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Delay-dependent Stability Criteria for Fuzzy Markovian Jumping Hopfield Neural Networks of Neutral Type with Time-varying Delays

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Abstract - This paper proposes delay-dependent stability conditions of the fuzzy Markovian jumping Hopfield neural networks of neutral type with time-varying delays. By constructing a suitable Lyapunov-Krasovskii's (L-K) functional and utilizing Finsler's lemma, new delay-dependent stability criteria for the systems are established in terms of linear matrix inequalities (LMIs) which can be easily solved by various effective optimization algorithms. A numerical example is given to illustrate the effectiveness of the proposed methods.

Key Words : Hopfield neural networks, Fuzzy systems, Neutral systems, Time-varying delays, Markovian jumping parameters, Lyapunov method, LMI

1. Introduction

Hopfield neural networks (HNNs) [1] have been extensively studied, and successfully employed in many areas such as combinatorial optimization, signal processing, pattern recognition, associate memory, knowledge acquisition, and so on. Such applications of neural networks heavily depend on the dynamical behaviors of the networks. In the implementation of neural networks, there exists time-delay due to the finite speed of information processing. It is well known that time-delay often cause undesirable dynamic network behaviors such as oscillation and instability. Therefore, stability analysis for HNNs with time-delay have been investigated in the literature [2, 3, 4]. Recently, Park and Kwon [5] addressed the problem of global stability for neural networks of neutral type with interval time-varying delays.

The time-delay systems with Markovian jumping parameters are a special sort of hybrid systems. The parameters usually jump among of finite modes, and the mode switching is limited by a Markovian process [6, 7, 8]. The system involving some kind of switching is promising in the mathematical modeling of the HNNs, and various results have been obtained. In [9], by delay partitioning idea to be less conservative, new stochastic stability criteria for Markovian jumping HNNs with constant and distributed delays were proposed. Also, the investigation of stochastic stability has been reported in [10] for uncertain stochastic neural networks with interval time-varying delays.

On the other hand, the well-known Takagi-Sugeno (T-S) fuzzy model [11] is recognized as an efficient tool in approximating a complex nonlinear system. The T-S fuzzy model approach is a multi-model approach in which some linear models are blended into an overall single model through nonlinear membership functions to represent the nonlinear dynamics. Based on the T-S fuzzy model, a number of important issues in fuzzy control systems are addressed in [12, 13, 14]. In [12], by using free-weighting matrix method, the stability criteria for the fuzzy bi-directional associative memory (BAM) neural networks with time-varying delays were expressed in the form of LMIs. Based on the parallel distributed compensation (PDC) method, new conditions for the existence of robust H_∞ controller for T-S fuzzy system with interval time-varying delay is presented in [13]. In [14], the problem of global stability for T-S fuzzy HNNs with time delay based on a generalized L-K functional and a parameterized model transformation was considered. Moreover, Balasubramaniam *et al.* [15] dealt with the delay-dependent asymptotic stability analysis problem for Markovian jumping fuzzy BAM neural networks with time-varying interval delays. However, to the best of authors' knowledge, delay-dependent stability analysis of fuzzy Markovian jumping HNNs of neutral type with

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time-varying delays has not considered in any other literature.

Motivated by the above discussions, we propose delay-dependent stability criteria for fuzzy Markovian jumping HNNs of neutral type with time-varying delays. By constructing a suitable L-K functional and utilizing Finsler's lemma, new delay-dependent criteria are derived in terms of LMIs which can be solved efficiently by using the interior-point algorithms [19]. A numerical example is included to show the effectiveness of the proposed method.

