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Related Fixed Point Theorem for Six Mappings on Three Fuzzy Metric Spaces

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Abstract. Related fixed point theorems on two or three metric spaces have been proved in different ways.However, so for the related fixed point theorem on fuzzy metric spaces have not been proved. Sharma, Deshpande and Thakur were the first who have established related fixed point theorem for four mappings on two complete fuzzy metric spaces. Their work was maiden in this line. In this paper we obtain a related fixed point theorem for six mappings on three complete fuzzy metric spaces. Of course this is a new result on this line.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [19] in 1965. Since then to use this concept in topology and analysis many authors have expansively developed the theory of fuzzy sets and applications. Especially, Deng [3], Erceg [4], Kaleva and Seikkala [12], Kramosil and Michalek [13] have introduced the concept of fuzzy metric spaces in different ways.

Related fixed point theorems on two or three metric spaces was proved by Fisher [5], [6], Nung [15], Popa [16], Jain, Sahu and Fisher [10], Jain, Shrivastava and Fisher [11], Cho, Kang and Kim [2], Fisher and Murthy [7] and many others.
Sharma, Deshpande and Thakur [18] established related fixed point theorem for four mappings on two complete fuzzy metric spaces. In this paper, we improve the results of Sharma, Deshpande and Thakur [18] and prove a related fixed point theorem for six mappings on three complete fuzzy metric spaces.

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2. Prelimiaries

Definition 2.1([17]). A binary operation $*$: $[0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if $\{[0,1], *\}$ is an abelian topological monoid with unit 1 such that $a * b \le c * d$ whenever $a \le c$ and $b \le d$, $a, b, c, d \in [0,1]$.

Examples of t-norm are $a * b = ab$ and $a * b = min\{a, b\}$.

Definition 2.2([13]). The triplet $(X, M, *)$ is a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$,
(FM-1) $M(x, y, 0) = 0$,
(FM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(FM-5) $M(x, y, \cdot) : [0,1] \to [0,1]$ is left continuous,
(FM-6) $\lim_{t\to\infty} M(x, y, t) = 1$for all x, y in X.

In what follows, $(X, M, *)$ will denote a fuzzy metric space. Note that $M(x, y, t)$ can be thought as the degree of nearness between x and y with respect to t. We identify $x = y$ with $M(x, y, t) = 1$ for all $t > 0$ and $M(x, y, t) = 0$ with ∞ and we can find some topological properties and examples of fuzzy metric spaces in George and Veeramani [8].

In the following example, we know that every metric induces a fuzzy metric.

Example 2.1([8]). Let (X, d) be a metric space. Define $a * b = ab$ or $a * b = min\{a, b\}$ and for all $x, y \in X$ and $t > 0$, let

$$M(x,\ y,\ t) \ = \ \frac{t}{t + d(x,\ y)}$$

Then $(X, M, *)$ is a fuzzy metric space. We call the fuzzy metric M induced by the metric d the standard fuzzy metric.

Definition 2.3([9]). Let $(X, M, *)$ is a fuzzy metric space:

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$), if

$$\lim_{n\to\infty} M(x_n, x, t) = 1,$$

for all $t > 0$.

(ii) A sequence $\{x_n\}$ in X called a Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1,$$

for all $t > 0$ and $p > 0$.

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Lemma 2.1([14]). *Let $\{y_n\}$ be a sequence in a fuzzy metric space $(X, M, *)$. If there exists a number $k \in (0,1)$ such that*

$$(1.a) \qquad M(y_{n+2},\ y_{n+1}, kt) \geq M(y_{n+1},\ y_n, t)$$

for all $t > 0$ and $n = 1, 2, \ldots..$ then $\{y_n\}$ is a Cauchy sequence in X.

