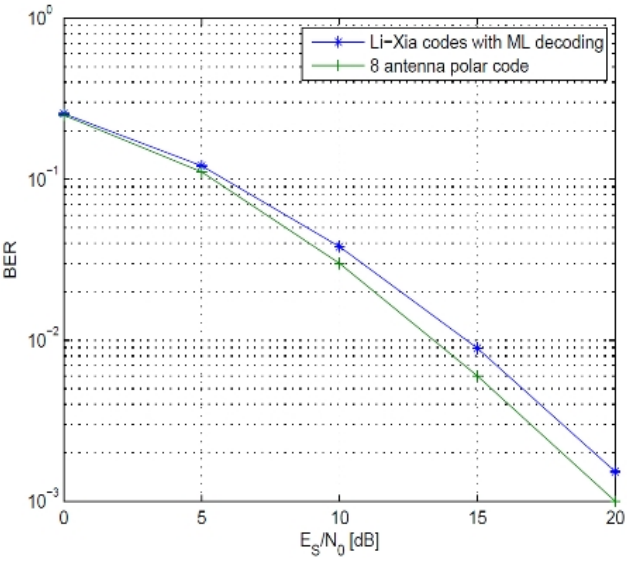

Fig. 5. The polar MIMO-OFDM relay system based on the OFDM polarizing for the FSF channels.

In Fig. 5, compared to the Li-Xia-Lee ML decoding MIMO-OFDM system, we can obtain 1 dB coding gain with 8 antenna up-down polar coding. In the proposed scheme, we do not require relays to decode. Only a simple encoding operation is done at each relay, which makes the transmission very simple and hence we can avoid imposing bottlenecks on the data rate. The complexity is then lower than that reported in [8].

Ⅵ. Conclusion

In this paper, we have presented a simple design for a cooperative relay scheme based on the 8 antenna polar relay switching system with two polarizing systems, which are shown as equation (19) and (23), the up-down polarizing system for full diversity. Two polar coding processes are available for each polarizing system, including source polarizing and relay polarizing sequences. The present polarizing relay system has a salient recursiveness feature and can be decoded with a SIC decoder, which renders the scheme analytically tractable and provides a low-complexity coding algorithm while multiple antennas are equipped. We analyzed the BER performance and diversity of such systems based on the Space Time Coding (STC) codes with the fixed size using the polarizing channels, which tend to polarize with respect to the increasing reliability under certain channel combining and splitting operations. Using this method, we achieved maximum diversity gains based on switching the state polar code with information bits. Each relay node only needs a simple processing operation to select and retransmit the partial signals without requiring any contemplation to decode the noisy signals. Simulation shows that the present scheme can provide an alternative solution for transmitting with higher reliability than the conventional relay schemes due to the occurrence of capacity-achieving OFDM sequences. This demonstrates that the proposed polar relay system has similar BER performance behaviors to the STC codes, but outperforms the STC codes in terms of the BER performance for large transmission power when the depolarizing algorithm is applied at the receiver.

Appendix

A. Proof of the Up-down Switch Analysis

From (11), we consider the following binary matrices in GF(2):

$$Q_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix},\ Q_2^T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{29}$$

It can be easily checked that

$$Q_2^2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 = (Q_2^T)^2 \tag{30}$$

$$\begin{aligned} Q_2 Q_2^T &= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = I_2 \end{aligned} \tag{31}$$

Furthermore,

$$\begin{pmatrix} Q_2 & Q_2^T \\ Q_2^T & \mathrm{Q}_2 \end{pmatrix} \times \begin{pmatrix} Q_2^T & Q_2 \\ Q_2 & Q_2^T \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{32}$$

Now we present a 4 by 4 polar matrix based on submatrix α and β defined. Let

$$Q_4 = \begin{pmatrix} Q_2 & Q_2^T \\ Q_2^T & Q_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}. \tag{33}$$

We have

$$Q_4^{-1} = \begin{pmatrix} Q_2^T & Q_2 \\ Q_2 & Q_2^T \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}^T \tag{34}$$

then

$$Q_4 Q_4^{-1} = I_4 . \tag{35}$$

Generator matrix $Q_N, N = 2^n$ is shown:

$$\begin{aligned} Q_N &= \left(Q_{\frac{N}{2}} \otimes Q_2 \right) \\ &= \left(I_{\frac{N}{2}} \otimes Q_2 \right) \left(Q_{\frac{N}{2}} \otimes I_2 \right) \\ &= \prod_{i=0}^{n} \left(I_{\frac{N}{2}} \otimes Q_2 \otimes I_{2^{(i-1)}} \right) \\ &= \prod_{i=0}^{n} \left(Q_N^i \right). \end{aligned} \tag{36}$$

where $\otimes$ is defined as Kronecker product.

