연료전지 시스템을 위한 헤머스테인-위너 모델기반의 모델예측제어

논 문
60-2-25

Hammerstein-Wiener Model based Model Predictive Control for Fuel Cell Systems

이 상 문*
(Sang-Moon Lee)

Abstract - In this paper, we consider Hammerstein-Wiener nonlinear model for solid oxide fuel cell (SOFC). A nonlinear model predictive control (MPC) is proposed to trace the constant stack terminal power by Hydrogen flow as control input. After the stability of the closed-loop system with static output feedback controller is analysed by Lyapunov method, a nonlinear model predictive control based on the Hammerstein-Wiener model is developed to control the stack terminal power of the SOFC system. Simulation results verify the effectiveness of the proposed control method based on the Hammerstein-Wiener model for SOFC system.

Key Words : SOFC, MPC, Hammerstein-Wiener model

1. Introduction

The Fuel Cell (FC) plant efficiency can be about high 40-50% because FC generates electrical energy directly from chemical reactions, unlike heat engine or gas turbine. Until now, various types of fuel cell is investigated, but recently among these types of fuel cell, solid oxide fuel cell (SOFC) has attracted considerable interest as it offers wide application ranges, flexibility in the choice of fuel, high system efficiency and possibility of operation with an internal reformer [1]. It is well known that SOFC systems are sealed, and work in a high-temperature (600-100℃) environment. So, the heat generation from the electrochemical reactions and the high-temperature environment can be used for co-generation applications increasing the efficiency up to 70℃.

During the last several years, SOFC modeling of the nonlinear dynamics have been investigated [2]-[4], for SOFC is a dynamic device which will affect the dynamic behavior of the power system to which it is connected. However, most of these models indicated the detailed electrochemical processes. These models are very useful to analyze the transient characteristics of the SOFC, but they are too complicated to be used in controller design. So, for developing effective control strategies, the system identification for SOFC is needed. Recently, a study which identified SOFC system to various system model ,e.g., Hammerstein model, neural network, nonlinear ARX model and so on,, has been introduced [5]-[6].

A special class of nonlinear model is block oriented one in which a linear time invariant dynamic block is preceded and followed by a static non-linearity. These models, such as Hammerstien and Wiener, do not require much fundamental knowledge about a system, but only require input-output data, and they are relatively easy to be constructed using process data. The Hammerstein-Wiener model consists of a linear dynamic block, input non-linearity and a static output non-linearity followed by the block. Although Hammerstein-Wiener models only represent a small subclass of all nonlinear model, they have appeared to be useful in modeling several nonlinear processed encountered in the process industry, such as distillation columns [7], a heat exchanger [8] and pH neutralization processed [9].

Since MPC can consider a finite horizon cost function, it can easily handle time varying tracking commands, input and output constraints and so on. For this reason, it has been widely investigated in academia and in industry [10]-[14].

In this paper, we propose a design method of MPC for SOFC. In order to design the model predictive controller, we identify SOFC model to Hammerstein-Wiener model. The proposed control law is based on integral action form to provide zero off-set for constant command signals and the closed loop stability is guaranteed under linear matrix inequality (LMI) conditions on the terminal weighting matrix using the decreasing monotonicity property of the performance. Through a simulation

† 정 회 원 : 대구대학교 전자공학부 전임강사
E-mail : moony@daegu.ac.kr
접수일자 : 2010년 11월 30일
최종완료 : 2010년 12월 20일

example, we show that the proposed schemes can be appropriate tracking controllers for Hammerstein-Wiener models.

2. Modeling of SOFC dynamic system

The dynamics modeling of a SOFC is comprised of the conservation equations: mass, momentum, and energy conservation equations. Besides these equations, the electrochemical reactions and different loss mechanisms are also considered. Different methods of modeling and simplifying these equations will be discussed. The output voltage is the most important variable in SOFC because most of control purpose is to make actual voltage trajectory and desired voltage trajectory be the same. As considering results in previous study, output voltage is consisted of partial pressure of hydrogen, oxide and water by Nernst's equation. Thus, to control the output voltage, we should know dynamics of each partial pressure. In many paper about SOFC, it is clearly developed. The Laplace transformed partial pressure inside the channel of hydrogen, oxygen and water are as follows:

