# A Study on Performance Analysis for Error Probability in SWSK Systems

Tae-Il Jeong, kwang-seok Moon, and Jong-Nam Kim, *Member, KIMICS*

***Abstract*— This paper presents a new method for shift keying using the combination of scaling function and wavelet named scaling wavelet shift keying (SWSK). An algorithm for SWSK modulation is carried out where the scaling function and the wavelet are encoded to 1 and 0 in accordance with the binary input, respectively. Signal energy, correlation coefficient and error probability of SWSK are derived from error probability of frequency shift keying(FSK). The performance is analyzed in terms of error probability and it is simulated in accordance with the kind of the wavelet. Based on the results, we can conclude that the proposed scheme is superior to the performance of the conventional schemes.**

***Index Terms*— Correlation Coefficients, Frequency Shift Keying(FSK), Error Probability, Scaling Wavelet Shift Keying(SWSK), Wavelet Transforms.**

## I. INTRODUCTION

**THE** wavelet transform is mainly used in the field of audio and image-signal processing. It is recently used in digital communications. A promising application of wavelet transforms is in the field of digital wireless communications where they can be used to generate waveforms that are suitable for transmission over wireless channels. This type of modulation is known as wavelet transform. The advantage of this scheme emerges from its diversity strategy; wavelet modulation allows transmission of the data signal at multiple rates simultaneously [1]. Wornell and Oppenheim outlined the design of the transmitter and receiver for wavelet modulation [2]. The performance of wavelet modulation in an additive white Gaussian noise (AWGN) channel was also evaluated in Wornell's work. Ptasinski and Fellman simulated wavelet modulation using the Daubechies wavelet and measured the performance of the bit error rate (BER) using an AWGN channel. Ptasinski's work showed the performance of wavelet modulation to be equivalent to that calculated by Wornell[3].

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The simplest wavelet is Haar [4][5] and the communication signals

considered are amplitude shift keying (ASK), frequency shift keying(FSK) and phases shift keying(PSK), which are transmitted over an AWGN channel.

Amplitude shift keying, frequency shift keying and phase shift keying are the conventional digital communication scheme. In frequency shift keying, the frequency of the carrier varies in accordance with the binary input. For binary transmission the carrier assumes one frequency for 1 and another frequency for 0. This type of on-off modulation is called frequency shift keying.

Recently some attempts have been made to apply the wavelet transform in digital communication [6][7]. Reference [8] provides the modulation scheme which uses the mother wavelet instead of the carrier frequency such as phase shift keying. That is, the phase of the mother wavelet is varied according to the source signal. The modulation schemes which use the scaling function and wavelet, and the demodulation schemes which use the binary matched filter are introduced in [9][10].

In this paper, we propose a new shift keying using the scaling function and the wavelet. The three parameters: signal energy, correlation coefficient and error probability are also introduced, and are derived from the formulas of the frequency shift keying. The performance of SWSK is analyzed in terms of the error probability, which is obtained from the average energy and correlation coefficient.

This paper is organized as follows: Section Ⅱ reviews the mathematical preliminaries. An algorithm for modulator and demodulator for SWSK is presented in Section Ⅲ. Simulation results and performance analysis are presented in Section Ⅳ. Finally, conclusions are drawn in Section Ⅴ.

## II. MATHEMATICAL PRELIMINARIES

### *A. Error probability of frequency shift keying*

In frequency modulation, the frequency of the carrier varies in accordance with the binary input. For binary transmission the carrier assumes one frequency to be 1 and another frequency to be 0. This type of on-off modulation is called frequency shift keying. The frequency shift keying signals are defined by [11][12]

$$S_{FSK}(t) = \begin{cases} s_0(t) = A\cos\omega_0(t),\ 0 \le t \le T_b,\ for\ 0 \\ s_1(t) = A\cos\omega_1(t),\ 0 \le t \le T_b,\ for\ 1 \end{cases} \quad (1)$$

where, $s_0(t)$ and $s_1(t)$ correspond to the binary symbols 0 and 1, respectively. The average energy($E$) and the correlation coefficient( $\rho$ ) per bit for coherent matched filter is given as, respectively

$$E = \frac{1}{2}\int_0^{T_b}\left[\ s_0^{\ 2}(t) + s_1^{\ 2}(t)\ \right]dt \quad (2)$$

