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On b-Locally Open Sets in Bitopological Spaces

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Abstract. In this article we introduce the notion of b-locally open sets, bLO^* sets, bLO^{**} sets in bitopological spaces and obtain several characterizations and some properties of these sets.

1. Introduction and preliminaries

In the recent past the notions of topological spaces have been generalized in many ways. Different types of open sets have been introduced and their properties have been investigated by many workers. The generalization of the open sets has attracted many workers to study different properties of the topological spaces. Andrijevic [1] introduced the notion of b-open sets in a topological space (X, τ).

The concept of bitopological spaces was introduced by Kelly [3]. A non-empty set X, equipped with two topologies τ_1 and τ_2 is called a bitopological space, denoted by (X, τ_1, τ_2). Later on several authors were attracted by the notion of bitopological space. Many notions of topological spaces were studied on considering bitopological space. Throughout the present paper, (X, τ_1, τ_2) denote a bitopological space.

Definition 1.1. Let $A \subset X$, then A is said to be b-open if $A \subset cl(intA) \cup int(clA)$, where $cl(A)$ and $int(A)$ denote the closure and interior of the set A respectively.

The notion of locally closed set in a topological space was introduced by Kuratowski and Sierpienski [4]. It is also found in Bourbaki [2]. Recently Nasef [5] have introduced and studies b-locally closed sets in topological space.

Definition 1.2. Let $A \subset X$, then A is said to be locally closed if $A = G \cap F$, where G is an open set in X and F is closed in X.

Definition 1.3. Let $A \subset X$, then A is said to be b-locally closed if $A = G \cap F$, where G is b-open set in X and F is b-closed set in X.

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In this paper we introduce the notions of locally open sets (in short LO sets), b-locally open sets (in short bLO sets), bLO^* sets, bLO^{**} sets in bitopological spaces.

The collection of all $(\tau_1, \tau_2) - LO$ (respectively $(\tau_1, \tau_2) - bLO$, $(\tau_1, \tau_2) - bLO^*$, $(\tau_1, \tau_2) - bLO^{**}$ sets of (X, τ_1, τ_2) will be denoted by $(\tau_1, \tau_2) - LO(X)$ (respectively $(\tau_1, \tau_2) - bLO(X)$, $(\tau_1, \tau_2) - bLO^*(X)$, $(\tau_1, \tau_2) - bLO^{**}(X)$).

2. b-locally open sets in bitopological space

Definition 2.1. A subset A of a space (X, τ_1, τ_2) is called (τ_1, τ_2)-locally open (in short (τ_1, τ_2)-LO) if $A = G \cup F$ where G is τ_1-closed and F is τ_2-open is (X, τ_1, τ_2).

Definition 2.2. A subset A of a space (X, τ_1, τ_2) is called (τ_1, τ_2)-b-locally open (in short (τ_1, τ_2)-bLO) if $A = G \cup F$, where G is τ_1-b-closed and F is τ_2-b-open in (X, τ_1, τ_2).

Definition 2.3. A subset A of a space (X, τ_1, τ_2) is called (τ_1, τ_2)-bLO^* if there exist a τ_1-b-closed set G and a τ_2-open set F of (X, τ_1, τ_2) such that $A = G \cup F$.

Definition 2.4. A subset A of a space (X, τ_1, τ_2) is called (τ_1, τ_2)-bLO^{**} if there exists a τ_1-closed set G and a τ_2-b-open set F of (X, τ_1, τ_2) such that $A = G \cup F$.

Theorem 2.1. *Let A be a subset of a space (X, τ_1, τ_2). Then if $A \in (\tau_1, \tau_2)$-$LO(X)$, then*

(*a*) $A \in (\tau_1, \tau_2)$-$bLO^*(X)$.

(*b*) $A \in (\tau_1, \tau_2)$-$bLO^{**}(X)$.