Notation: $\mathbf{R}^n$ is the n-dimensional Euclidean space, $\mathbf{R}^{m\times n}$ denotes the set of m by n real matrix. $\mathbf{C}_{n,h}=\mathbf{C}([-h,\mathbf{R}^n)$ denotes the Banach space of continuous functions mapping the interval $[-h,0]$ into $\mathbf{R}^n$, with the topology of uniform convergence. $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation. For symmetric matrices X and Y, the notation $X>Y$ (respectively, $X\geq Y$) means that the matrix $X-Y$ is positive definite (respectively, nonnegative). I and 0 denote the identity matrix and zero matrix with appropriate dimensions. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $diag\{\cdots\}$ denotes the block diagonal matrix. $\lambda_{\min}\{\cdot\}$ means the smallest eigenvalues of a given square matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. For a given matrix $X\in\mathbf{R}^{m\times n}$, such that $rank(X)=r$, we define $X^{\perp}\in\mathbf{R}^{n\times(n-r)}$ as the right orthogonal complement of X; i.e., $XX^{\perp}=0$.

2. Problem Statements

Given a complete probability space $(\Omega,\mathbf{F},P)$ where Ω is the sample space, $\mathbf{F}$ is the algebra of events, P is the probability measure defined on $\mathbf{F}$, and ρ_t is a finite state Markov jump process representing the system mode; that is, ρ_t takes values in a given finite set $S=\{1,2,\ldots,N\}$.

Let $\Pi=\{\pi_{ij}\}$, $i,j\in S$, $\pi_{ij}\geq 0$, $j\neq i$, $\pi_{ii}=-\sum_{j\neq i}\pi_{ij}$. The transition probability can be described as

$$\Pr\{\rho_{t+\delta}=j|\rho_t=i\}=\begin{cases}\pi_{ij}\delta+o(\delta) & j\neq i\\ 1+\pi_{ii}\delta+o(\delta), & j=i,\end{cases} \tag{1}$$

where $\lim_{\delta\to 0}o(\delta)/\delta=0$.

A model of Markovian jumping HNNs of neutral type with time-varying delays can be described as

$$\dot{y}(t)=-A(\rho_t)y(t)+C(\rho_t)\dot{y}(t-\tau(t))+W(\rho_t)g(y(t-h(t)))+b, \tag{2}$$

where $y(t)=[y_1(t),\ldots,y_n(t)]^T\in\mathbf{R}^n$ is the neuron state vector, $g(\cdot)=[g_1(\cdot),\ldots,g_n(\cdot)]^T\in\mathbf{R}^n$ denotes the neuron activation function vector with initial condition $g(0)=0$, $b=[b_1,\ldots,b_n]^T\in\mathbf{R}^n$ means the external bias at time t, $A(\rho_t)$, $W(\rho_t)$ and $C(\rho_t)$ are matrices functions of the random jumping process $\{\rho_t\}$, n denotes the number of neurons in a neural network, and $h(t)$ and $\tau(t)$ are the time-varying delays satisfying

$$0\leq h(t)\leq h_m,\ \dot{h}(t)\leq h_D,\ \dot{\tau}(t)\leq\tau_D<1. \tag{3}$$

In Eq.(2),

$$\begin{aligned}&A(\rho_t)=diag\{a_p(\rho_t)\},\ C(\rho_t)=(c_{pq}(\rho_t))_{n\times n},\\ &W(\rho_t)=(w_{pq}(\rho_t))_{n\times n},\qquad p,q=1,\ldots,n,\end{aligned} \tag{4}$$

where $A(\rho_t)$ is a diagonal matrix with $a_p(\rho_t)>0$, $W(\rho_t)$ and $C(\rho_t)$ are the interconnection matrices representing the weight coefficients of the neurons.

In this paper, it is assumed that the activation functions satisfy the following condition:

Assumption 1. The neurons activation functions $g_p(\cdot)$ are assumed to be non-decreasing, bounded and globally Lipschiz; that is

$$0\leq\frac{g_p(\xi_1)-g_q(\xi_2)}{\xi_1-\xi_2}\leq l_p,\ \xi_1,\xi_2\in\mathbf{R},\ \xi_1\neq\xi_2, \tag{5}$$

where l_p are positive constants.