$$p_{H_2}(s)=\frac{1/K_{H_2}}{1+\tau_{H_2}s}(q_{H_2}^{in}-2K_{rI}), \tag{1}$$

$$p_{O_2}(s)=\frac{1/K_{O_2}}{1+\tau_{O_2}s}(q_{O_2}^{in}-2K_{rI}), \tag{2}$$

$$p_{H_2O}(s)=2K_rI\frac{1/K_{H_2O}}{1+\tau_{H_2O}s}, \tag{3}$$

where I is stack current (A), K_{H_2}, K_{H_2O}, K_{o_2} are value molar constants for hydrogen, water and oxygen $(mol\,s^{-1}Pa)$, K_r is constant, p_{H_2}, p_{H_2O}, p_{O_2} are partial pressure of hydrogen, water and oxygen (atm), $q_{H_2}^{in}$, $q_{O_2}^{in}$ are input hydrogen and oxygen flow $(mol\,s^{-1}Pa)$, τ_{H_2}, τ_{H_2O}, τ_{O_2} are response time for hydrogen, water and oxygen low (s).

Also, the SOFC consists of hundreds of cells connected in series or in parallel. By regulating the fuel valve, the amount of fuel into the SOFC can be adjusted, and the output voltage of the SOFC can be controlled. The Nernst's equation determine the average voltage magnitude of the fuel cell stack, In addition, if we consider terms of voltage loss then we can get more perfectly voltage equation of SOFC. Hence, applying Nernst's equation and terms of voltage loss, the output of the SOFC can be modeled as follows:

$$V_{dc}=E-\eta_{act}-\eta_{conc}-\eta ohm, \tag{4}$$

$$E=N_0(E^0+\frac{RT}{2F}ln\frac{p_{H_2}p_{O_2^{1/2}}}{p_{H_2O}}), \tag{5}$$

$$\eta_{act}=\partial+\beta\log I, \tag{6}$$

$$\eta_{conc}=\frac{RT}{2F}ln(1-I/I_L), \tag{7}$$

$$\eta_{ohm}=IR_{ohm}, \tag{8}$$

where E is open-circuit reversible potential (V), E^0 is standard reversible cell potential (V), F is Faraday's constant $(c\ mol^{-1})$, I_L is limiting current (A), N_0 is number of cells in the stack, R is gas constant $(J\ mol^{-1}K)$, R_{ohm} is Ohmic resistance (Ω), T is cell temperature (K), β is Tafel slope, η_{act} is activation losses (V), η_{conc} is concentration losses (V), η_{ohm} is Ohmic losses (V), ∂ is a Tafel constant.

Fuel utilization is one of the most important operating variables affecting the performance of fuel cells. The fuel utilization is defined as:

$$u_f=\frac{q_{H_2}^{in}-q_{H_2}^{0}}{q_{H_2}^{in}}=\frac{q_{H_2}^{r}}{q_{H_2}^{in}}=\frac{N_0I}{2Fq_{H_2}^{in}}, \tag{9}$$

where $q_{H_2}^{0}$ is output hydrogen flow $(mol\,s^{-1})$, $q_{H_2}^{r}$ is hydrogen flow that reacts $(mol\,s^{-1})$, u_f is fuel utilization. When the stack is operated at a high fuel utilization, the voltage density decreases. Furthermore, if fuel utilization is too large, it becomes impossible for the SOFC sustain the voltage across the load. However, it is a waste under a low fuel utilization when there is no cycling of the anode gas flow. Therefore the fuel utilization should be carefully selected to achieve the high SOFC performance. From Eq. (9), the SOFC stack is operated with constant steady-state utilization by controlling the natural gas input flow to the stack as:

$$q_f=\frac{N_0I}{2F\ u_{fs}}, \tag{10}$$

where q_f is natural gas flow rate $(mol\,s^{-1})$, u_{fs} is desired utilization in steady-state.