Proof. (a) Since $A \in (\tau_1, \tau_2)$-$LO(X)$, so there exist a τ_1-closed set G and a τ_2-open set F such that $A = G \cup F$. Since G is τ_1-closed, we have $int(clG) \subset G$ and $cl(intG) \subset G$. Therefore $int(clG) \cap cl(intG) \subset G$. Hence G is τ_1-b-closed. Thus we have $A = G \cup F$, where G is τ_1-b-closed and F is τ_2-open. Hence $A \in (\tau_1, \tau_2)$-$bLO^*(X)$.

(b) Let $A \in (\tau_1, \tau_2)$-$LO(X)$. Then we have $A = G \cup F$, where G is τ_1-closed and F is τ_2-open. Since F is τ_2-open , we have $F \subset int(clF)$ and $F \subset cl(intF)$. Therefore $F \subset cl(intF) \cup int(clF)$. Hence F is τ_2-b-open. Now we have $A = G \cup F$, where G is τ_1-closed and F is τ_2-b-open. Hence $A \in (\tau_1, \tau_2)$-$bLO^{**}(X)$.

This completes the proof. □

Remark 2.1. The converse of Theorem 2.1 is not necessarily true. It is clear from the following example.

Example 2.1. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$. Here, $\{c\} \in (\tau_1, \tau_2)$-$bLO^*(X)$ but $\{c\} \notin (\tau_1, \tau_2)$-$LO(X)$ and

$\{a, b\} \in (\tau_1, \tau_2)$-$bLO^{**}(X)$ but $\{a, b\} \notin (\tau_1, \tau_2)$-$LO(X)$.

Theorem 2.2. *Let A be a subset of the bitopological space (X, τ_1, τ_2). If $A \in (\tau_1, \tau_2)$-$bLO^*(X)$, then $A \in (\tau_1, \tau_2)$-$bLO(X)$.*

Proof. Let $A \in (\tau_1, \tau_2)$-$bLO^*(X)$. Then there exists a τ_1-b-closed set P and a τ_2-open set Q such that $A = P \cup Q$. Since Q is τ_2-open, we have $Q \subset int(clQ)$.

Further we have $Q \subset cl(intQ)$. Thus we have $Q \subset cl(intQ) \cup int(clQ)$. Hence Q is a τ_2-b-open set. Thus there exist a τ_1-b-closed set P and τ_2-b-open set Q such that $A = P \cup Q$.

Therefore $A \in (\tau_1, \tau_2)$-$bLO(X)$. □

Remark 2.2. The converse of Theorem 2.2 is not always true. It follows from the following example.

Example 2.2. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$. Here $\{a, b\} \in (\tau_1, \tau_2)$-$bLO(X)$ but $\{a, b\} \notin (\tau_1, \tau_2)$-$bLO^*(X)$.

Theorem 2.3. *Let A be a subset of a space (X, τ_1, τ_2). If $A \in (\tau_1, \tau_2)$-$bLO^{**}(X)$, then $A \in (\tau_1, \tau_2)$-$bLO(X)$.*

Proof. The proof is easy, so omitted. □

Remark 2.3. The converse of Theorem 2.3 is not always true. It follows from the following example.

Example 2.3. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, \{b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$. Here $\{c\} \in (\tau_1, \tau_2)$-$bLO(X)$ but $\{c\} \notin (\tau_1, \tau_2)$-$bLO^{**}(X)$.

Theorem 2.4. *Let P and Q be any two subsets of a space (X, τ_1, τ_2). If $P \in (\tau_1, \tau_2)$-$bLO(X)$ and Q is τ_1-b-closed and τ_2-b-open, then $P \cap Q \in (\tau_1, \tau_2)$-$bLO(X)$.*

Proof. Since $P \in (\tau_1, \tau_2)$-$bLO(X)$, then there exist a τ_1-b-closed set A and a τ_2-b-open set B such that $P = A \cup B$. We have $P \cap Q = (A \cup B) \cap Q = (A \cap Q) \cup (B \cap Q)$. Since Q is τ_1-b-closed, then $A \cap Q$ is τ_1-b-closed. Since Q is τ_2-b-open, then $B \cap Q$ is τ_2-b-open. Then there exist a τ_1-b-closed set $A \cap Q$ and a τ_2-b-open set $B \cap Q$ such that $P \cap Q = (A \cap Q) \cup (B \cap Q)$