Note that by using the Brouwer's fixed-point theorem, it can be easily proved that there exists at least one equilibrium point for system (2) [5]. For simplicity, in stability analysis of the system (2), the equilibrium point $y^*=[y_1^*,\ldots,y_n^*]^T$ is shifted to the origin by utilizing the transformation $x(\cdot)=y(\cdot)-y^*$, which leads the system (2) to the following form

$$\dot{x}(t)=-A(\rho_t)x(t)+C(\rho_t)\dot{x}(t-\tau(t))+W(\rho_t)f(x(t-h(t))), \tag{6}$$

where $x(t)=[x_1(t),\ldots,x_n(t)]^T\in\mathbf{R}^n$ is the state vector of the transformed system.
Note that $f(\cdot)=[f_1(\cdot),\ldots,f_n(\cdot)]^T$ and $f_q(\cdot)=g_q(\cdot+y_q^*)-g_q(y_q^*)$ with $f_q(0)=0$, and f_q satisfies Assumption 1.

Next, the system (2) will be represented by T-S fuzzy model.

Plant Rule l:
IF $\theta_1(t)$ is Θ_{l1}, ... , and $\theta_s(t)$ is Θ_{ls}, $l=1,2,\ldots,r$,
THEN

$$\begin{aligned}&\dot{x}(t)=-A_l(\rho_t)x(t)+C_l(\rho_t)\dot{x}(t-\tau(t))+W_l(\rho_t)f(x(t-h(t))),\\ &x(t)=\phi(t),\ -h_m\leq t\leq 0,\end{aligned} \tag{7}$$

where $\theta_1(t),\ldots,\theta_s(t)$ are the premise variables, and $\Theta_{1s},\ldots,\Theta_{ls}$ are fuzzy sets, r is the number of IF-THEN rules, and $\phi(t)\in \mathbf{C}_{n,h_m}$ denotes the initial conditions. For simplicity of notations, in this paper we denote the matrices associated with the ith mode ($\rho_t=i$) by

$$A_l^i=A_l(\rho_t),\ C_l^i=C_l(\rho_t),\ W_l^i=W_l(\rho_t), \tag{8}$$

where A_l^i, C_l^i and W_l^i are known constant matrices of appropriate dimensions.

Using the center-average defuzzifier, product interference and singleton fuzzifier, the defuzzified output of system (7) can be inferred as follow

$$\dot{x}(t)=\sum_{l=1}^{r}\mu_l(\theta(t))[-A_l^i x(t)+C_l^i\dot{x}(t-\tau(t))+W_l^i f(x(t-h(t)))], \tag{9}$$

where

$$\omega_l(\theta(t))=\prod_{k=1}^{s}\Theta_{lk}(\theta_k(t)),\ \mu_l(\theta(t))=\frac{\omega_l(\theta(t))}{\sum_{l=1}^{r}\omega_l(\theta(t))}, \tag{10}$$

and $\Theta_{lk}(\theta_k(t))$ is the grade of membership of $\theta_k(t)$ in Θ_{lk}. It is assumed that

$$\omega_l(\theta(t))\ge 0,\ \sum_{l=1}^{r}\omega_l(\theta(t))>0,\ \forall t. \tag{11}$$

Then, we have the following condition

$$\mu_l(\theta(t))\ge 0,\ \sum_{l=1}^{r}\mu_l(\theta(t))=1. \tag{12}$$

For stability analysis, the system (9) can be written as

$$\dot{x}(t)=-A(t)x(t)+C(t)\dot{x}(t-\tau(t))+W(t)f(x(t-h(t))), \tag{13}$$

where

$$A(t)=\sum_{l=1}^{r}\mu_l(\theta(t))A_l^i,\ C(t)=\sum_{l=1}^{r}\mu_l(\theta(t))C_l^i,$$
$$W(t)=\sum_{l=1}^{r}\mu_l(\theta(t))W_l^i. \tag{14}$$

As we have mentioned in Section 1, the HNNs perform important roles in many practical systems, such as signal processing, pattern recognition, optimization, system fusion and so on. We extend the T-S fuzzy models to describe the HNNs which are subjected to environmental noise. Moreover, in real nervous systems, synaptic transmission is a noisy process brought on by random change from the freedom of neurotransmitters, and other probabilistic causes. So, the form of HNNs (13) are considered.

In order to investigate the delay-dependent stability analysis for fuzzy Markovian jumping HNNs of neutral type with time-varying delays (13), we introduce the following definition and lemmas.