Proof. For $t > 0$ and $k \in (0,1)$, we have

$$M(y_2, y_3, kt) \geq M(y_1, y_2, t) \geq M(y_0, y_1, t/k)$$

or

$$M(y_2, y_3, t) \geq M(y_0, y_1, t/k^2).$$

By simple induction with the condition (1.a), we have for all $t > 0$ and $n = 1, 2, \ldots$

$$(1.b) \qquad M(y_{n+1}, y_{n+2}, t) \geq M(y_1, y_2, t/k^n)$$

Thus by (1.b) and (FM-4), for any positive integer p and real number $t > 0$, we have

$$\begin{aligned} M(y_n, y_{n+p}, t) &\geq M(y_n, y_{n+1}, t/p) * \overset{p-times}{\ldots\ldots} * \ M(y_{n+p-1}, y_{n+p}, t/p) \\ &\geq M(y_1, y_2, t/pk^{n-1}) * \overset{p-times}{\ldots\ldots} * \ M(y_1, y_2, t/pk^{n+p-2}). \end{aligned}$$

Therefore, by (FM-6), we have

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * \overset{p-times}{\ldots\ldots} * 1 \geq 1,$$

which implies that $\{y_n\}$ is a Cauchy sequence in X. This completes the proof. □

Lemma 2.2([14]). *If for all $x,\ y \in X,\ t > 0$ and for a number $k \in (0,1)$*

$$M(x, y, kt) \geq M(x, y, t)$$

then $x = y$.

Sharma, Deshpande and Thakur [18] established the following related fixed point theorem for four mappings on two complete fuzzy metric spaces.

Theorem 2.1. *Let $(X, M_1, *)$ and $(Y, M_2, *)$ be two complete fuzzy metric spaces. Let A, B be mappings from X into Y and let S, T be mappings from Y into X satisfying the inequalities:*

$$(2.1) \qquad \begin{aligned} M_1(SAx, TBx', kt) \geq &M_1(x, x', t) * M_1(x, SAx, t) * M_1(x', TBx', t) \\ &* M_1(SAx, TBx', t), \end{aligned}$$

$$(2.2) \qquad \begin{aligned} M_2(BSy, ATy', kt) \geq &M_2(y, y', t) * M_2(y, BSy, t) * M_2(y', ATy', t) \\ &* M_2(BSy, ATy', t), \end{aligned}$$

for all x, $x\prime$ in X and y, $y\prime$ in Y. If one of the mappings A, B, S, T is continuous, then SA and TB have a unique common fixed point z in X and BS and AT have a unique common fixed point w in Y. Further $Az = Bz = w$ and $Sw = Tw = z$.

Now, we prove a related fixed point theorem for six mappings on three complete fuzzy metric spaces.

3. Main result

Theorem 3.1. *Let $(X, M_1, *)$,$(Y, M_2, *)$ and $(Z, M_3, *)$ be three complete fuzzy metric spaces with $t*t \geq t$ for all $t \in [0, 1]$. Let A, B be mappings from X into Y, S, T be mappings from Y into Z and P, Q be mappings from Z into X satisfying the inequalities:*

$$(3.1)\quad M_1(PSAx, QTBx\prime, kt) \geq M_1(x, x\prime, t) * M_1(x, PSAx, t) * M_1(x\prime, QTBx\prime, t) * M_1(PSAx, QTBx\prime, t)$$

$$(3.2)\quad M_2(APSy, BQTy\prime, kt) \geq M_2(y, y\prime, t) * M_2(y, APSy, t) * M_2(y\prime, BQTy\prime, t) * M_2(APSy, BQTy\prime, t)$$

$$(3.3)\quad M_3(SAPz, TBQz\prime, kt) \geq M_3(z, z\prime, t) * M_3(z, SAPz, t) * M_3(z\prime, TBQz\prime, t) * M_3(SAPz, TBQz\prime, t)$$

for all x, $x\prime$ in X, y, $y\prime$ in Y and z, $z\prime$ in Z. If one of the mappings A, B, S, T, P or Q is continuous, then PSA and QTB have a unique common fixed point u in X, APS and BQT have a unique common fixed point v in Y and SAP and TBQ have a unique common fixed point w in Z. Further $Au = Bu = v$, $Sv = Tv = w$ and $Pw = Qw = u$.