For example, $N = 2^2$ case:

$$\begin{aligned} Q_4 &= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = (I_2 \otimes Q_2)(Q_2 \otimes I_2) \\ &= \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \right] \left[\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \end{aligned} \tag{37}$$

We have, $N = 2^3$ case:

$$\begin{aligned} Q_8 &= \prod_{i=1}^{3} \left(I_{2^{3-i}} \otimes Q_2 \otimes I_{2^{(i-1)}} \right) \\ &= (I_4 \otimes Q_2 \otimes I_1)(I_2 \otimes Q_2 \otimes I_2)(I_1 \otimes Q_2 \otimes I_4) \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}. \end{aligned} \tag{38}$$

B. SC (Successive Cancellation) Recursive Algorithm

Proof: From (25), we have log-likelihood ratio(LLR) [9], [10].

$$L(d) = \log_e\left[\frac{P(d=0)}{P(d=1)}\right] = \log_e\left[\frac{P(d=0)}{1-P(d=0)}\right] \tag{39}$$

we can see that

$$e^{L(d)} = \left[\frac{P(d=0)}{1-P(d=0)}\right] \tag{40}$$

$$e^{L(d)} - e^{L(d)}P(d=0) = P(d=0) \tag{41}$$

$$P(d=0)\left[1+e^{L(d)}\right] = e^{L(d)} \tag{42}$$

$$P(d=0) = \frac{e^{L(d)}}{1+e^{L(d)}} \tag{43}$$

$$P(d=1) = 1-P(d=0) = 1-\frac{e^{L(d)}}{1+e^{L(d)}} = \frac{1}{1+e^{L(d)}} \tag{44}$$

$$\begin{aligned} &L(d_1 \oplus d_2) = \\ &\log_e\left[\frac{P(d_1=0)P(d_1=1)+(1-P(d_1=0))(1-P(d_2=1))}{P(d_1=0)P(d_1=0)+(1-P(d_1=0))(1-P(d_2=0))}\right] \\ &= \log_e\left[\frac{\left(\frac{e^{L(d_1)}}{1+e^{L(d_1)}}\right)\left(\frac{1}{1+e^{L(d_2)}}\right)+\left(\frac{1}{1+e^{L(d_1)}}\right)\left(\frac{e^{L(d_2)}}{1+e^{L(d2)}}\right)}{\left(\frac{e^{L(d_1)}}{1+e^{L(d_1)}}\right)\left(\frac{e^{L(d_2)}}{1+e^{L(d_2)}}\right)+\left(\frac{1}{1+e^{L(d_1)}}\right)\left(\frac{1}{1+e^{L(d2)}}\right)}\right] \\ &= \log_e\left[\frac{\frac{e^{L(d_1)}+e^{L(d_2)}}{\left(1+e^{L(d_1)}\right)\left(1+e^{L(d_2)}\right)}}{\frac{e^{L(d_1)}e^{L(d_2)}+1}{\left(1+e^{L(d_1)}\right)\left(1+e^{L(d_2)}\right)}}\right] \end{aligned} \tag{45}$$

where the $\oplus$ sign is used to denote the modulo-2 sum of data expressed as binary digits. The $\Leftrightarrow$ denotes log likelihood addition.

This gives:

$$\left(L(d_1) \Leftrightarrow L(d_2)\right) = L(d_1 \oplus d_2) = \log_e \frac{e^{L(d_1)}+e^{L(d_2)}}{e^{L(d_1)}e^{L(d_2)}+1} \tag{46}$$

$$e^{L(d_1 \oplus d_2)} = \frac{e^{L(d_1)}+e^{L(d_2)}}{e^{L(d_1)}e^{L(d_2)}+1} \tag{47}$$

$$(e^{L(d_1 \oplus d_2)})^{-1} = \frac{e^{L(d_1)}e^{L(d_2)}+1}{e^{L(d_1)}+e^{L(d_2)}} \tag{48}$$

Defining $e^{L(d_1)}, e^{L(d_2)}, (e^{L(d_1 \oplus d_2)})^{-1}$ as follows:

$$e^{L(d_1)} = L_{N/2}^{(i)}(y_1^{N/2}, \hat{u}_{1,0}^{2i-2}), \tag{49}$$

$$e^{L(d_2)} = L_{N/2}^{(i)}(y_{N/2+1}^{N/2}, \hat{u}_{1,e}^{2i-2}), \tag{50}$$

$$(e^{L(d_1 \oplus d_2)})^{-1} = L_{N/2}^{(2i-1)}(y_1^N, \hat{u}_1^{2i-2}), \tag{51}$$

Finally, the recursive decoding is specified as:

$$\begin{aligned} &L_{N/2}^{2i-1}(y_1^N, \hat{u}_1^{2i-2}) = \\ &\frac{1+L_{N/2}^{(i)}(y_1^{N/2}, \hat{u}_{1.o}^{2i-2} \oplus \hat{u}_{1,e}^{2i-2}) \cdot L_{N/2}^{(i)}(y_{N/2+1}^N, \hat{u}_{1.e}^{2i-1})}{L_{N/2}^{(i)}(y_1^{N/2}, \hat{u}_{1.o}^{2i-2} \oplus \hat{u}_{1,e}^{2i-2}) + L_{N/2}^{(i)}(y_{N/2+1}^N, \hat{u}_{1.e}^{2i-1})} \end{aligned} \tag{52}$$

From equation (52), we can easily generate the 8 polar decoding formula (28).