Definition 1. [8] The system (13) is said to be stochastically stable, if for any finite $\phi(t)\in\mathbf{C}_{n,h_m}$, and the initial condition of the mode $\rho_0\in S$ the following condition is satisfied

$$\lim_{t\to\infty}\mathbf{E}\left\{\int_0^t x^T(s)x(s)ds|\phi,\rho_0\right\}<\infty. \tag{15}$$

Lemma 1. (Finsler's lemma) [18] *Let $\zeta\in\mathbf{R}^n$, $\Phi=\Phi^T\in\mathbf{R}^{n\times n}$, and $B\in\mathbf{R}^{m\times n}$ such that $rank(B)<n$. The following statements are equivalent:*

(i) $\zeta^T\Phi\zeta<0,\ \forall B\zeta=0,\ \zeta\neq 0,$

(ii) $B^{\perp T}\Phi B^{\perp}<0,$

(iii) $\exists X\in\mathbf{R}^{n\times m}:\Phi+XB+B^TX^T<0.$

Lemma 2. *For any constant matrix $M>0$, the following inequality holds:*

$$-h(t)\int_{t-h(t)}^{t}\dot{x}^T(s)M\dot{x}(s)ds$$
$$\le\begin{bmatrix}x(t)\\x(t-h(t))\end{bmatrix}^T\begin{bmatrix}-M & M\\ \star & -M\end{bmatrix}\begin{bmatrix}x(t-h(t))\\x(t-h_m)\end{bmatrix}. \tag{16}$$

Proof. According to Jensen's inequality [17], one obtains

$$\left(\int_\beta^\alpha\dot{x}(s)ds\right)^TM\left(\int_\beta^\alpha\dot{x}(s)ds\right)\le(\alpha-\beta)\int_\beta^\alpha\dot{x}^T(s)M\dot{x}(s)ds. \tag{17}$$

If we choose $\alpha=t$ and $\beta=t-h(t)$ in (17), inequality (16) can be obtained. ■

3. Main Results

In this section, we propose stability criteria for system (13). Before introducing our main result, the notations are defined for simplicity:

$$\zeta^T(t)=\left[\ x^T(t)\ x^T(t-h(t))\ \dot{x}^T(t)\ x^T(t-h_m)\ \dot{x}^T(t-\tau(t))\right.$$
$$\left.f^T(x(t-h(t)))\ \right],$$
$$B(t)=\left[\ -A(t)\ 0\ -I\ \ 0\ C(t)\ W(t)\ \right],$$

$$\Phi^i = \begin{bmatrix} \Phi_{11}^i & R & P^i & 0 & 0 & 0 \\ \star & \Phi_{22}^i & 0 & R & 0 & 0 \\ \star & \star & h_m^2 R + T & 0 & 0 & 0 \\ \star & \star & \star & -Q_1^i - R & 0 & 0 \\ \star & \star & \star & \star & -(1-\tau_D)T & 0 \\ \star & \star & \star & \star & \star & -\epsilon I \end{bmatrix},$$

$$\Phi_{11}^i = \sum_{j=1}^{N} \pi_{ij} P^j + Q_1^i + Q_2^i + h_m(Q_1 + Q_2) - R,$$

$$\Phi_{22}^i = -(1-h_D)Q_2^i - 2R + \epsilon L^T L. \quad (18)$$

Now, we have the following theorem.

Theorem 1. *For given $h_m > 0$, h_D, τ_D and $L = diag\{l_1, l_2, \ldots, l_n\}$, the* system *(13) is stochastically stable for $0 \le h(t) \le h_m$, $\dot{h}(t) \le h_D$ and $\dot{\tau}(t) \le \tau_D < 1$, if $\| C(t) \| < 1$ and there exist positive definite matrices P^i, Q_1^i, Q_2^i, Q_1, Q_2, R, T, and positive scalar ϵ satisfying the following LMIs:*

$$\left(B_l^{i\perp}\right)^T \Phi^i \left(B_l^{i\perp}\right) < 0,\ l = 1, \cdots, r,\ i = 1, \cdots, N, \quad (19)$$

$$\sum_{j=1}^{N} \pi_{ij} Q_1^j \le Q_1, \quad (20)$$

$$\sum_{j=1}^{N} \pi_{ij} Q_2^j \le (1-h_D) Q_2, \quad (21)$$

where $B_l^{i\perp}$ is the right orthogonal complement of $B_l^i = \left[-A_l^i \ \ 0 - I \ \ 0 \ \ C_l^i \ \ W_l^i \right]$.

