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OVERRINGS OF t-COPRIMELY PACKED DOMAINS

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Abstract. It is well known that for a Krull domain R, the divisor class group of R is a torsion group if and only if every subintersection of R is a ring of quotients. Thus a natural question is that under what conditions, for a non-Krull domain R, every (t-)subintersection (resp., t-linked overring) of R is a ring of quotients or every (t-)subintersection (resp., t-linked overring) of R is flat. To address this question, we introduce the notions of $*$-compact packedness and $*$-coprime packedness of (an ideal of) an integral domain R for a star operation $*$ of finite character, mainly t or w. We also investigate the t-theoretic analogues of related results in the literature.

1. Introduction

The notion of compact packedness of (an ideal of) a commutative ring with identity, as a generalization of the Prime Avoidance Theorem, was first introduced by C. Reis and T. Viswanathan in [32], and further investigated by J. V. Pakala and T. S. Shores, N. Popescu, and W. Smith in [30, 31, 34]. This notion was generalized to that of coprime packedness of (an ideal of) a commutative ring with identity by V. Erdoğdu in [9] and further extensively studied by V. Erdoğdu, S. McAdam, D. E. Rush and L. J. Wallace in [10, 11, 12, 13, 33]. Recently in [4], the t-analogue notions of compact packedness and related properties were introduced by G. W. Chang and C. J. Hwang and generalized weakly factorial domains and weakly Krull domains were characterized in terms of these notions.

Let T be an overring of an integral domain R. Then the following chain of implications is well known: T is a ring of quotients of $R \Rightarrow T$ is flat over $R \Rightarrow$ T is t-flat over $R \Rightarrow T$ is a t-subintersection of $R \Rightarrow T$ is a generalized ring of quotients of $R \Rightarrow T$ is t-linked over R (We will define these concepts later). It was shown in [25, Proposition 2.10] that a domain R is a PvMD if and only if every t-linked overring of R is t-flat. Thus the last four conditions are equivalent

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for PvMDs, but in general none of the implications can be reversed ([16, p. 287]). On the other hand, it is also well known that for a Krull domain R, the divisor class group of R is a torsion group if and only if every subintersection of R is a ring of quotients ([15, Proposition 6.8]). Thus a natural question is that under what conditions, for a non-Krull domain R, every (t-)subintersection (resp., t-linked overring) of R is a ring of quotients or every (t-)subintersection (resp., t-linked overring) of R is flat. To address this question, we introduce the notions of $*$-compact packedness and $*$-coprime packedness of (an ideal of) an integral domain R for a star operation $*$ of finite character, mainly t or w. We also investigate the t-theoretic analogues of related results of [11, 27, 29, 35], using methods developed there.

Throughout this paper, R denotes an integral domain with quotient field K. Let $\mathcal{F}(R)$ denote the set of nonzero fractional ideals of R. A $*$-*operation* (*star operation*) on R is a mapping $A \to A_*$ from $\mathcal{F}(R)$ to $\mathcal{F}(R)$ which satisfies the following conditions for all $a \in K \setminus \{0\}$ and $A, B \in \mathcal{F}(R)$: (1) $(a)_* = (a)$ and $(aA)_* = aA_*$, (2) $A \subseteq A_*$; if $A \subseteq B$, then $A_* \subseteq B_*$, and (3) $(A_*)_* = A_*$. For details on star operations, the reader may consult [18, Sections 32 and 34]. Yet for our purposes, we include some of the definitions. An $A \in \mathcal{F}(R)$ is called a $*$-*ideal* if $A_* = A$. Recall that the function on $\mathcal{F}(R)$ defined by $A \mapsto A_v := (A^{-1})^{-1}$ is a star operation called the *v-operation*, where $A^{-1} := R :_K A = \{x \in K \mid xA \subseteq R\}$. The *t-operation* on R is the star operation defined by $A \mapsto A_t := \cup\{J_v | J \subseteq A \text{ with } J \in \mathcal{F}(R) \text{ finitely generated}\}$. The *w-operation* on R is the star operation defined by $A \mapsto A_w := \{x \in K \mid Jx \subseteq A$ for some finitely generated ideal J of R with $J^{-1} = R\}$ ([36]). Finally the identity mapping on $\mathcal{F}(R)$ is obviously a star operation; it is called the *d-operation*. Given a star operation $*$, we have $(AB)_* = (A_*B_*)_*$ for all $A, B \in \mathcal{F}(R)$. This equation is said to define the $*$-multiplication. For any star operation $*$ and for any $A \in \mathcal{F}(R)$, we have $A \subseteq A_* \subseteq A_v$, and hence $(A_*)_v = A_v$. In particular, a v-ideal (divisorial ideal) is a $*$-ideal for any $*$. A star operation $A \mapsto A_*$ is said to be *of finite character* if $A_* = \bigcup (A_i)_*$ for each $A \in \mathcal{F}(R)$, where $\{A_i\}$ is the family of nonzero finitely generated fractional ideals of R contained in A. The t-operation, w-operation, and d-operation on R are the most important examples of star operations of finite character.