Now, we consider the Hammerstein-Wiener model to apply the MPC algorithm. Hammerstein-Wiener model consists of a linear dynamic block input non-linearity and a static output non-linearity followed by the block. Let us consider the following identified Hammerstein-Wiener model equation described by

$$\begin{aligned}x(k+1)&=Ax(k)+B\phi(u(k)),\\ y(k)&=Cx(k)+Du(k),\\ z(k)&=h(y(k)),\end{aligned} \tag{11}$$

where A,B,C and D are the system matrices of the linear dynamic block, $x(k)\in R^n$ is the state, $u(k)\in R^m$ and $y(k)\in R^l$ are the input and output of the linear block respectively. $\phi(u(k))$ is sector bounded nonlinear function, $z(k)\in R^l$ is the output of the nonlinear block and $h(y(k))$ is the nonlinear mapping from $y(k)$ to $z(k)$. The static nonlinear function $h(\cdot)$ is assumed to be known and invertible. The Hammerstein_Wiener model is easily obtained by using the nonlinear system identification toolbox in MATLAB.

3. Model predictive control

In this section, we introduce the model predictive control method for Hammerstein-Wiener model. The goal of this paper is to obtain output feedback model predictive tracking control law which stabilize (11) and makes outputs follow given command signals.

Consider a static output feedback controller with the following structure

$$u(k) = F(k)z(k) + N(k), \quad (12)$$

where $F(k)$ and $N(k)$ are design variables. We consider integral action form because it provides zero-offset for constant command signals. We define the increments $\delta u(k) = u(k+1) - u(k)$, $\delta x(k) = x(k+1) - x(k)$, $\delta y(k) = y(k+1) - y(k)$, and replace $u(k)$ with $\delta u(k)$, then we obtain the incremental model as follows:

$$\begin{aligned} x^e(k+1) &= \begin{bmatrix} I & C \\ 0 & A \end{bmatrix} x^e(k) + \begin{bmatrix} D \\ 0 \end{bmatrix} \delta u(k) \\ &\quad + \begin{bmatrix} 0 \\ B \end{bmatrix} (\phi(u(k+1)) - \phi(u(k))), \\ y(k) &= [I \quad 0]\, x^e(k), \\ z(k) &= h(y(k)). \end{aligned} \quad (13)$$

The nonlinear function $\phi(\bullet)$ is a sector bounded function and we assume that the nonlinear function $\phi(\bullet)$ also satisfies a slope constraint such that

$$a_i \le \frac{\phi_i(\sigma_i(k+1)) - \phi_i(\sigma_i(k))}{\sigma_i(k+1) - \sigma_i(k)} \le \beta_i, \quad (14)$$

where $\phi_{i(\sigma_{i(k)})}$ is the ith element of the $\sigma(\bullet)$, α_i and β_i are the lower and upper bounds of the sector. Let $\overline{\Delta_i}(u_i(k))$ be an element of a convex hull $Co\{\alpha_i, \beta_i\}$ and define

$$\Theta_1 = diag(\alpha_1, \ldots, \alpha_m), \quad (15)$$

$$\Theta_2 = diag(\beta_1, \ldots, \beta_m), \quad (16)$$

$$\overline{\Delta}(k) = diag(\overline{\Delta_1}(u_1(k)), \ldots, \overline{\Delta_m}(u_m(k))), \quad (17)$$

then

$$\Theta_1 \le \overline{\Delta}(k) \le \Theta_2, \quad (18)$$

and the increment of the nonlinear function can be represented as

$$\phi_i(\sigma_i(k+1)) - \phi_i(\sigma_i(k)) = \overline{\Delta}(k)\delta u(k). \quad (19)$$

With the convex representation of the nonlinearity (19), the incremental model (13) can be rewritten as:

$$\begin{aligned} x^e(k+1) &= A^e x^e(k) + B^e(k)\delta u(k), \\ y(k) &= C^e x^e(k), \\ z(k) &= h(y(k)), \end{aligned} \quad (20)$$

where

$$A^e = \begin{bmatrix} I & C \\ 0 & A \end{bmatrix}, \quad B^e(k) = \begin{bmatrix} D \\ B\overline{\Delta}(k) \end{bmatrix},$$

$$C^e = [I \ 0], \qquad x^e = \begin{bmatrix} y(k) \\ \delta x(k) \end{bmatrix},$$

and the performance index is considered as follows:

$$\begin{aligned} \Delta J(k) &= \sum_{i=k}^{k+N-1} \big[\{z(k+i|k) - z_r(k+i|k)\}^T Q \times \\ &\{z(k+i|k) - z_r(k+i|k)\} \\ &+ \delta u(k+i|k)^T R \delta u(k+i|k) \big] + \\ &\begin{bmatrix} y(k+N|k) - y_r(k+N|k) \\ \delta x(k+N|k) \end{bmatrix}^T P^e(k+N) \\ &\times \begin{bmatrix} y(k+N|k) - y_r(k+N|k) \\ \delta x(k+N|k) \end{bmatrix}, \end{aligned} \quad (21)$$