Proof. For positive definite matrices $P^i, Q_1^i, Q_2^i, Q_1, Q_2, R$, T and each $i \in S$, let us consider the following L-K functional candidate as

$$V(x(t), i) = V_1 + V_2 + V_3 + V_{4,} \quad (22)$$

where

$$V_1 = x^T(t) P^i x(t),$$

$$V_2 = \int_{t-h_m}^{t} x^T(s) Q_1^i x(s) ds + \int_{t-h(t)}^{t} x^T(s) Q_2^i x(s) ds + \int_{t-\tau(t)}^{t} \dot{x}^T(s) T \dot{x}(s) ds,$$

$$V_3 = \int_{t-h_m}^{t} \int_{s}^{t} x^T(u) Q_1 x(u) du ds + \int_{t-h(t)}^{t} \int_{s}^{t} x^T(u) Q_2 x(u) du ds,$$

$$V_4 = h_m \int_{t-h_m}^{t} \int_{s}^{t} \dot{x}^T(u) R \dot{x}(u) du ds. \quad (23)$$

According to the work [7], it is known that the random process $\{(x(t), \rho_t, t \ge 0)\}$ is a $\mathbf{C}_{n,h_m} \times S$-vauled Markov jump process with initial state $(\phi(\cdot), \rho_t)$. Applying on $V(x(t), i): \mathbf{C}_{n,h_m} \times S \times \mathbf{R}^+ \to \mathbf{R}$, its weak infinitesimal operator $\Im$ is defined by

$$\Im V(x(t), i) = \lim_{\delta \to 0_+} \frac{1}{\delta} [\, \mathbf{E}\{ V(x(t+\delta), \rho_{t+\delta} | x(t), \rho_t = i\} - V(x(t), \rho_t = i) \,]. \quad (24)$$

Then, we have

$$\Im V_1^i = 2x^T(t) P^i \dot{x}(t) + x^T(t) \left(\sum_{j=1}^{N} \pi_{ij} P^j \right) x(t),$$

$$\begin{aligned} \Im V_2^i \le\ & x^T(t)(Q_1^i + Q_2^i) x(t) + \dot{x}(t) T \dot{x}(t) \\ & - \begin{bmatrix} x(t-h(t)) \\ x(t-h_m) \\ \dot{x}(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} (1-h_D) Q_2^i & 0 & 0 \\ \star & Q_1^i & 0 \\ \star & \star & (1-\tau_D) T \end{bmatrix} \begin{bmatrix} x(t-h(t)) \\ x(t-h_m) \\ \dot{x}(t-\tau(t)) \end{bmatrix} \\ & + \int_{t-h_m}^{t} x^T(s) \left(\sum_{j=1}^{N} \pi_{ij} Q_1^j \right) x(s) ds \\ & + \int_{t-h(t)}^{t} x^T(s) \left(\sum_{j=1}^{N} \pi_{ij} Q_2^j \right) x(s) ds, \end{aligned}$$

$$\begin{aligned} \Im V_3 \le\ & x^T(t) h_m (Q_1 + Q_2) x(t) - \int_{t-h_m}^{t} x^T(s) Q_1 x(s) ds \\ & - (1-h_D) \int_{t-h(t)}^{t} x^T(s) Q_2 x(s) ds, \end{aligned}$$

$$\begin{aligned} \Im V_4 =\ & h_m^2 \dot{x}^T(t) R \dot{x}(t) - h_m \int_{t-h_m}^{t} \dot{x}^T(s) R \dot{x}(s) ds \\ =\ & h_m^2 \dot{x}^T(t) R \dot{x}(t) - h_m \int_{t-h(t)}^{t} \dot{x}^T(s) R \dot{x}(s) ds \\ & - h_m \int_{t-h_m}^{t-h(t)} \dot{x}^T(s) R \dot{x}(s) ds. \end{aligned} \quad (25)$$