$$\rho = \frac{\int_0^{T_b}\left[s_0(t)s_1(t)\right]dt}{E} \quad (3)$$

The probability of error yields

$$Pe = \frac{1}{2}erfc\left(\sqrt{\frac{E(1-\rho)}{2\eta}}\right) = \frac{1}{2}erfc\left(\sqrt{\frac{A^2T_b}{4\eta}}\right) \quad (4)$$

where, *erfc* is complementary error function, and denotes the noise power in *watt/Hz*. A is the amplitude of carrier signals. It is assumed that the noise is additive white Gaussian noise with a two-sided power spectral density of $\eta/2$, mean zero and variance $\sigma^2 = \eta/2$. For the coherent detection, matched filter detection is optimum as illustrated in [11][12].

*B. Discrete Wavelet Transforms*

The discrete wavelet transform algorithms make use of bank of quadrature filters. The filter coefficients are based on the mother wavelet, ψ(t), and the scaling function, φ(t) derived from the mother wavelet. The mother wavelet and scaling function are used to implement high pass filters and low pass filters respectively. The procedure for decomposition of a signal using discrete wavelet transform at first, involves the convolution of the signal with a pair of quadrature filters to obtain the approximation coefficient from the low pass filter and to obtain the detailed coefficients from the the high pass filter. The outputed data sequences are then decimated by a factor of two. To compute the next level of resolution, the decimated sequence of detailed coefficients are inputted to the next set of quadrature filters. In this fashion the wavelet coefficients at the n$^{th}$ level of decomposion can be obtained[4][5].

Let the signal processing functions of the high pass filters and the low pass filters be denoted by the operators H and G. Now let {h(k)} be a square-summable sequence of coefficients which defines the linear operator H, and similarly {g(k)} for G. The relationship between the mother wavelet, the scaling function and the filter coefficients are defined as [7]

$$\Psi(t) = \sqrt{2\sum_k g(k)\ \varphi(2t-k)}$$

$$\varphi(t) = \sqrt{2\sum_k h(k)\ \varphi(2t-k)}$$

This methodology for implementing the discrete wavelet transform is known as multiresolution analysis. The filter coefficients have to satisfy the conditions as follows[4][5]:

$$\sum_n h[n] = \sqrt{2} \quad (5a)$$

$$\sum_n h^2[n] = 1 \quad (5b)$$

$$\sum_n g[n] = 0 \quad (6a)$$

$$\sum_n g^2[n] = 1 \quad (6b)$$

where $h[n]$ is the scaling function and corresponds to the coefficient of the low pass filter, $g[n]$ is the wavelet and corresponds to the coefficient of the high pass filter.

We will now discuss a mothod for construction of scaling function using successive approximation.

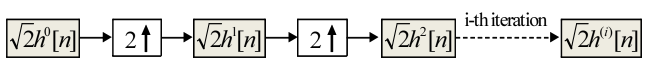



Fig. 1. Discrete impulse response for scaling functions.

The discrete impulse response

$$h^{(i)}[n] = \prod_{k=0}^{i} {}^{*}\, h_k[n] \quad (7)$$

can be obtained in the *i*-th iteration step by *i*-fold convolution of the dyadic upsampled inpulse response

$$h_k[n] = \begin{cases} \sqrt{2}h[m] & if\ n = 2^k m \\ 0 & otherwise \end{cases}$$

The impulse response $h^{(i)}[n]$ has $(N-1)(2^{i+1}-1)+1$ coefficients, where $N$ is the number of the coefficient of $h[n]$. If the number of iteration is incresed $i=\infty$ then the discrete impulse response is considerd to be a continuous signal. Hence equ.(7) can be written as[4][5]

$$h^{(\infty)}[n] = \prod_{k=0}^{\infty} {}^{*}\, h_k[n] \cong \varphi(t) \quad (8)$$

Similarly

$$g^{(\infty)}[n] = \prod_{k=0}^{\infty} {}^{*}\, g_k[n] \cong \Psi(t) \quad (9)$$

where, * means a convolution.

In practical applications, few iteration steps are enough to obtain the scaling function in Fig.1.

*C. Cross-correlation Receiver*

The optimum receiver for a known signal in an additive white gaussian noise is the correlator or matched filter. The correlator performs a cross-correlation of the received signal *r(t)*, with each of the prototype of the transmitted signal $s_m(t)$, on the interval $0 \le t \le T_b$, producing *m* outputs which are then compared by the detector. The decision circuit determines the largest magnitude output from the output of the sampler and declares it as the transmitted symbol [4].