Proof. Let $x = x_o$ be an arbitrary point in X and define sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in X, Y and Z respectively as follows:

Choose a point $z_1 = Sy_1$, a point $y_1 = Ax_0$, a point $x_1 = Pz_1$, a point $z_2 = Ty_2$, a point $y_2 = Bx_1$ and a point $x_2 = Qz_2$. In general, having chosen x_{2n-2}, in X, choose a point $y_{2n-1} = Ax_{2n-2}$, a point $y_{2n} = Bx_{2n-1}$, a point $z_{2n-1} = Sy_{2n-1}$, a point $z_{2n} = Ty_{2n}$, a point $x_{2n-1} = Pz_{2n-1}$ and a point $x_{2n} = Qz_{2n}$ for all $n = 1, 2, ...$

Applying inequality (3.1), we have

$$(3.4)\quad \begin{aligned} &M_1(x_{2n+1}, x_{2n}, kt) \\ &= M_1(PSAx_{2n}, QTBx_{2n-1}, kt) \\ &\geq M_1(x_{2n}, x_{2n-1}, t) * M_1(x_{2n}, PSAx_{2n}, t) * M_1(x_{2n-1}, QTBx_{2n-1}, t) \\ &\quad * M_1(PSAx_{2n}, QTBx_{2n-1}, t) \\ &= M_1(x_{2n}, x_{2n-1}, t) * M_1(x_{2n}, x_{2n+1}, t) \\ &\quad * M_1(x_{2n-1}, x_{2n}, t) * M_1(x_{2n+1}, x_{2n}, t) \\ &\geq M_1(x_{2n}, x_{2n-1}, t) * M_1(x_{2n}, x_{2n+1}, t) \end{aligned}$$

Similarly, we have

(3.5) $$M_1(x_{2n+2}, x_{2n+1}, kt) \geq M_1(x_{2n+1}, x_{2n}, t) * M_1(x_{2n+1}, x_{2n+2}, t).$$

Thus from (3.4) and (3.5), it follows that

$$M_1(x_{n+1}, x_{n+2}, kt) \quad \geq M_1(x_n, x_{n+1}, t) * M_1(x_{n+1}, x_{n+2}, t)$$

for all $n = 1, 2, \ldots$.

Consequently, for positive integers n, p and using proof as in Lemma 2.1, we have

$$M_1(x_{n+1}, x_{n+2}, kt) \quad \geq \; M_1(x_n, x_{n+1}, t) * M_1(x_{n+1}, x_{n+2}, t/k^p).$$

Thus since $M_1(x_{n+1}, x_{n+2}, t/k^p) \to 1$ *as* $p \to \infty$,

(3.6) $$M_1(x_{n+1}, x_{n+2}, kt) \geq M_1(x_n, x_{n+1}, t).$$

Similarly applying inequality (3.2), we have

(3.7) $$\begin{aligned} &M_2(y_{2n}, y_{2n+1}, kt) \\ &= M_2(APSy_{2n-1}, BQTy_{2n}, kt) \\ &\geq M_2(y_{2n-1}, y_{2n}, t) * M_2(y_{2n-1}, APSy_{2n-1}, t) * M_2(y_{2n}, BQTy_{2n}, t) \\ &\quad * M_2(APSy_{2n-1}, BQTy_{2n}, kt) \\ &= M_2(y_{2n-1}, y_{2n}, t) * M_2(y_{2n-1}, y_{2n}, t) * M_2(y_{2n}, y_{2n+1}, t) * M_2(y_{2n}, y_{2n+1}, t) \\ &\geq M_2(y_{2n-1}, y_{2n}, t) * M_2(y_{2n}, y_{2n+1}, t). \end{aligned}$$

Similarly, we have

(3.8) $$M_2(y_{2n+1}, y_{2n+2}, kt) \geq M_2(y_{2n}, y_{2n+1}, t) * M_2(y_{2n+1}, y_{2n+2}, t).$$

Thus from (3.7) and (3.8), it follows that

$$M_2(y_{n+1}, y_{n+2}, kt) \geq M_2(y_n, y_{n+1}, t) * M_2(y_{n+1}, y_{n+2}, t).$$

