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Resonance of Continued Fractions Related to ${}_2\psi_2$ Basic Bilateral Hypergeometric Series

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Abstract. In this paper, making use of transformation due to S. N. Singh [21], an attempt has been made to establish certain results involving basic bilateral hypergeometric series and continued fractions.

1. Introduction

Continued fractions offer a useful means of expressing numbers and functions. The indian mathematician Arya Bhatt(475-550AD) used a continued fraction to solve a linear equation[15]. Continued fraction were used only in specific examples for more than one thousand years. In the begining of the 20th century, the theory had advanced due to the Indian genius S. Ramanujan[17, 18]. Generalized hypergeometric series both ordinary and basic have been a very significant tool in the derivation of continued fraction representation. In the chapter 16 of Ramanujan's second notebook, first seventeen sections are devoted mainly to basic hypergeometric functions and chapter 12 of his second notebook is entirely devoted to the study of continued fractions. Ramanujan has made significant contribution to the theory of continued fractions making use of basic hypergeometric function as significant tool and provided a new dimension to this interesting branch of analysis. During the end of the 20th century continued fraction had been center of attraction for the mathematicians working in the field of q-series namely R. P. Agarwal [1], George E. Andrews [2], Bruce C. Berndt[7], S. Bhargava and C. S., Adiga[9], R. Y. Denis [10, 11, 12, 13], C. S. Adiga and D. D. Somashekara [3, 4], S. N. Singh [19, 20], Pankaj Srivastava [22], Maheshwar Pathak and Pankaj Srivastava [16], Bhaskar Srivastava [23], L. C. Zhang[24] etc. To investigate continued fraction representation for the ratio of two hypergeometric function ${}_2F_1$, basic hypergeometric series ${}_2\phi_1$, and ${}_3\phi_2$, several mathematicians namely R. P. Agarwal [1], R. Y. Denis [10, 11, 12, 13], S. N. Singh [19, 20], N. A. Bhagirathi [5, 6], S. Bhargava, C. S. Adiga

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and D. D. Somashekara [8] established a number of results. In the present paper, transformation due to S. N. Singh [21] has been used to established new continued fraction representation for the ratio of basic bilateral hypergeometric series ${}_2\psi_2$. S. N. Singh [21] establish the following general transformation for basic bilateral series into basic series:

$$ (1.1) \qquad {}_{r+3}\psi_{r+3}\left[\begin{matrix} \frac{a}{b}, & cq, & dq, & \frac{b_1}{b}q^{m_1}, & ...., & \frac{b_r}{b}q^{m_r} \\ & & & & & & ;q; \quad z \\ \frac{q}{b}, & c, & d, & \frac{b_1}{b}, & ...., & \frac{b_r}{b} \end{matrix}\right] = $$

$$ \frac{\left(q, \frac{bq}{az}, \frac{az}{b}, \frac{q}{a}; q\right)_\infty}{\left(\frac{q}{b}, \frac{q}{az}, az, \frac{bq}{a}; q\right)_\infty} \times \frac{(1-bc)(1-bd)(b_1;q)_{m_1}.......(b_r;q)_{m_r}}{(1-c)(1-d)(b_1/b;q)_{m_1}.......(b_r/b;q)_{m_r}} $$

$$ \times\, {}_{r+3}\phi_{r+2}\left[\begin{matrix} a, & bcq, & bdq, & b_1q^{m_1}, & ...., & b_rq^{m_r} \\ & & & & & & ;q; \quad z \\ bc, & bd, & b_1, & ...., & b_r \end{matrix}\right] $$

2. Definitions and notations

A basic hypergeometric series is defined as:

$$ (2.1) \qquad {}_r\phi_s\left[\begin{matrix} a_1, & ...., & a_r \\ & & & ;q;z \\ b_1, & ...., & b_s \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n(a_2)_n.......(a_r)_n}{(b_1)_n(b_2)_n.......(b_s)_n}\frac{z^n}{(q)_n} $$

subject to appropriate convergence series conditions, where $|q| < 1$.