## Ⅲ. SCALING WAVELET SHIFT KEYING

*A. SWSK modulation*

The conventional modulation schemes for frequency shift keying require two carrier frequencies. The high frequency is encoded to 1 and the low frequency is encoded to 0 for an input binary data. SWSK system requires a scaling function and wavelet instead of using two carrier frequencies in frequency shift keying. The scaling function is encoded to 1 and the wavelet is encoded to 0 for input binary data. It is important that scaling function and wavelet satisfy equ.(8) and (9), if the number of iteration of scaling function and wavelet are more than that obtained from the smoothing waveforms.

The shift keying system using the scaling function and the wavelet is called scaling wavelet shift keying(SWSK). We are defined as

$$S_{SWSK}(t) = \begin{cases} s_1(t) = \varphi(t), & for\ binary\ 1 \\ s_0(t) = \psi(t), & for\ binary\ 0 \end{cases} \quad (10)$$

For binary transmission the carrier assumes scaling function for 1 and wavelet for 0 as represented in equ.(10). Transmitter and receiver of SWSK systems are presented in Fig. 2. If the binary data is applied to SWSK modulator, it is encoded to the scaling function and wavelet by SWSK modulator. The modulated signal is added to the additive white Gaussian noise before transmission by the transmitter.

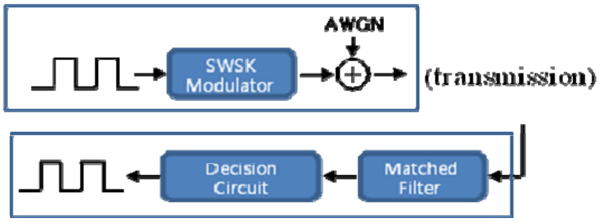



Fig.2. Block diagram of SWSK systems.

The SWSK receiver consists of a matched filter and a comparator (decision circuit). Coherent detection of SWSK is accomplished by comparing the outputs of two matched filters. The modulation algorithm for SWSK is represented in Fig. 3 which is similar to frequency shift keying. If the binary input is '1' then the modulator is encoded to scaling function, and if the binary input is '0' then encoded to the wavelet.

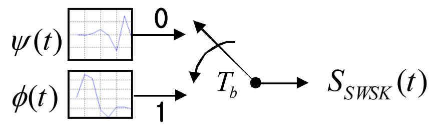



Fig. 3. SWSK system modulator.

*B. SWSK demodulation*

In SWSK demodulation, we find the sum of the correlation for the interval $0 \le t \le T_b$, later we compare it with the given threshold. If the sum is larger than the given threshold, the result would be 1 by the comparator, and if it is smaller, it would be 0 [9]. The algorithm for SWSK demodulation is described as follows:

1) Receive the SWSK signal which is included in additive white Gaussian noise.
2) Compute simultaneously the correlation by each correlator.
3) Obtain the matched filter outputs.
4) Reconstruct the binary output by the comparator (decision circuit).
5) Iterate step 1) – 4) until the last signal is achieved.

*C. Calculation of the signal energy, correlation coefficient and probability of error*

In this section, we derive the signal energy, the correlation coefficients and the probability of error for SWSK. To determine the probability of error for the matched filter, we need the signal energy and the correlation coefficients of the signal to be calculated in SWSK schemes. The average energy(*E*) and the correlation coefficients($\rho$) per bit for SWSK system is derived as[10]

$$E = \frac{1}{2}\int_0^{T_b} [\,\psi^2(t) + \varphi^2(t)]dt \quad (11)$$

$$\rho = \frac{\int_0^{T_b} [\,\varphi(t)\cdot\psi(t)]dt}{E} \quad (12)$$

From equ.(5a) ~ (6b), (8), (9) and Fig. 1, we can define the scaling function and wavelet as follows:

$$\int_0^{Tb} \varphi(t) = \sqrt{2} \quad (13a)$$

$$\int_0^{Tb} \varphi^2(t) = 1 \quad (13b)$$

$$\int_0^{Tb} \Psi(t) = 0 \quad (14a)$$

$$\int_0^{Tb} \Psi^2(t) = 1 \tag{14b}$$

By substituting equ.(13a)~(14b) in equ.(11) and (12) respectively, we get the average energy and the correlation coefficients per bit for SWSK system. Therefore, the probability of error ($Pe$) for SWSK can be derived as[10]

$$Pe = \frac{1}{2} erfc\left(\sqrt{\frac{E(1-\rho)}{2\eta}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{T_b}{2\eta}}\right) \tag{15}$$