By using Lemma 2, an upper bound of $\Im V_4$ can be obtained as

$$\Im V_4 \le h_m^2 \dot{x}^T(t) R \dot{x}(t) + \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_m) \end{bmatrix}^T \begin{bmatrix} -R & R & 0 \\ \star & -2R & R \\ \star & \star & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_m) \end{bmatrix}. \quad (26)$$

For any scalar $\epsilon > 0$, from Assumption 1, there exist that

$$0 \le \epsilon \{ x^T(t-h(t)) L^T L x(t-h(t)) - f^T(x(t-h(t)) f(x(t-h(t))) \}. \quad (27)$$

From (22)-(27), the $\Im V(x(t), i)$ has a new upper bound as

$$\begin{aligned} & \Im V(x(t), i) \\ & \le \zeta^T(t) \Phi^i \zeta(t) + \int_{t-h_m}^{t} x^T(s) \left(\sum_{j=1}^{N} \pi_{ij} Q_1^j - Q_1 \right) x(s) ds \\ & \quad + \int_{t-h(t)}^{t} x^T(s) \left(\sum_{j=1}^{N} \pi_{ij} Q_2^j - (1-h_D) Q_2 \right) x(s) ds, \end{aligned} \quad (28)$$

where Φ^i and $\zeta(t)$ are defined in (18).
If inequalities (20) and (21) hold, an upper bound of the inequality (28) can be obtained as

$$\Im V(x(t), i) \le \zeta^T(t) \Phi^i \zeta(t). \quad (29)$$

Also, the system (13) with the augmented vector $\zeta(t)$ and each $i \in S$ can be rewritten as

$$\sum_{l=1}^{r}\mu_l(\theta(t))B_l^i\zeta(t)=0,\ l=1,\ldots,r, \tag{30}$$

where $B_l^{i\perp}$ is defined in Theorem 1.
Therefore, a stability condition for system (13) is given as

$$\zeta^T(t)\Phi^i\zeta(t)<0,\ \sum_{l=1}^{r}\mu_l(\theta(t))B_l^i\zeta(t)=0. \tag{31}$$

From Lemma 1 (iii), inequality (31) is equivalent to the following condition where X is any matrix satisfying

$$\Phi^i+X\left(\sum_{l=1}^{r}\mu_l(\theta(t))B_l^i\right)+\left(\sum_{l=1}^{r}\mu_l(\theta(t))B_l^i\right)^T X^T<0. \tag{32}$$

By considering the conditions (12), at each plant rule, the condition (32) holds if

$$\begin{aligned}&\Phi^i+XB_1^i+B_1^{iT}X^T<0,\\&\quad\vdots\\&\Phi^i+XB_r^i+B_r^{iT}X^T<0.\end{aligned} \tag{33}$$

By utilizing Lemma 1 (ii), the inequalities (33) are equivalent to the following LMIs, respectively.

$$\begin{aligned}&\left(B_1^{i\perp}\right)^T\Phi^i\left(B_1^{i\perp}\right)<0,\\&\quad\vdots\\&\left(B_r^{i\perp}\right)^T\Phi^i\left(B_r^{i\perp}\right)<0.\end{aligned} \tag{34}$$

From the inequalities (34) and the convex-hull properties, if the LMI (19) satisfies, then inequality (31) holds.
Furthermore, from Eq. (29), for any $h_m\le t$, we have

$$\Im V(x(t),i)\le -x^T(t)\min_{i\in S}\left\{\lambda_{\min}\left(-\Phi^i\right)\right\}x(t). \tag{35}$$