Hence $P \cap Q \in (\tau_1, \tau_2)$-$bLO(X)$. □

Theorem 2.5. *Let $A \in (\tau_1, \tau_2)$-$bLO^*(X)$ and B be a τ_1-closed and τ_2-open subsets of (X, τ_1, τ_2), then $A \cap B \in (\tau_1, \tau_2)$-$bLO^*(X)$.*

Proof. Since $A \in (\tau_1, \tau_2)$-$bLO^*(X)$. Then there exist a τ_1-b-closed set P and a τ_2-open set Q such that $A = P \cup Q$. We have $A \cap B = (P \cup Q) \cap B = (P \cap B) \cup (Q \cap B)$. Since B is τ_1-closed, $P \cap B$ is τ_1-b-closed set.

Further B is τ_2-open, therefore $Q \cap B$ is τ_2-open. Thus there exist a τ_1-b-closed set $P \cap B$ and a τ_2-open set $Q \cap B$ such that $A \cap B = (P \cap B) \cup (Q \cap B)$. Hence $A \cap B \in (\tau_1, \tau_2)$-$bLO^*(X)$. □

Theorem 2.6. *Let $A \in (\tau_1, \tau_2)$-$bLO^{**}(X)$ and B is τ_1-closed and τ_2-open subsets of (X, τ_1, τ_2), then $A \cap B \in (\tau_1, \tau_2)$-$bLO^{**}(X)$.*

Proof. Since $A \in (\tau_1, \tau_2)$-$bLO^{**}(X)$, then there exist a τ_1-closed set P and a τ_2-b-open set Q such that $A = P \cup Q$. Clearly $A \cap B = (P \cup Q) \cap B = (P \cap B) \cup (Q \cap B)$. Since B is τ_1-closed, therefore $P \cap B$ is τ_1-closed.

Again B is τ_2-open, therefore $Q \cap B$ is τ_2-b-open. Then there exist a τ_1-closed set $P \cap B$ and a τ_2-b-open set $Q \cap B$ such that $A \cap B = (P \cap B) \cup (Q \cap B)$. Hence $A \cap B \in (\tau_1, \tau_2)$-$bLO^{**}(X)$. □

Theorem 2.7. *Let A be a subset of a space (X, τ_1, τ_2). Then $A \in (\tau_1, \tau_2)$-$bLO^*(X)$ if and only if $A = G \cup \tau_2$-$int(A)$ for some τ_1-b-closed set G.*

Proof. Let $A \in (\tau_1, \tau_2)$-$bLO^*(X)$. Then $A = G \cup F$, where G is τ_1-b-closed and F is τ_2-open set in (X, τ_1, τ_2). Since $G \subset A$ and τ_2-$int(A) \subset A$. We have

$$G \cup \tau_2 - int(A) \subset A. \tag{2.1}$$

Further $\tau_2 - int(A) \supset F$. Therefore

$$G \cup \tau_2 - int(A) \supset G \cup F = A. \tag{2.2}$$

From (2.1) and (2.2) we have $A = G \cup \tau_2$-$int(A)$.

Conversely, given that G is τ_1-b-closed. We have τ_2-$int(A)$ is τ_2-open. Thus there exist a τ_1-b-closed set G and a τ_2-open set τ_2-$int(A)$ in (X, τ_1, τ_2) such that $A = G \cup \tau_2$-$int(A)$. Hence $A \in (\tau_1, \tau_2)$-$bLO^*(X)$. □

Theorem 2.8. *Let A and B be any two subsets of the bitopological space (X, τ_1, τ_2). If $A \in (\tau_1, \tau_2)$-$bLO(X)$ and B is either τ_1-b-closed or τ_2-b-open, then $A \cup B \in (\tau_1, \tau_2)$-$bLO(X)$.*

Proof. Since $A \in (\tau_1, \tau_2)$-$bLO(X)$, then there exist a τ_1-b-closed set P and a τ_2-b-open set Q such that $A = P \cup Q$. We have $A \cup B = (P \cup Q) \cup B = (P \cup B) \cup Q$. If B is τ_1-b-closed, then $P \cup B$ is also τ_1-b-closed. Hence $A \cup B \in (\tau_1, \tau_2)$-$bLO(X)$.