Consequently, for positive integers n, q we have

$$M_2(y_{n+1}, y_{n+2}, kt) \geq M_2(y_n, y_{n+1}, t) * M_2(y_{n+1}, y_{n+2}, t/k^q).$$

Thus since $M_2(y_{n+1}, y_{n+2}, t/k^q) \to 1$ as $q \to \infty$, we have

(3.9) $$M_2(y_{n+1}, y_{n+2}, kt) \quad \geq M_2(y_n, y_{n+1}, t).$$

Now, applying inequality (3.3), we have

(3.10) $$\begin{aligned} &M_3(z_{2n+1}, z_{2n}, kt) \\ &= M_3(SAPz_{2n}, TBQz_{2n-1}, kt) \\ &\geq M_3(z_{2n}, z_{2n-1}, t) * M_3(z_{2n}, SAPz_{2n}, t) \\ &\quad * M_3(z_{2n-1}, TBQz_{2n-1}, t) * M_3(SAPz_{2n}, TBQz_{2n-1}, t) \\ &= M_3(z_{2n}, z_{2n-1}, t) * M_3(z_{2n}, z_{2n+1}, t) * M_3(z_{2n-1}, z_{2n}, t) \\ &\quad * M_3(z_{2n+1}, z_{2n}, t) \\ &\geq M_3(z_{2n}, z_{2n-1}, t) * M_3(z_{2n}, z_{2n+1}, t) \end{aligned}$$

Similarly, we have

$$M_3(z_{2n+2}, z_{2n+1}, kt) \geq M_3(z_{2n+1}, z_{2n}, t) * M_3(z_{2n+1}, z_{2n+2}, t) \tag{3.11}$$

Thus from (3.10) and (3.11), it follows that

$$M_3(z_{n+1}, z_{n+2}, kt) \geq M_3(z_n, z_{n+1}, t) * M_3(z_{n+1}, z_{n+2}, t)$$

for all $n = 1, 2, \ldots$.

Consequently, for positive integers n, r we have

$$M_3(z_{n+1}, z_{n+2}, kt) \geq M_3(z_n, z_{n+1}, t) * M_3(z_{n+1}, z_{n+2}, t/k^r).$$

Thus since $M_3(z_{n+1}, z_{n+2}, t/k^r) \to 1$ as $r \to \infty$,

$$M_3(z_{n+1}, z_{n+2}, kt) \geq M_3(z_n, z_{n+1}, t). \tag{3.12}$$

By Lemma 2.1, $\{x_n\}$ is a Cauchy sequence in complete fuzzy metric space X and so has a limit u in X. It follows similarly that sequences $\{y_n\}$ and $\{z_n\}$ are also Cauchy sequence in complete fuzzy metric space Y and Z and so have limits v in Y and w in Z.

Using (3.1), we have

$$\begin{aligned}
&M_1(PSAx_{2n}, u, kt)\\
\geq &M_1(PSAx_{2n}, x_{2n}, \frac{kt}{2}) * M_1(x_{2n}, u, \frac{kt}{2})\\
= &M_1(PSAx_{2n}, QTBx_{2n-1}, \frac{kt}{2}) * M_1(x_{2n}, u, \frac{kt}{2})\\
\geq &M_1(x_{2n}, x_{2n-1}, \frac{t}{2}) * M_1(x_{2n}, PSAx_{2n}, \frac{t}{2}) * M_1(x_{2n-1}, QTBx_{2n-1}, \frac{t}{2})\\
&* M_1(PSAx_{2n}, QTBx_{2n-1}, \frac{t}{2}) * M_1(x_{2n}, u, \frac{kt}{2})\\
\geq &M_1(x_{2n}, x_{2n-1}, \frac{t}{2}) * M_1(x_{2n}, x_{2n+1}, \frac{t}{2}) * M_1(x_{2n-1}, x_{2n}, \frac{t}{2})\\
&* M_1(x_{2n+1}, x_{2n}, \frac{t}{2}) * M_1(x_{2n}, u, \frac{kt}{2})
\end{aligned}$$