where $z_r(k+i)$ is given reference signals, N is fixed finite horizon and Q and R are positive definite diagonal weighting matrices. For our goal, the above performance index (21) is to be minimized at the time k. Using Norquay's WMPC algorithm which is done by inverting the static output nonlinearity, the performance index (21) can be changed into

$$\begin{aligned} \Delta J(k) &= \sum_{i=0}^{N-1} \big[\{y(k+i|k) - y_r(k+i|k)\}^T \times \\ &\hat{Q}(k+i)\{y(k+i|k) - y_r(k+i|k)\} \\ &+ \delta u(k+i|k)^T R \delta u(k+i|k) \big] + \\ &\begin{bmatrix} y(k+N|k) - y_r(k+N|k) \\ \delta x(k+N|k) \end{bmatrix}^T P^e(k+N) \\ &\times \begin{bmatrix} y(k+N|k) - y_r(k+N|k) \\ \delta x(k+N|k) \end{bmatrix}, \end{aligned} \quad (22)$$

where

$$\begin{aligned} \hat{Q}(k+i) &= \left(\frac{\delta h(y)}{\delta y} \Big|_{y=h^{-1}(z_r(k+i))} \right)^T Q \times \\ &\left(\frac{\delta h(y)}{\delta y} \Big|_{y=h^{-1}(z_r(k+i))} \right) \\ &= H(k+i)^T Q H(k+i), \end{aligned}$$

and the nonlinearity output and the control input can be rewritten as:

$$\begin{aligned} z(k+i) &= h(y(k+i)) = H(k+i)y(k+i), \\ \delta u(k+i) &= F(k+i)H(k+i)y(k+i) + N(k+i). \end{aligned}$$

Since the minimization problem can be solved by transformation to a stand regulator problem, we augment the system states and reference signals, and we change the performance index as well. The augmented system states witch are from (2) and command signals are as follows:

$$\overline{x^e}(k+i+1|k) = \overline{A^e}(k+i)\overline{x^e}(k+i|k), \quad (23)$$

where $\overline{x^e} = \begin{bmatrix} x^e(k) \\ 1 \end{bmatrix}$,

$$\overline{A^e}(k+i) = \begin{bmatrix} A^e + B^e(k)F(k+i)H(k+i)C^e & B^e(k)N(k+i) \\ 0 & 1 \end{bmatrix}.$$

Consequently, the performance index from (22) is

$$\begin{aligned} \Delta J(k) &= \sum_{i=0}^{N-1} \overline{x^e}(k+i|k)^T \overline{Q^e}(k+i)\overline{x^e}(k+i|k) \\ &+ \overline{x^e}(k+i|k)^T \overline{P^e}(k+N)\overline{x^e}(k+i|k), \end{aligned} \quad (24)$$

where

$$\overline{Q^e}(k+i)=\begin{bmatrix}\overline{Q^e}(k+1)\{1,1\} & \star \\ \overline{Q^e}(k+1)\{2,1\} & \overline{Q^e}(k+1)\{2,2\}\end{bmatrix},$$
$$\overline{Q^e}(k+1)\{1,1\}= C^{e^T}\hat{Q}(k+i)C^e+C^{e^T}H(k+i)^T\times F(k+i)^TRF(k+i)H(k+i)C^e,$$
$$\overline{Q^e}(k+1)\{2,1\}=-y_r(k+i)^T\hat{Q}C^e+N(k+i)^TRF(k+i)H(k+i)C^e$$
$$\overline{Q^e}(k+1)\{2,2\}=y_r(k+i)^T\hat{Q}(k+i)y_r(k+i)+N(k+i)^TRN(k+i),$$
$$\overline{P^e}(k+N)=\begin{bmatrix}P^e(k+N) & -P^e(k+N)C^{e^T}y_r(k+N) \\ -y_r(k+N)^TC^eP^e(k+N) & y_r(k+N)^TC^eP^e(k+N)C^{e^T}y_r(k+N)\end{bmatrix}$$

From now, we derive the control law from the augmented system (23) with the performance index (24). Using the following theorem, we solve the minimization problem.