$$ (2.2) \qquad (a)_n = (a;q)_n = \begin{cases} (1-a)(1-aq).......(1-aq^{n-1}), & n \geq 1 \\ 1, & n = 0 \end{cases} $$

A basic bilateral hypergeometric series is defined as:

$$ (2.3) \qquad {}_r\psi_r\left[\begin{matrix} a_1, & ...., & a_r \\ & & & ;q;z \\ b_1, & ...., & b_r \end{matrix}\right] = \sum_{n=-\infty}^{\infty} \frac{(a_1,a_2,...,a_r;q)_n}{(b_1,b_2,...,b_r;q)_n} z^n $$

where $|b_1b_2...b_r/a_1a_2...a_r| < |z| < 1$ for convergence. An expression of the form

$$ (2.4) \qquad \frac{a_1}{b_1+}\,\frac{a_2}{b_2+}\,\frac{a_3}{b_3+.....}\,\frac{a_n}{b_n}. $$

is said to be a terminating continued fraction and as $n \to \infty$, it is said to be an infinite continued fraction.

We shall make use of following results in our analysis:

$$\frac{{}_2\phi_1\left[\begin{matrix}\alpha, & \beta & \\ & & ;xq \\ \gamma & & \end{matrix}\right]}{{}_2\phi_1\left[\begin{matrix}\alpha, & \beta & \\ & & ;x \\ \gamma & & \end{matrix}\right]} = \frac{1}{1+}\ \frac{(1-\alpha)(1-\beta)x}{(1-x)+}\ \frac{\alpha\beta xq-\gamma}{1+}\ \frac{(1-\alpha q)(1-\beta q)x}{(1-x)+} \tag{2.5}$$

$$+\frac{\alpha\beta xq^3-\gamma q}{1+}\ \frac{(1-\alpha q^2)(1-\beta q^2)x}{(1-x)+\ldots\ldots.}$$

[Denis, [10]]

$$\frac{{}_2\phi_1\left[\ a,\quad b\quad ;cq;\quad x\ \right]}{{}_2\phi_1\left[\ a,\quad b\quad ;c;\quad x\ \right]} = \frac{1}{1+}\ \frac{xc(1-a)(1-b)/(1-c)(1-cq)}{(1-abx/cq)+} \tag{2.6}$$

$$+\frac{x(a-cq)(b-cq)/cq(1-cq)(1-cq^2)}{1+}\ \frac{cxq^2(1-aq)(1-bq)/(1-cq^2)(1-cq^3)}{(1-abx/cq)+\ldots\ldots}$$

[R. P. Agarwal, [1]]

$$\frac{{}_2\phi_1\left[\ a,\quad bq\quad ;c;\quad x\ \right]}{{}_2\phi_1\left[\ a,\quad b\quad ;c;\quad xq\ \right]} = 1+\frac{x(1-a)}{(1-x)(1-b)+}\ \frac{(b-c)/(1-axq)}{1+} \tag{2.7}$$

$$+\frac{x(1-aq)}{(1-x)(1-b)+}\ \frac{(b-cq)(1-axq^2)}{1+\ldots\ldots}$$

[R. P. Agarwal, [1]]

$$\frac{{}_2\phi_1\left[\ a,\quad b\quad ;c;\quad xq\ \right]}{{}_2\phi_1\left[\ aq,\quad bq\quad ;cq;\quad x\ \right]} = \frac{(1-x)}{(1-c)-}\ \frac{c(1-abxq/c)}{(1-c)+}\ \frac{x(1-aq)(1-bq)}{(1-x)/(1-c)-} \tag{2.8}$$

$$-\frac{cq(1-abxq^2/c)}{(1-c)+}\ \frac{x(1-aq^2)(1-bq^2)}{(1-x)/(1-c)-}\ \frac{cq^2(1-abxq^3/c)}{(1-c)+\ldots\ldots}$$