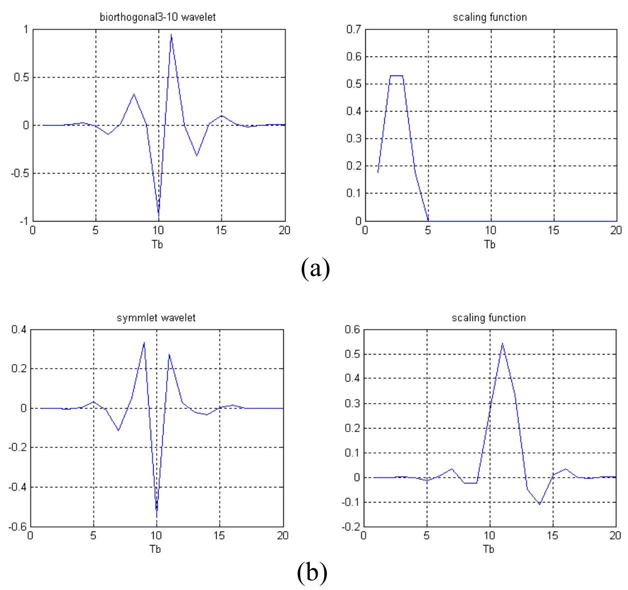
## Ⅳ. SIMULATION AND PERFORMANCE ANALYSIS

Computer simulations were performed to examine the probability of error for the SWSK system in the additive white Gaussian noise channel. The simulation was mainly performed by Daubechies5 wavelet. The results were obtained using the MATLAB software. Equations of average energy and correlation coefficient are used in equ.(11) and (12) in order to obtain the probability of error, which is used in equ.(15). The $SNR$ is between 0 to 12dB, and we set the given threshold to 0. We considered the following three modulation schemes:

1) ASK, A=1, ρ = 0;
2) FSK, A=1, ρ = 0;
3) PSK, A=1, ρ = -1;

where, A is the amplitude of signal

Fig. 4 shows the different kinds of wavelet used in this paper. The number of tap is 20. If the number of iteration is incresed then each figures are considered to be a continuous signal which satisfied equ.(13a)-(14b),



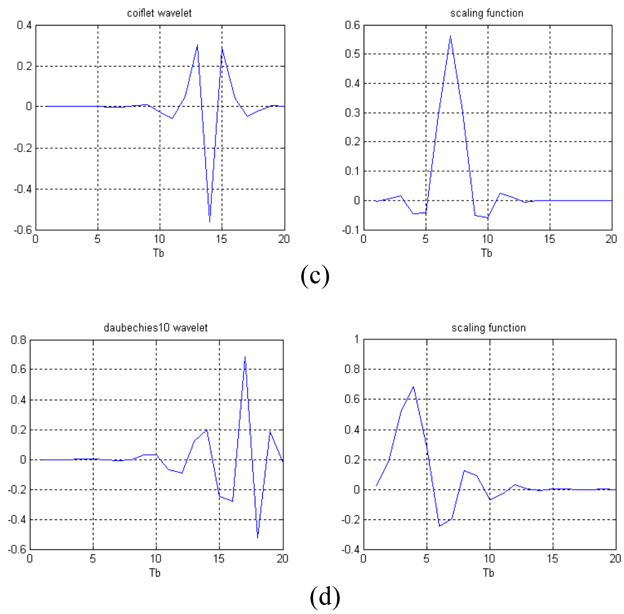




Fig. 4. Examples of different wavelet and scaling function: (a)Biorthogonal, (b) Symlet, (c) Coiflet3, (d) Daubechies10.

The experimental results are given in Tables Ⅰ, Ⅱ and Fig. 5~8. Fig. 5 shows the output waveform for the bit stream 1011001001 at $SNR$=6[dB] by SWSK scheme. The output waveforms are related as shown in Fig. 2. The waveform of the original data is observed in Fig. 5 (upper). It is then passed through the matched filter and received by the SWSK's receiver as shown in Fig. 5 (middle). Finally, the waveform data is reconstructed as depicted in Fig. 5 (bottom). We confirm that the inputted transmitted signal is perfectly reconstructed by the SWSK receiver.