Let B be τ_2-b-open, then $A \cup B = (P \cup Q) \cup B = P \cup (Q \cup B)$, where $Q \cup B$ is τ_2-b-open. Thus $A \cup B \in (\tau_1, \tau_2)$-$bLO(X)$. □

Theorem 2.9. *If $A \in (\tau_1, \tau_2)$-$bLO^*(X)$ and B is either τ_1-closed or τ_2-open subset of (X, τ_1, τ_2) then $A \cup B \in (\tau_1, \tau_2)$-$bLO^*(X)$.* □

Proof. Since $A \in (\tau_1, \tau_2)$-$bLO^*(X)$, then $A = P \cup Q$, where P is τ_1-b-closed set and Q is τ_2-open set of (X, τ_1, τ_2), Now $A \cup B = (P \cup Q) \cup B = (P \cup B) \cup Q$. Let B be τ_1-closed, then $P \cup B$ is also τ_1-b-closed, where P is τ_1-b-closed set. Hence $A \cup B \in (\tau_1, \tau_2)$-$bLO^*(X)$. If B is τ_2-open, then $Q \cup B$ is τ_2-open. Now $A \cup B = (P \cup Q) \cup B = P \cup (Q \cup B)$. Thus $A \cup B \in (\tau_1, \tau_2)$-$bLO^*(X)$. □

Theorem 2.10. *If $A \in (\tau_1, \tau_2)$-$bLO^{**}(X)$ and B is either τ_1-closed or τ_2-open subset of (X, τ_1, τ_2) then $A \cup B \in (\tau_1, \tau_2)$-$bLO^{**}(X)$.*

Proof. The proof is easy, so omitted. □

Theorem 2.11. *If $A, B \in (\tau_1, \tau_2)$-$bLO(X)$, then $A \cup B \in (\tau_1, \tau_2)$-$bLO(X)$.*

Proof. Let $A, B \in (\tau_1, \tau_2)$-$bLO(X)$. Then there exist τ_1-b-closed sets P, R and

τ_2-b-open sets Q, S such that $A = P \cup Q$ and $B = R \cup S$. We have $A \cup B = (P \cup Q) \cup (R \cup S) = (P \cup R) \cup (Q \cup S)$, where $P \cup R$ is τ_1-b-closed set and $Q \cup S$ is τ_2-b-open. Hence $A \cup B \in (\tau_1, \tau_2)$-$bLO(X)$. □

Theorem 2.12. *If* $A, B \in (\tau_1, \tau_2)$-$bLO^*(X)$, *then* $A \cup B \in (\tau_1, \tau_2)$-$bLO^*(X)$.

Proof. Since $A, B \in (\tau_1, \tau_2)$-$bLO^*(X)$, then by Theorem 2.7 there exist τ_1-b-closed sets P and Q such that $A = P \cup \tau_2$-$int(A)$ and $B = Q \cup \tau_2$-$int(B)$. We have

$$\begin{aligned} A \cup B &= [P \cup \tau_2 - int(A)] \cup [Q \cup \tau_2 - int(B)] \\ &= (P \cup Q) \cup (\tau_2 - int(A) \cup \tau_2 - int(B)), \end{aligned}$$

where $P \cup Q$ is τ_1-b-closed and τ_2-$int(A) \cup \tau_2$-$int(B)$ is τ_2-open set. Hence $A \cup B \in (\tau_1, \tau_2)$-$bLO^*(X)$.

Theorem 2.13. *If* $A, B \in (\tau_1, \tau_2)$-$bLO^{**}(X)$, *then* $A \cup B \in (\tau_1, \tau_2)$-$bLO^{**}(X)$.

Proof. Easy, so omitted. □

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