By Dynkin's formula [16], we obtain

$$\begin{aligned}&\mathbf{E}\{V(x(t),i)\}-\mathbf{E}\{V(\phi,\rho_0)\}\\&\le -\min_{i\in S}\left\{\lambda_{\min}\left(-\Phi^i\right)\right\}\mathbf{E}\left\{\int_0^t x^T(s)x(s)ds\right\},\end{aligned} \tag{36}$$

which yields

$$\mathbf{E}\left\{\int_0^t x^T(s)x(s)ds\right\}\le\frac{V(\phi,\rho_0)}{\min_{i\in S}\left\{\lambda_{\min}\left(-\Phi^i\right)\right\}}. \tag{37}$$

According to the work [8], from (22) and (37), it is easy to prove that there exists a scalar σ, such that

$$\lim_{i\to\infty}\mathbf{E}\left\{\int_0^t x^T(s)x(s)ds|\phi,\rho_0\right\}\le\sigma\sup_{-h_m\le s\le 0}|\phi(s)|^2. \tag{38}$$

From Eq. (38) and Definition 1, it is concluded that the system (13) is stochastically stable. This completes our proof. ■

When the value of the time-derivative of time-delay, $\dot{h}(t)$, is unknown, then, by setting $Q_2^i=0$ and $Q_2=0$ in (23), we can obtain the following theorem.

Theorem 2. *For given* $h_m>0$, τ_D *and* $L=diag\{l_1,l_2,\ldots,l_n\}$, *the* system *(13) is stochastically stable for* $0\le h(t)\le h_m$ *and* $\dot{\tau}(t)\le\tau_D<1$, *if* $\|C(t)\|<1$ *and there exist positive definite matrices* P^i, Q_1^i, Q_1, R, T, *and positive scalar* ϵ *satisfying the following LMIs:*

$$B_l^{i\perp T}\hat{\Phi}^i B_l^{i\perp}<0,\ l=1,\cdots,r,\ i=1,\cdots,N, \tag{39}$$

$$\sum_{j=1}^{N}\pi_{ij}Q_1^j\le Q_1, \tag{40}$$

where $B_l^{i\perp}$ is defined in Theorem 1, and

$$\hat{\Phi}^i=\begin{bmatrix}\hat{\Phi}_{11}^i & R & P^i & 0 & 0 & 0\\ \star & -2R+\epsilon L^T L & 0 & R & 0 & 0\\ \star & \star & h_m^2 R+T & 0 & 0 & 0\\ \star & \star & \star & -Q_1^i-R & 0 & 0\\ \star & \star & \star & \star & -(1-\tau_D)T & 0\\ \star & \star & \star & \star & \star & -\epsilon I\end{bmatrix},$$

$$\hat{\Phi}_{11}^i=\sum_{j=1}^{N}\pi_{ij}P^j+Q_1^i+h_m Q_1-R. \tag{41}$$

4. Numerical Examples

In this section, we provide a numerical example to show the effectiveness of the proposed stability criterion.

Example 1. Consider the following system with two modes

$$\dot{x}(t)=\sum_{l=1}^{2}\mu_l(\theta(t))\left[-A_l^i x(t)+C_l^i\dot{x}(t-\tau(t))+W_l^i f(x(t-h(t)))\right], \tag{42}$$

where

$$A_1^1=\begin{bmatrix}1.4 & 0\\ 0 & 1.0\end{bmatrix},\ C_1^1=\begin{bmatrix}0.2 & 0\\ 0 & 0.1\end{bmatrix},\ W_1^1=\begin{bmatrix}0.5 & -0.2\\ 0.1 & -0.5\end{bmatrix},$$

$$A_2^1=\begin{bmatrix}0.9 & 0\\ 0 & 1.5\end{bmatrix},\ C_2^1=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\ W_2^1=\begin{bmatrix}0.3 & -0.6\\ 0.6 & 0.5\end{bmatrix},$$

$$A_1^2=\begin{bmatrix}1.0 & 0\\ 0 & 0.8\end{bmatrix},\ C_1^2=\begin{bmatrix}0.2 & 0\\ 0 & 0.4\end{bmatrix},\ W_1^2=\begin{bmatrix}0.3 & 0.8\\ -0.2 & 0.3\end{bmatrix},$$

$$A_2^2=\begin{bmatrix}1.0 & 0\\ 0 & 1.2\end{bmatrix},\ C_2^2=\begin{bmatrix}0.2 & 0\\ 0 & 0.3\end{bmatrix},\ W_2^2=\begin{bmatrix}0.2 & -0.2\\ 0.3 & 0.3\end{bmatrix},$$

$$f(x)=\frac{1}{2}(|x+1|-|x-1|),$$

$$\mu_1(\theta(t))=\sin^2(x_1),\ \mu_2(\theta(t))=\cos^2(x_1), \tag{43}$$

with $\Pi = \begin{bmatrix} -7 & 7 \\ 4 & -4 \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