Theorem 1. If there exist $\overline{P^e}(k+i)$ satisfying such that

$$\overline{A^e}(k+i)^T\overline{P^e}(k+i+1)\overline{A^e}(k+i)+\overline{Q^e}(k+i)-\overline{P^e}(k+i)<0, \tag{25}$$

for $i=k,\ldots,k+N+1$, then

$$\Delta J(k)<\overline{x^e}(k)^T\overline{P^e}\,\overline{x^e}(k). \tag{26}$$

Proof. We can derive the following relation:

$$\Delta J(k)-\overline{x^e}(k)\overline{P^e}\,\overline{x^e}= \sum_{i=0}^{N-1}\left\{\overline{x^e}(k+i|k)^T\overline{Q^e}(k+i)\overline{x^e}(k+i|k)\right\} + \overline{x^e}(k+i+1|k)^T\overline{P^e}(k+i+1)\overline{x^e}(k+i+1|k) - \overline{x^e}(k+i|k)^T\overline{P^e}(k+i)\overline{x^e}(k+i|k)\}. \tag{27}$$

If we find a proper $\overline{P^e}(k+i)$, we obtain the following inequality:

$$\begin{aligned}&\overline{x^e}(k+i|k)\overline{Q^e}(k+i)\overline{x^e}(k+i|k)\\&+\overline{x^e}(k+i+1|k)^T\overline{P^e}(k+i+1|k)\overline{x^e}(k+i+1|k)\\&-\overline{x^e}(k+i|k)^T\overline{P^e}(k+i)\overline{x^e}(k+i|k)\\&=\overline{x^e}(k+i|k)^T\{\overline{A^e}(k+i)^T\overline{P^e}(k+i+1)\overline{A^e}(k+i)\\&+\overline{Q^e}(k+i)-\overline{P^e}(k+i)\}\overline{x^e}(k+i|k)<0,\end{aligned} \tag{28}$$

and then it gives an upper bound of the performance index:

$$\Delta J(k)<\overline{x^e}(k)^T\overline{P^e}\,\overline{x^e}(k). \quad \blacksquare \tag{29}$$

We can obtain the static output feedback model predictive tracking control law $F(k+i)$ and $N(k+i)$ recursively that minimize the performance $\Delta J(k)$ by minimizing $trace(\overline{P^e}(k+i))$ satisfying the inequality (25) with given $\overline{P^e}(k+i+1)$ during the fixed finite horizon.

For the fixed finite horizon from k to $k+N$, we can find $\overline{P^e}(k+N)=P_f^e$ by following relation from (25):

$$A_j^{e^T}P_f^eA_j^e+Q^e-P_f^e \tag{30}$$
$$\begin{bmatrix}Q^e-P_f^e & (A_j^e)^TP_f^e \\ \star & -P_f^e\end{bmatrix}<0,$$

for $j=1,2$, where

$$A_j^e=A^e+B_j^eFHC^e,$$
$$B_j^e=\begin{bmatrix}D \\ B\Theta_j\end{bmatrix},$$
$$Q^e=C^{e^T}\hat{Q}C^e+C^{e^T}H^TF^TRFHC^e,$$
$$\hat{Q}=H^TQH,$$
$$H=\left(\frac{\partial h(y)}{\partial y}\Big|_{y=h^{-1}(0)}\right),$$

and

$$\min_{P_f^e,Y,S}P_f^e \text{ subject to } P_f^e>0,$$
$$\begin{bmatrix}-Y & \star & \star & \star \\ A_j^eY+B^eS-Y & \star & \star & \\ C^eY & 0 & \hat{Q}^{-1} & \star \\ S & 0 & 0 & R^{-1}\end{bmatrix}<0, \tag{31}$$

for $j=1,2$, where $Y=P_f^{e^{-1}}$ and $S=FHC^eY$. Actually, the calculated $P^e(k+N)=P_f^e$ is terminal weighting matrix to stabilize the system.

Next, we find $\overline{P^e}(k+i)$ recursively with given $\overline{P^e}(k+i+1)$ from $i=N-1$ to 0.