[R. P. Agarwal, [1]]

$$\frac{{}_2\phi_1\left[\ \alpha,\quad \beta\quad ;\gamma;\quad z\ \right]}{{}_2\phi_1\left[\ \alpha,\quad \beta q\quad ;\gamma;\quad z\ \right]} = 1-\frac{z\beta(1-\alpha)}{(1-\gamma/\beta q)+}\ \frac{\gamma(1-\beta q)(1-\alpha\beta zq/\gamma)/\beta q}{1-} \tag{2.9}$$

$$-\frac{z\beta q(1-\alpha q)}{(1-\gamma/\beta q)+}\ \frac{\gamma(1-\beta q^2)(1-\alpha\beta zq^2/\gamma)/\beta q}{1-}\ \frac{z\beta q^2(1-\alpha q^2)}{(1-\gamma/\beta q)+\ldots\ldots.}$$

[S. N. Singh, [19]]

$$\frac{{}_2\phi_1\left[\ \alpha,\quad \beta\quad ;\gamma;\quad z\ \right]}{{}_2\phi_1\left[\ \alpha,\quad \beta\quad ;\gamma q;\quad z\ \right]} = 1+\frac{z\gamma(1-\alpha)(1-\beta)/(1-\gamma)(1-\gamma q)}{(1-\alpha\beta z/\gamma q)+} \tag{2.10}$$

$$+\frac{\mu(\alpha-\gamma q)/(\beta-\gamma q)/(1-\gamma q)(1-\gamma q^2)}{1+......}$$

[S. N. Singh, [19]]

where $\mu = \dfrac{z}{\gamma q}$

$$\frac{{}_2\phi_1\left[\ \alpha,\quad \beta q\quad ;\gamma;\quad x\ \right]}{{}_2\phi_1\left[\ \alpha,\quad \beta\quad ;\gamma;\quad xq\ \right]} = 1+\frac{x(1-\alpha)}{(1-x)(1-\beta)+}\ \frac{(\beta-\gamma)/(1-\alpha xq)}{1+} \tag{2.11}$$

$$+\frac{x(1-\alpha q)}{(1-x)(1-\beta)+}\ \frac{(\beta-\gamma q)/(1-\alpha xq^2)}{1+}\ \frac{x(1-\alpha q^2)}{(1-x)(1-\beta)+......}$$

[S. N. Singh, [20]]

$$\frac{(1-cq^i){}_2\phi_1\left[\begin{matrix} aq^i, & bq^i & \\ & & ;x \\ cq^i & & \end{matrix}\right]}{(1-c){}_2\phi_1\left[\begin{matrix} aq^{i+1}, & bq^{i+1} & \\ & & ;x/q \\ cq^{i+1} & & \end{matrix}\right]} = \frac{(1-x/q)}{(1-c)} - \frac{(1-abxq^i/c)cq^i}{(1-c)+} \tag{2.12}$$

$$+\frac{x(1-aq^{i+1})(1-bq^{i+1})/b}{(1-x/q)(1-c)-}\ \frac{(1-abxq^{i+1}/c)cq^{i+1}}{(1-c)+}\ \frac{x(1-aq^{i+2})(1-bq^{i+2})/q}{(1-x/q)/(1-c)-......}$$

[Bhagirathi, [5]]

$$\frac{(1-c){}_2\phi_1\left[\begin{matrix} aq^{i+1}, & bq^{i+1} & \\ & & ;x/q \\ cq^{i+1} & & \end{matrix}\right]}{{}_2\phi_1\left[\begin{matrix} aq^{i+1}, & bq^{i+1} & \\ & & ;x \\ cq^{i+1} & & \end{matrix}\right]} = \tag{2.13}$$

$$(1-c)+\frac{(1-aq^{i+1})(1-bq^{i+1})/q}{(1-x/q)(1-c)-}\ \frac{(1-abxq^{i+1}/c)cq^{i+1}}{(1-c)+}$$

$$+\frac{x(1-aq^{i+2})(1-bq^{i+2})/q}{(1-x/q)/(1-c)-}\ \frac{(1-abxq^{i+2}/c)cq^{i+2}}{(1-c)+}\ \frac{x(1-aq^{i+3})(1-bq^{i+3})/q}{(1-x/q)/(1-c)-......}$$