Taking the limit, we have

$$\lim_{n\to\infty} M_1(PSAx_{2n}, u, kt) \to 1.$$

Thus we have

$$\lim_{n\to\infty} PSAx_{2n} = u = PSy_{2n+1}. \tag{3.13}$$

Similarly using (3.1), we have

$$
\begin{aligned}
&M_1(QTBx_{2n-1}, u, kt)\\
\geq &M_1(QTBx_{2n-1}, x_{2n-1}, \frac{kt}{2}) * M_1(x_{2n-1}, u, \frac{kt}{2})\\
= &M_1(QTBx_{2n-1}, PSAx_{2n-2}, \frac{kt}{2}) * M_1(x_{2n-1}, u, \frac{kt}{2})\\
\geq &M_1(x_{2n-1}, x_{2n-2}, \frac{t}{2}) * M_1(x_{2n-2}, PSAx_{2n-2}, \frac{t}{2}) * M_1(x_{2n-1}, QTBx_{2n-1}, \frac{t}{2})\\
&* M_1(QTBx_{2n-1}, PSAx_{2n-2}, \frac{t}{2}) * M_1(x_{2n-1}, u, \frac{kt}{2})\\
\geq &M_1(x_{2n-1}, x_{2n-2}, \frac{t}{2}) * M_1(x_{2n-2}, x_{2n-1}, \frac{t}{2}) * M_1(x_{2n-1}, x_{2n}, \frac{t}{2})\\
&* M_1(x_{2n-1}, x_{2n}, \frac{t}{2}) * M_1(x_{2n-1}, u, \frac{kt}{2})
\end{aligned}
$$

Taking the limit, we have

$$\lim_{n\to\infty} M_1(QTBx_{2n-1}, u, kt) \to 1.$$

Thus we have

$$\lim_{n\to\infty} QTBx_{2n-1} = u = QTy_{2n}. \tag{3.14}$$

Similarly, we have

$$\lim_{n\to\infty} APSy_{2n-1} = v = APz_{2n-1}, \tag{3.15}$$

$$\lim_{n\to\infty} BQTy_{2n} = v = BQz_{2n}, \tag{3.16}$$

$$\lim_{n\to\infty} SAPz_{2n} = w = SAx_{2n}, \tag{3.17}$$

$$\lim_{n\to\infty} TBQz_{2n-1} = w = TBx_{2n-1}. \tag{3.18}$$

Now, suppose A is continuous.
Thus

$$\lim_{n\to\infty} Ax_{2n} = Au = v. \tag{3.19}$$

Using inequality (3.1), we have

$$
\begin{aligned}
M_1(PSAu, QTBx_{2n-1}, kt) \geq &M_1(u, x_{2n-1}, t) * M_1(u, PSAu, t)\\
&* M_1(x_{2n-1}, QTBx_{2n-1}, t) * M_1(PSAu, QTBx_{2n-1}, t)
\end{aligned}
$$

Letting $n \to \infty$ and using (3.14), we have

$$M_1(PSAu, u, kt) \geq M_1(u, PSAu, t).$$

Therefore by Lemma 2.2, We have $PSAu = u = PSv$.

Using inequality (3.1), we have

$$\begin{aligned} M_1(PSAx_{2n}, QTBu, kt) \geq & M_1(x_{2n}, u, t) * M_1(x_{2n}, PSAx_{2n}, t) \\ & * M_1(u, QTBu, t) * M_1(PSAx_{2n}, QTBu, kt) \end{aligned}$$

Letting $n \to \infty$ and using (3.13), we have

$$M_1(u, QTBu, kt) \geq M_1(u, QTBu, t).$$

Therefore, by Lemma 2.2 and using (3.19), We have $QTBu = u$.

Now, suppose B is continuous. Thus $\lim_{n\to\infty} Bx_{2n-1} = Bu = v$.

Therefore, We have $QTBu = u = QTv$.