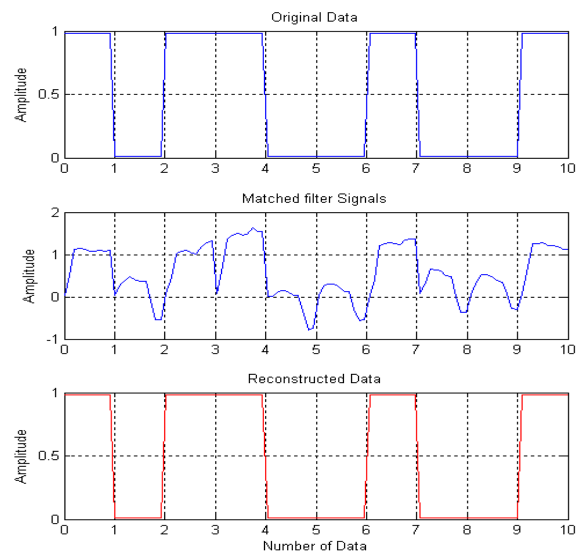



Fig. 5. The output waveform for bit stream 1011001001.

Table Ⅰ presents the performance of error probability for SWSK against the conventional communication schemes with *Tb*=0.01*sec*. It is observed that the performance of SWSK scheme is superior to the conventional schemes as bit error rate (BER) is 0.26 at *SNR* of 6*dB*. The error probability of phase shift keying and amplitude shift keying used equ.(16) and (17), respectively [11][12].

$$Pe = \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T_b}{2\eta}}\right) \tag{16}$$

$$Pe = \frac{1}{2} erfc\left(\sqrt{\frac{A^2 T_b}{8\eta}}\right) \tag{17}$$

TABLE Ⅰ
THE COMPARISON OF BER PERFORMANCE AGAINST SNR

| SNR (dB) | SWSK (ρ=0) | ASK (ρ=0) | FSK (ρ=0) | PSK (ρ=-1) |
|---|---|---|---|---|
| 0 | 0.33 | 0.46 | 0.46 | 0.44 |
| 2 | 0.29 | 0.46 | 0.45 | 0.43 |
| 4 | 0.24 | 0.44 | 0.44 | 0.41 |
| 6 | 0.16 | 0.43 | 0.42 | 0.39 |
| 8 | 0.13 | 0.39 | 0.4 | 0.36 |
| 10 | 0.079 | 0.36 | 0.38 | 0.33 |
| 12 | 0.038 | 0.33 | 0.35 | 0.29 |

Fig. 6 shows the BER against *SNR* for SWSK scheme at *Tb*=0.1*sec*. The performances of conventional schemes are also included in the figure for comparison. It is observed from the figure that the performance of SWSK is better than the conventional schemes. For example, at *SNR* of 6*dB*, the BER of SWSK using coherent matched filter is 0.023, where the BER of ASK, FSK, PSK are 0.27, 0.27 and 0.19, respectively.

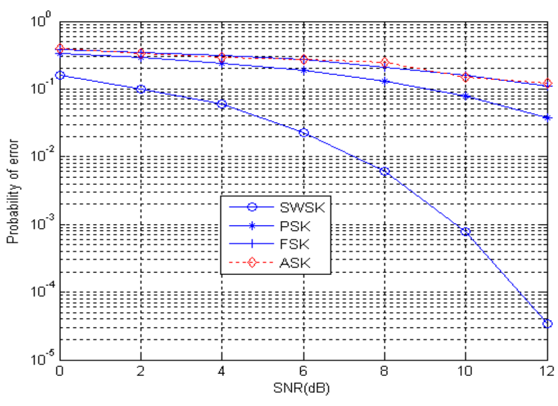



Fig. 6. BER versus *SNR* for different modulation schemes.

Fig. 7 shows the performance comparison for error probability at *SNR* of 6*dB*. Comparing with the conventional schemes, if the duration of the period is longer, the performance of SWSK is improved. For example, at *Tb*=1*sec*, the BER of SWSK using coherent matched filter is $1.4 \times 10^{-10}$. The BER of ASK, FSK, PSK are also $1.9 \times 10^{-2}$, $2.4 \times 10^{-2}$ and $2.5 \times 10^{-3}$, respectively.