[Bhagirathi, [5]]

$$\frac{\frac{(1-cq^i)}{(1-c)}{}_2\phi_1\left[\begin{matrix} aq^i, & bq^i & \\ & & ;x \\ cq^i & & \end{matrix}\right]}{{}_2\phi_1\left[\begin{matrix} aq^{i+1}, & bq^i & \\ & & ;x/q \\ cq^{i+1} & & \end{matrix}\right]} = \frac{(1-c/b)}{(1-c)+}\ \frac{c(1-bq^i)(1-abxq^i/c)/b}{(1-c)-} \tag{2.14}$$

$$-\frac{(1-aq^{i+1})xbq^i}{(1-c/b)/(1-c)+}\ \frac{c(1-bq^{i+1})(1-abxq^{i+1}/c)/b}{(1-c)-}\ \frac{(1-aq^{i+2})xbq^{i+1}}{(1-c/b)/(1-c)+\ldots\ldots}$$

[Bhagirathi, [5]]

3. Main results

The following results are to be established in this section:

$$\frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}q\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]}=\frac{b}{1+}\ \frac{(1-a)(1-b_1q^{m_1})z}{(1-z)+}\ \frac{b_1(azq^{m_1+1}-1)}{1+}\tag{3.1}$$

$$+\frac{(1-aq)(1-b_1q^{m_1+1})z}{(1-z)+}\ \frac{b_1q(zaq^{m_1+3}-1)}{1+}\ \frac{(1-aq^2)(1-b_1q^{m_1+2})z}{(1-z)+\ldots..}$$

$$\frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1+1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}q\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]}=\tag{3.2}$$

$$\frac{(b-b_1)/b(1-b_1)}{1+}\frac{zb_1(1-a)(1-b_1q^{m_1})/(1-b_1)(1-b_1q)}{(1-azq^{m_1-1})+}\frac{z(a-b_1q)(q^{m_1-1}-1)/(1-b_1q)(1-b_1q^2)}{1+}$$

$$+\frac{b_1zq^2(1-aq)(1-b_1q^{m_1+1}/b)/(1-b_1q^2)(1-b_1q^3)}{(1-azq^{m_1-1})+\ldots\ldots}$$

$$\frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1+1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}q\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & zq \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]}=\frac{(b-b_1)}{b^2(1-b_1)}\Bigg[1+\frac{z(1-a)}{(1-z)(1-b_1q^{m_1})+}\tag{3.3}$$

$$+\frac{b_1(q^{m_1}-1)/(1-azq)}{1+}\ \frac{z(1-aq)}{(1-z)(1-b_1q^{m_1})+}\ \frac{b_1q(q^{m_1-1}-1)(1-azq^2)}{1+\ldots..}\Bigg]$$

$$(3.4)\qquad \frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & zq \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{aq}{b}, & \frac{b_1}{b}q^{m_1+1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}q\end{matrix}\right]} =$$

$$\frac{b(1-b_1)(a-b)}{(b-b_1)(a-1)}\Bigg[\frac{(1-z)}{(1-b_1)-}\ \frac{b_1(1-azq^{m_1+1})}{(1-b_1)+}\ \frac{z(1-aq)(1-b_1q^{m_1+1})}{(1-z)/(1-b_1)-}$$
$$-\frac{b_1q(1-azq^{m_1+2})}{(1-b_1)+}\ \frac{z(1-aq^2)(1-b_1q^{m_1+2})}{(1-z)/(1-b_1)-}\ \frac{b_1q^2(1-azq^{m_1+3})}{(1-b_1)+.....}\Bigg].$$