Theorem 2. Suppose that the problem (22) subject to (25) for $i=k+N$ is feasible at each time k, if there exists $P_f^e(k+i)=P_f^e(k+i)^T>0$, $F(k+i)$ and $N(k+i)$ satisfying such that

$$\begin{bmatrix}\Omega\{1,1\} & \star & \star & \star & \star \\ \Omega\{2,1\} & \Omega\{2,1\} & \star & \star & \star \\ \Omega\{3,1\} & \Omega\{3,2\} & \Omega\{3,3\} & \star & \star \\ \Omega\{4,1\} & \Omega\{4,2\} & \Omega\{4,3\} & \Omega\{4,4\} & \star \\ \Omega\{5,1\} & \Omega\{5,2\} & \Omega\{5,3\} & \Omega\{5,4\} & \Omega\{5,5\}\end{bmatrix}<0, \tag{32}$$

where

$$\overline{P^e}(k+i)=\begin{bmatrix}\overline{P^e}(k+i)\{1,1\} & \star \\ \overline{P^e}(k+i)\{2,1\} & \overline{P^e}(k+i)\{2,2\}\end{bmatrix},$$
$$\Omega\{1,1\}=C^{e^T}\hat{Q}(k+i)C^e-\overline{P^e}(k+i)\{1,1\},$$
$$\Omega\{2,1\}=-y_r(k+i)^T\hat{Q}(k+i)C^e-\overline{P^e}(k+i)\{2,1\},$$
$$\Omega\{2,2\}=y_r(k+i)^T\hat{Q}(k+i)y_r(k+i)-\overline{P^e}(k+i)\{2,2\},$$
$$\Omega\{3,1\}=F(k+i)H(k+i)C^e,$$
$$\Omega\{3,2\}=N(k+i),$$
$$\Omega\{3,3\}=-R^{-1},$$
$$\Omega\{4,1\}=\overline{P^e}(k+i+1)\{1,1\}A^e+\overline{P^e}(k+i+1)\{1,1\}B^eF(k+i)H(k+i)C^e,$$
$$\Omega\{4,2\}=\overline{P^e}(k+i+1)\{1,1\}B_j^eN(k+i)+\overline{P^e}(k+i+1)\{1,2\},$$
$$\Omega\{4,3\}=0,$$
$$\Omega\{4,4\}=-\overline{P^e}(k+i+1)\{1,1\},$$

$$\Omega\{5,1\} = \overline{P^e}(k+i+1)\{2,1\}A^e + \overline{P^e}(k+i+1)\{2,1\}B_j^e F(k+i)H(k+i)C^e,$$
$$\Omega\{5,2\} = \overline{P^e}(k+i+1)\{2,1\}B_j^e N(k+i) + \overline{P^e}(k+i+1)\{2,2\},$$
$$\Omega\{5,3\} = 0,$$
$$\Omega\{5,4\} = -\overline{P^e}(k+i+1)\{2,1\},$$
$$\Omega\{5,5\} = -\overline{P^e}(k+i+1)\{2,2\}.$$

then, the tracking error of the closed-loop system (23) goes to zero. Furthermore, the minimum value of the performance index can be obtained by

$$\min_{P^e(k+i),F(k+i),N(k+i)} trace(\overline{P^e}(k+i))$$

Proof. If the terminal weighting matrix which satisfies the inequality (30) exist, then the following terminal inequality satisfies:

$$\Delta J^*(k,k+N+1) < \Delta J^*(k,k+N), \qquad (33)$$

because

$$\delta\Delta J^*(k+N) = \Delta J^*(k,k+N+1) - \Delta J^*(k,k+N)$$
$$= \sum_{i=k}^{k+N-1}\{x^e(i)^{1T}Q^e x^e(i)^1\} + \Delta J^*(k+N,k+N+1)$$
$$- \sum_{i=k}^{k+N+1}\{x^{e(i)^{2T}}Q^e x^e(i)^2\} - x^e(k+N)^{2T}P_f^e x^e(k+N)^2,$$

and if we replace $x^e(i)^{1T}$ by $x^e(i)^{2T}$, then we have

$$\delta\Delta J^*(k+N) = \Delta J^*(k+N,k+N+1) \qquad (34)$$
$$- x^e(k+N)^{2T}P_f^e x^e(k+N)^2$$
$$= x^e(k+N)^{2T}Q^e x^e(k+N)^2$$
$$+ x^e(k+N+1)^{2T}P_f^e x^e(k+N+1)$$
$$- x^e(k+N)^{2T}P_f^e x^e(k+N+1)^2$$
$$= x^e(k+N)^{2T}\{A^{e^T}P_f^e A^e + Q^e - P_f^e\}x^e(k+N)^2 < 0,$$