The results of the upper bound of time-delay with different values of h_D and τ_D provided by Theorem 1 are listed as Table 1. When the value of the time-derivative of time-delay is unknown, then, by applying Theorem 2 to the above system (42) for $\tau_D = 0$, it can be obtained that the upper bound of time-delay is 0.314.

The simulation results for the state responses of the system (42) with two Markovian jumping modes $(i = 1, 2)$ are shown in Figures 1 and 2, respectively. We assume that the state-delay $h(t) = 0.5\sin^2(0.4t)$ with $h_m = 0.5$ and $h_D = 0.2$, and the neutral-delay $\tau(t) = \max\{h(t)\}$ with $\tau_D = 0$. To solve the above system (42) employ Fourth-order Runge-Kutta method with sampling time 0.0001[sec].

Figures 1 and 2 show that the system (42) with two modes responses converge to zero when initial values of the state are $x(0) = [1 \ -0.5]^T$. By selecting one of the results in Table 1, the simulation results confirm that the effectiveness of the proposed methods are demonstrated.

표 1 h_D와 τ_D의 다른 값들을 고려한 지연 한계값 h_m.
Table 1 Delay bounds h_m with different values of h_D and τ_D.

τ_D	0			0.5		
h_D	0.2	0.5	0.9	0.1	0.5	0.9
Theorem 1	0.560	0.314	0.314	0.167	0.110	0.110

5. Conclusions

In this paper, the delay-dependent stability criteria for the fuzzy Markovian jumping HNNs of neutral type with time-varying delays are proposed. To do this, the suitable L-K functional is used to investigate the feasible region of stability criteria. One numerical example has been given to show the effectiveness and usefulness of the presented criteria.

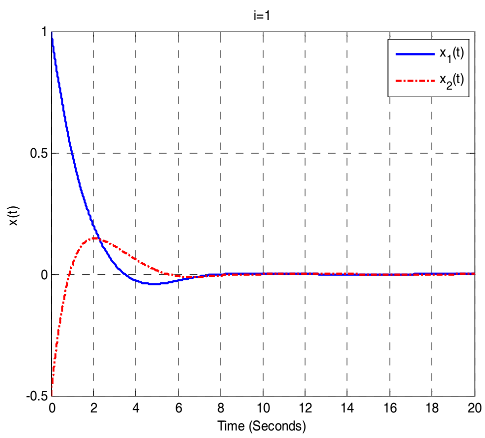

그림 1 모드 1을 고려한 시스템의 상태 응답.
Fig. 1 State responses of the system with $i = 1$.

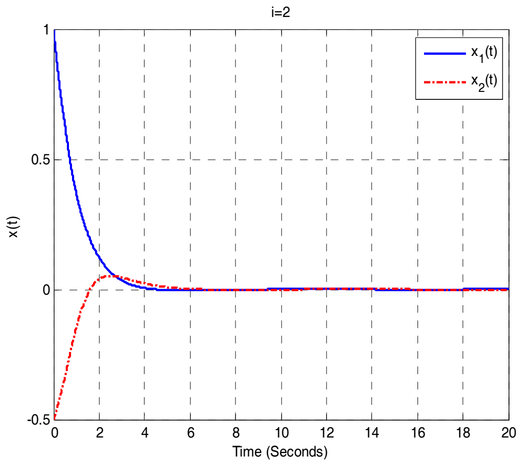

그림 2 모드 2를 고려한 시스템의 상태 응답.
Fig. 2 State responses of the system with $i = 2$.

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