Now, suppose S is continuous. Thus

$$\lim_{n\to\infty} Sy_{2n-1} = Sv = w. \tag{3.20}$$

Using inequality (3.2), we have

$$\begin{aligned} M_2(APSv, BQTy_{2n}, kt) \geq & M_2(v, y_{2n}, t) * M_2(v, APSv, t) * M_2(y_{2n}, BQTy_{2n}, t) \\ & * M_2(APSv, BQTy_{2n}, t). \end{aligned}$$

Letting $n \to \infty$ and using (3.16), we have

$$M_2(APSv, v, kt) \geq M_2(v, APSv, t).$$

Therefore by Lemma 2.2, We have $APSv = v = APw$. Using inequality (3.2), we have

$$\begin{aligned} M_2(APSy_{2n-1}, BQTv, kt) \geq & M_2(y_{2n-1}, v, t) * M_2(y_{2n-1}, APSy_{2n-1}, t) \\ & * M_2(v, BQTv, t) * M_2(APSy_{2n-1}, BQTv, kt) \end{aligned}$$

Letting $n \to \infty$ and using (3.15), we have

$$M_2(v, BQTv, kt) \geq M_2(v, BQTv, t).$$

Therefore, by Lemma 2.2 and using (3.20), We have $BQTv = v$.

Now, suppose T is continuous. Thus $\lim_{n\to\infty} Ty_{2n} = Tv = w$. Therefore, We have $BQTv = v = BQw$.

Now, suppose P is continuous. Thus

$$\lim_{n\to\infty} Pz_{2n} = Pw = u \tag{3.21}$$

Using inequality (3.3), we have

$$M_3(SAPw, TBQz_{2n-1}, kt) \geq M_3(w, z_{2n-1}, t) * M_3(w, SAPw, t) * M_3(z_{2n-1}, TBQz_{2n-1}, t) * M_3(SAPw, TBQz_{2n-1}, t)$$

Letting $n \to \infty$ and using (3.18), we have

$$M_3(SAPw, w, kt) \geq M_3(w, SAPw, t).$$

Therefore by Lemma 2.2, We have $SAPw = w = SAu$. Using inequality (3.3), we have

$$M_3(SAPz_{2n}, TBQw, kt) \geq M_3(z_{2n}, w, t) * M_3(z_{2n}, SAPz_{2n}, t) * M_3(w, TBQw, t) * M_3(SAPz_{2n}, TBQw, t)$$

Letting $n \to \infty$ and using (3.17), we have

$$M_3(w, TBQw, kt) \geq M_3(w, TBQw, t).$$

Therefore, by Lemma 2.2 and using (3.21), We have $TBQw = w$.

Now, suppose Q is continuous. Thus $\lim_{n\to\infty} Qz_{2n-1} = Qw = u$. Therefore, We have $TBQw = w = TBu$. Thus, we have

$$(3.22) \qquad \begin{cases} PSAu = QTBu = PSv = QTv = Pw = Qw = u, \\ APSv = BQTv = APw = BQw = Au = Bu = v, \\ SAPw = TBQw = SAu = TBu = Sv = Tv = w. \end{cases}$$

By the symmetry (3.22) holds if one of the mappings B, S, T, P, Q is continuous instead of A.

To prove the uniqueness suppose that PSA and QTB have a common fixed point $u\prime$ also.

Using inequality (3.1), we have

$$M_1(PSAu, QTBu\prime, kt) \geq M_1(u, u\prime, t) * M_1(u, PSAu, t) * M_1(u\prime, QTBu\prime, t) * M_1(PSAu, QTBu\prime, t).$$

Therefore, we have

$$M_1(u, u\prime, kt) \geq M_1(u, u\prime, t).$$

By Lemma 2.2, we have $u = u\prime$. Similarly we can prove that v and w are unique common fixed point of APS and BQT and of SAP and TBQ. This completes the proof. □

Remark 3.1. On putting $P = Q = Ix$ (the identity mapping on X) our result reduces to Sharma, Deshpande and Thakur [18].

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