Thus, Eq. (34) is guaranteed in the following relation:

$$\Delta J^*(k,k+N) > \Delta J^*(k,k+N+1) \qquad (35)$$
$$= x^e(k)^T Q^e(k) x^e(k)$$
$$+ \Delta J^*(k+1,k+N+1)$$
$$> \Delta J^*(k+1,k+N+1).$$

For a given $\overline{P}^e(k+i)$, from the inequality (25), the following inequality can be obtained:

$$\begin{bmatrix} \overline{Q^e}(k+i) - \overline{P^e}(k+i) & \star \\ \overline{P^e}(k+i+1)\overline{A^e}(k+i) & -\overline{P^e}(k+i+1) \end{bmatrix} < 0. \qquad (36)$$

The inequality (32) can be obtained using Schur complement. Moreover, if the condition (32) is feasible, then the closed-loop system is asymptotic stable. ■

4. Simulation Results

To establish the Hammerstein-Wiener model, we use the SOFC simulator in MATLAB [15]. The parameters of the Hammerstein-Wiener model is as follows:

$$A = \begin{bmatrix} 0.9995 & -0.2824 & -0.0752 \\ 0.0143 & 0.9896 & 0.6329 \\ 0.0001 & -0.0076 & 0.9536 \end{bmatrix}, B = \begin{bmatrix} -47.0236 \\ 11.0719 \\ 0.0128 \end{bmatrix}$$
$$C = [-0.2750\ -0.4587\ 0.5252], D = 2.9167e+003,$$
$$h(y) = 0.0937y^3 - 0.3156y^2 + 1.0193y + 0.0348,$$

and the input nonlinear function $\phi(u)$ is a saturation function described as following:

$$\phi(u) = \begin{cases} 0.1 & u \geq 0.1 \\ -0.1 & u \leq -0.1 \\ u & \text{otherwise} \end{cases},$$

which is slope bounded in $[0,1]$.

The estimation error of identified Hammerstein-Wiener model can be calculated by the VAF which computes the percentage variance accounted for (VAF) between two signals. The VAF of the same signals is 100% and the lower VAF means more different signals. Thus, the VAF is often used to verify the correctness of a model, by comparing the real output with the estimated output of the model. The VAF is defined as follows:

$$VAF = (1 - \frac{y - y_{est}}{y}) \times 100[\%].$$

The VAF of two signals, the output voltage of the SOFC simulator and the output voltage of the Hammerstein-Wiener model, is 99.6%. On the other hand, the VAF of linear dynamic model is 89.3%. Fig. 1 and Fig. 2 show the system identification result of linear and nonlinear model that are compared actual controlled output with estimated output. It means that the identified Hammerstein-Wiener model for SOFC can be reasonably replaced the actual SOFC dynamic system.

With the control strategies, we can regulate the input hydrogen flow by using Eq.(10). To prove the effectiveness of the control strategies, we choose the current disturbance as a multiple step signal which increase from 4.9kW to 5.1kW at 120s. In order to prove the validity of our MPC algorithm based on Hammerstein-Wiener model for SOFC, we conduct constant output voltage control at the same current disturbance. For this simulation, following parameter for MPC method is selected

$$Q = 10, \quad R = 0.01, \quad N = 3.$$

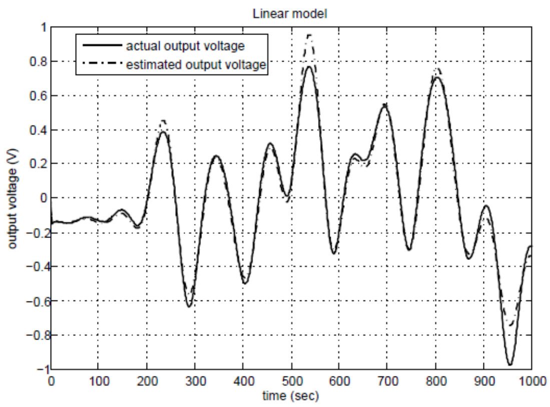

Fig. 1 Estimated output of linear model (VAF=89.3%)