$$(3.5)\qquad \frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1+1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]} =$$

$$1-\frac{zb_1q^{m_1}(1-a)}{(1-q^{-m_1-1})+}\ \frac{b_1(1-b_1q^{m_1+1})(1-azq^{m_1+1})/b_1q^{m_1+1}}{1-}$$
$$-\frac{zb_1q^{m_1+1}(1-aq)}{(1-q^{-m_1-1})+}\ \frac{b_1(1-b_1q^{m_1+2})(1-zab_1q^{m_1+1})/b_1q^{m_1+1}}{1-.....}.$$

$$(3.6)\qquad \frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1+1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}q\end{matrix}\right]} =$$

$$\frac{b(1-b_1)}{(b-b_1)}\Bigg[1+\frac{zb_1(1-a)(1-b_1q^{m_1})/(1-b_1)(1-b_1q)}{(1-azq^{m_1-1})+}$$
$$+\frac{z/b_1q\times(a-b_1q)/[b_1(q^{m_1}-q)/(1-b_1q)(1-bq^2)]}{1+.....}\Bigg].$$

$$(3.7)\qquad \frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1+1} \\ & & ;q; & z \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; & zq \\ \frac{q}{b}, & \frac{b_1}{b}\end{matrix}\right]} =$$

$$\frac{1}{b}\Bigg[1+\frac{z(1-a)}{(1-z)(1-b_1q^{m_1})+}\ \frac{b_1(q^{m_1}-1)/(1-azq)}{1+}$$

$$+\frac{z(1-aq)}{(1-z)(1-bq^{m_1})+}\ \frac{b_1q(q^{m_1-1}-1)(1-azq^2)}{1+}\ \frac{z(1-aq^2)}{(1-z)(1-b_1q^{m_1})+\ldots..}\Bigg].$$

$$(3.8)\qquad \frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}q^i, & \frac{b_1}{b}q^{m_1+i} \\ & & ;q;\quad z \\ \frac{q}{b}, & \frac{b_1}{b}q^i\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}q^{i+1}, & \frac{b_1}{b}q^{m_1+i+1} \\ & & ;q;\quad z/q \\ \frac{q}{b}, & \frac{b_1}{b}q^{i+1}\end{matrix}\right]}=\frac{b(aq^i-b)(1-b_1q^i)(1-z/q)}{(aq^i-1)(b-b_1q^i)(1-b_1)}$$

$$-\frac{(1-azq^{m_1+i})b_1q^i}{(1-b_1)+}\ \frac{z(1-aq^{i+1})(1-b_1q^{m_1+i+1})/b_1q^{m_1}}{(1-z/q)/(1-b_1)-}$$

$$-\frac{(1-azq^{m_1+i+1})b_1q^{i+1}}{(1-b_1)+}\ \frac{z(1-aq^{i+2})(1-b_1q^{m_1+i+2})/q}{(1-z/q)/(1-b_1)-\ldots..}$$

$$(3.9)\qquad \frac{{}_2\psi_2\left[\begin{matrix}aq^{i+1}, & \frac{b_1}{b}q^{m_1+i+1} \\ & & ;q;\quad z/q \\ \frac{q}{b}, & \frac{b_1}{b}q^{i+1}\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}q^{i+1}, & \frac{b_1}{b}q^{m_1+i+1} \\ & & ;q;\quad z \\ \frac{q}{b}, & \frac{b_1}{b}q^{i+1}\end{matrix}\right]}=\frac{1}{b(1-b_1)}\Bigg[(1-b_1)+$$

$$\frac{(1-aq^{i+1})(1-b_1q^{m_1+i+1})/q}{(1-z/q)/(1-b_1)-}\ \frac{(1-azq^{m_1+i+1})b_1q^{i+1}}{(1-b_1)+}\ \frac{z(1-aq^{i+2})(1-b_1q^{m_1+i+2})/q}{(1-z/q)/(1-b_1)-}$$