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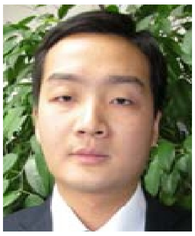
Jun Li

received his B.S. degree in electronic engineering from South Central University for Nationalities, P.R. China, in 2009, and his M.S. degree in electronic engineering from Chonbuk National University, Jeonju, Republic of Korea, in 2011. He has been doing research on the digital mobile wireless communications based on polar codes, MIMO-OFDM since September 2011 participating in the World Class University (WCU) project sponsored by the National Research Foundation (NRF) as a researcher studying for his doctorate who is presently affiliated to the Institute of Information and Communications in Chonbuk National University.

Moon Ho Lee

received both his B.S. and M.S. degrees in electrical engineering from Chonbuk National University, Jeonju, Republic of Korea, in 1967 and 1976, respectively, and his Ph.D. degrees in electronics engineering from Chonnam National University, Gwangju, Republic of Korea, in 1984 and from the University of Tokyo, Japan, in 1990, respectively. From 1970 to 1980, he was a chief engineer with Namyang Moonhwa Broadcasting Corporation. Since 1980, he has been a professor in the department of electronics engineering at Chonbuk National University. From 1985 to 1986, he was also with the University of Minnesota, as a postdoctoral fellow. He held visiting positions at the University of Hanover, Germany, in 1990, the University of Aachen, Germany in 1992 and 1996, respectively, and the University of Munich, Germany, in 1998. He has authored 34 books, including Digital Communication (Youngil, 1999, Republic of Korea), Information and Coding (Bokdu, 1998, Republic of Korea), Digital Image Processing (Daeyoung, 1994, Republic of Korea), and Digital Filter Design (Daeyoung, 1995, Republic of Korea). His current research interests include multi-dimensional source and channel coding, mobile communications, image processing, information security, signal processing, digital communications, and polar codes. He is a Registered Telecommunication Professional Engineer and a member of the National Academy of Engineering in Korea. He was the recipient of the paper prize awards from the Korean Institute of Communication Science in 1986 as well as in 1997, the Institute of Electronics Engineers of Korea in 1987, Chonbuk Province in 1992, and the commendation of the Prime Minister in 2002 for inventing the Jacket matrix, respectively. In addition, he won the Best Paper Awards from ICSEA, France, in 2006, ICSNC, France, in 2006, and EITC, Republic of Korea, in 2006, respectively. Furthermore, he won the National Order of Merit, the Doyak Medal, from the Korean government in 2007, the Haedong Information and Communications Best Prize from the Korea Information and Communications Society in 2007, the Award for Scientist and Engineer of the Month from the Ministry of Science and Technology and the Korea Science and Engineering Foundation in January, 2008, the Best Paper Awards from CEIC 2009 and CEIC 2010 sponsored by IEEE's Korea Daejeon Section, and the Grand Alumni Award from the Chonbuk National University's alumni association in 2009, respectively. Moreover, he has been a principal of the World Class University project since 2008 and an honor professor of Central South University, P. R. China, since 2009.

Yier Yan



received the B.S. degree of Applied Electronic Engineering from South Central University for Nationalities, Wuhan, Hubei, P. R. China, in 2004. He received both his M.S. degree of communication engineering and Ph.D. of electronic engineering from Chonbuk National University, Jeonju, Republic of Korea, in 2006 and 2010, respectively. He has been a professor in the Mechanical and electronic school of Guangzhou University, Guangzhou, P. R. China, since 2010. His research interests include information theory, signal processing, and OFDM in MIMO system.

Bu Shi Peng

received his B.S. degree of Electrical Engineering from Guizhou University, Guiyang, P. R. China, in 2009. He is currently affiliated with the Institute of Information & Communications as a researcher toward M. S. degree, in Chonbuk National University, Jeonju, Republic of Korea. And he has been doing research on wireless mobile communications. His main research interests are MIMO, OFDM, ECC, and network coding.

Gun Joon Hwang

received the B.S. degree from the Division of Information and Telecommunication Engineering, Howon University, Kunsan, Republic of Korea, in 1999. He received his M.S degree of Department of Information Science from Chonbuk National University, Jeonju, Republic of Korea in 2001. He completed his doctoral coursework in 2005, and he has been still doing research toward PhD degree. He is currently a chief of supporting center of Information and Communication, in Jeonju University, Jeonju, Republic of Korea. His interests areas include internet website design, and software design.