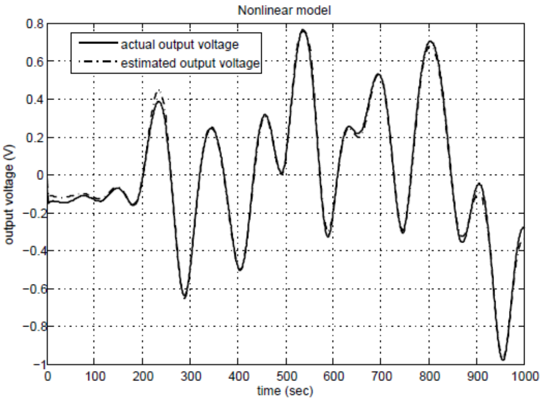

Fig. 2 Estimated output of Hammerstein-Wiener model (VAF=99.6%)

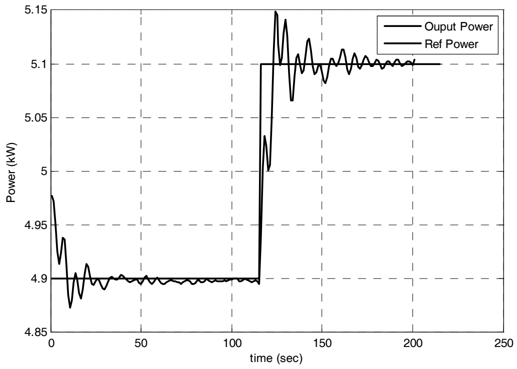

Fig. 3 Controlled output power

If the finite horizon step N increases, then performance of the controller gets better, however, the amount of calculation of MPC controller increases as well. On the other side, relatively smaller N has lower probability to get feasible MPC controller. Fig. 3. shows that the output power can be tracked the desired value by using MPC controller.

5. Conclusions

To effectively control the SOFC system, the Hammerstein-Wiener model is used to identify nonlinear dynamic behavior of SOFC system. By regulating the input hydrogen flow, a constrained nonlinear model predictive control method is presented for Hammerstein-Wiener model. The simulation shows that the effectiveness of the proposed method.

Acknowledgement

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0011460)

References

[1] W. Sangtongkitcharoen, S. Vivanpatarakij, et al., Chem. Eng. J. 138 (2008) 436.
[2] D.J. Hall, R.G. Colclaser, IEEE Trans. Energy Convers. 14 (1999) 749.
[3] C. Wang, M. Hashem Nehrir, IEEE Trans. Energy Convers. 22 (2007) 887.
[4] A. Selimovic, M. Kemm, et al., Journal Power Sources 145 (2005) 463.
[5] H.H Huo, X.J. Zhu, ea al., Journal Power Sources 185 (2008) 338.Differential Equatins, Springer-Verlag, New York, 1993.
[6] X.J. Wu, X.J. Zhu et al., Journal Power Sources 179 (2008) 232.
[7] M. Verhaegen, Chemical Engineering Communications, 163 (1998) 111.
[8] E. Eskinat, S.H. Johnson, and W.L. Luyben, AIChE Journal, 37 (1991) 255.
[9] S. J. Norquay, A. Palazoglu, and J.A. Romagnoli,IEEE Trans. Control Systems Tech. 7 (1999) 437.
[10] M.V. Kothare, V. Balakrishnan, and M. Morari, Automatica, vol. 32 (1996) 1361.
[11] J.B. Rawlings, K.R. Muske, IEEE Trans. Automat. Contr. 38 (1993) 1512.
[12] M.J. Kim, W.H. Kwon, et al., Journal of Guidance, Control, and Dynamics, 20 (1997).
[13] S.M. Lee, D.H. Lee, S.C. Won, SICE Annual Conference in Fukui, August 4-6 (2003).
[14] S.J. Norquay, A. Palazoglu, and J.A. Romagnoli, Chemical Engineering Science, 53 (1998) 75.
[15] http://www.coe.montana.edu/ee/fuelcell/

저 자 소 개

이 상 문 (李 相 文)

1973년 6월 15일생. 1999년 경북대학교 전자공학과 졸업(공학). 2006년 포항공과대학교 전기전자공학부 졸업(공박). 현재 대구대학교 전자공학부 전임강사.

Tel : 053-850-6647
Fax : 053-850-6619
E-mail : moony@daegu.ac.kr