$$-\frac{(1-azq^{m_1+i+2})b_1q^{i+2}}{(1-b_1)+}\ \frac{z(1-aq^{i+3})(1-b_1q^{m_1+i+3})/q}{(1-z/q)/(1-b_1)-\ldots..}\Bigg].$$

$$(3.10)\qquad \frac{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}q^i, & \frac{b_1}{b}q^{m_1+i} \\ & & ;q;\quad z \\ \frac{q}{b}, & \frac{b_1}{b}q^i\end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix}\frac{a}{b}q^{i+1}, & \frac{b_1}{b}q^{m_1+i} \\ & & ;q;\quad z/q \\ \frac{q}{b}, & \frac{b_1}{b}q^{i+1}\end{matrix}\right]}=$$

$$\frac{b(1-b_1)(aq^i-b)}{(b-b_1q^i)(aq^i-1)}\Bigg[\frac{(1-1/q^{m_1})}{(1-b_1)+}\frac{b_1(1-b_1q^{m_1+i})(1-azq^{m_1+i})/b_1q^{m_1}}{(1-b_1)-}\frac{(1-aq^{i+1})zb_1q^{m_1+i}}{(1-1/q^{m_1})/(1-b_1)+}$$

$$+\frac{b_1(1-b_1q^{m_1+i+1})(1-azq^{m_1+i+1})/b_1q^{m_1}}{(1-b_1)-}\ \frac{(1-aq^{i+2})zb_1q^{m_1+i+1}}{(1-1/q^{m_1})/(1-b_1)+\ldots..}\Bigg].$$

for $i = 0, 1, 2, 3, ...$

Proof of (3.1)-(3.10).
As an illustration, we shall prove (3.1).
Taking c=d=0 and r=1 in (1.1), we have

$$ {}_2\psi_2 \left[\begin{matrix} \frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; \quad z \\ \frac{q}{b}, & \frac{b_1}{b} \end{matrix} \right] = \frac{(b_1;q)_{m_1}}{(b_1/b;q)_{m_1}} \tag{3.11} $$

$$ \times \frac{\left(q, \frac{bq}{az}, \frac{az}{b}, \frac{q}{a}; q\right)_\infty}{\left(\frac{q}{b}, \frac{q}{az}, az, \frac{bq}{a}; q\right)_\infty} {}_2\phi_1 \left[\begin{matrix} a, & b_1 q^{m_1} \\ & & ;q; \quad z \\ b_1, \end{matrix} \right] $$

Now, replacing z by zq in (3.11) and finally taking the ratio of these two, we have

$$ \frac{{}_2\psi_2 \left[\begin{matrix} \frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; \quad zq \\ \frac{q}{b}, & \frac{b_1}{b} \end{matrix} \right]}{{}_2\psi_2 \left[\begin{matrix} \frac{a}{b}, & \frac{b_1}{b}q^{m_1} \\ & & ;q; \quad z \\ \frac{q}{b}, & \frac{b_1}{b} \end{matrix} \right]} = b \frac{{}_2\phi_1 \left[\begin{matrix} a, & b_1 q^{m_1} \\ & & ;q; \quad zq \\ b_1 \end{matrix} \right]}{{}_2\phi_1 \left[\begin{matrix} a, & b_1 q^{m_1} \\ & & ;q; \quad z \\ b_1 \end{matrix} \right]} $$

Now, using the result due to Denis[10], (3.1) can be obtained easily.

Proceeding in the same way and using the results due to R. P. Agarwal [[1], (3.23), (3.21), (3.18) pages[69-71]], S. N. Singh [19, 20] and N. Bhagirathi [5], one can easily establish the results (3.2) to (3.10), respectively. □

Note: It is important to mention that a number of results for the basic hypergeometric series ${}_2\phi_1$ can be deduced from the results established in §3.

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