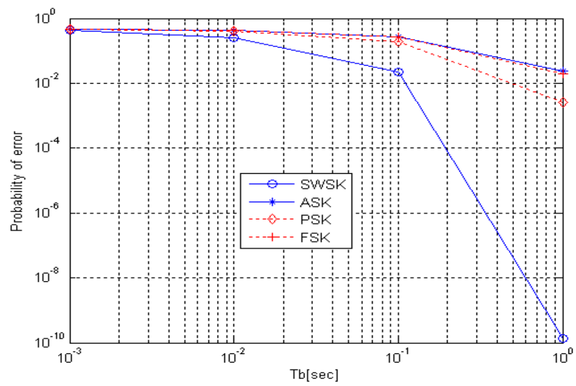



Fig. 7. BER versus period for different modulation schemes.

If amplitude (A) in equ.(16) is 1 then equ.(15) and (16) are equal. But the simulation experiment is different. Although the formula of error probability for SWSK is similar to phase shift keying, the error probabilities are different, and it is mainly due to the noise power.

Table Ⅱ shows the noise power against SNR for SWSK and phase shift keying schemes. It is noticed that the noise power of SWSK is about 10 times smaller when compared with the phase shift keying. Note that because the scaling function and wavelet are satisfied from (13a) to (14b), the noise power for SWSK signal is robust compared with phase shift keying scheme.

TABLE Ⅱ
THE COMPARISON OF NOISE POWER

| SNR(dB) | SWSK | PSK |
|---|---|---|
| 0 | 0.05 | 0.51 |
| 2 | 0.03 | 0.32 |
| 4 | 0.02 | 0.2 |
| 6 | 0.013 | 0.13 |
| 8 | 0.008 | 0.08 |
| 10 | 0.005 | 0.05 |
| 12 | 0.003 | 0.032 |

The comparison of BER performance for different wavelets for $T_b$=0.1*sec* is shown in Fig. 8. In this paper, the Daubechies, Coiflet3, Symlet5, and Biorthogonal wavelets are used. The results of Fig. 8 show experimentally that the Coiflet3 wavelet was superior to the other wavelets. The number of filter taps for the Coiflet3 wavelet is 18, Daubechies10 and Biorthogonal3_10 wavelets have 20, and the Daubechies5, Biorthogonal1_5 and Symlet5 wavelets have 10 taps. Increasing the number of filter taps causes the differences in error probability, which also depends on the wavelet characteristics.

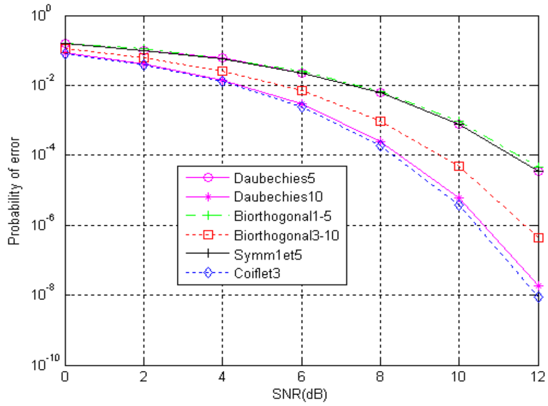



Fig. 8. The comparison of BER performance for wavelets, for *Tb*=0.1sec.

## V. CONCLUSIONS

The conventional communication schemes are presented as ASK, FSK and PSK. In this paper, we have theoretically and experimentally investigated the error probability for SWSK scheme which is a new shift keying using scaling function and wavelet. The formula of error probability for SWSK was derived from the frequency shift keying. The SWSK performance was compared with the conventional communication scheme in terms of error probability. The simulation results show that the performance of BER is given as 0.023 at $T_b$=0.1*sec* for SWSK scheme. Based on the results achieved we conclude that the error probability of SWSK scheme is superior to the conventional schemes. Regarding Fig. 7, more theoretical confirmation will be added in future papers.

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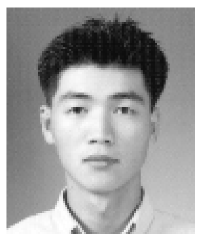

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