Let $*$ be a star operation of finite character on an integral domain R. Then the set of integral proper $*$-ideals has maximal elements under inclusion, called $*$-*maximal ideals*, and these ideals are prime. A $*$-ideal which is prime is also called a $*$-*prime ideal*. We denote by $*$-Spec(R) the set of $*$-prime ideals of R and by $*$-Max(R) the set of $*$-maximal ideals of R. In [21], E. G. Houston defined the t-dimension of a domain R to be the supremum of the lengths of all chains of t-primes in R. The w-dimension of R can be also defined similarly. In particular, if each t-maximal ideal of R has height one, we may say that R has t-dimension one. We will often use the following well-known facts.

- A minimal prime ideal of a t-ideal (resp., w-ideal) is a t-ideal (resp., w-ideal), e.g., a height-one prime ideal is a t-ideal (resp., w-ideal).
- t-Max(R)=w-Max(R).

In [36, 37], F. Wang and R. L. McCasland defined an integral domain R to be a *strong Mori domain* (for short, SM domain) if R satisfies the ascending chain condition on w-ideals. Note that the class of Noetherian domains is properly contained in the class of SM domains and the class of SM domains is properly contained in the class of Mori domains (An integral domain R is called a *Mori domain* if R satisfies the ascending chain condition on v-ideals). Recall that a domain R is a *Prüfer v-multiplication domain* (or *PvMD* for short) if R_M is a valuation domain for each t-maximal ideal M of R. Note that both SM domains and PvMDs are generalizations of Krull domains.

A nonempty set $\mathscr{F}$ of nonzero ideals of R is called a *multiplicative system of ideals* if $IJ \in \mathscr{F}$ for each $I, J \in \mathscr{F}$. The ring $R_{\mathscr{F}} := \{x \in K \mid xI \subseteq R$ for some $I \in \mathscr{F}\}$ is called a *generalized ring of quotients* of R.

A particular type of multiplicative system is a *localizing system*. This is a set $\mathscr{F}$ of ideals of R such that (1) if $I \in \mathscr{F}$ and J is an ideal of R with $I \subseteq J$, then $J \in \mathscr{F}$, and (2) if $I \in \mathscr{F}$ and J is an ideal of R with $(J :_R a) \in \mathscr{F}$ for every $a \in I$, then $J \in \mathscr{F}$. If Λ is a subset of Spec(R), then $\mathscr{F}(\Lambda) := \{I \mid I$ is an ideal of R such that $I \nsubseteq P$ for each $P \in \Lambda\}$ is a localizing system. It is easy to see that $R_{\mathscr{F}(\Lambda)} = \bigcap_{P \in \Lambda} R_P$.

These notions have t-analogues. A set of t-ideals is a *t-multiplicative system* if it is closed under t-multiplication; a t-multiplicative system $\mathscr{T}$ is a *t-localizing system* if it satisfies the closure operations (1) and (2) above. Denoting the set of t-ideals of R by $t(R)$, it is easy to see that if $\mathscr{F}$ is a localizing system, then $\mathscr{T} := \mathscr{F} \cap t(R)$ is a t-localizing system and $R_{\mathscr{T}} = R_{\mathscr{F}}$ ([16, p. 287]).

An overring T of R is called a *t-subintersection* of R if it has the form $\bigcap R_P$, where the intersection is taken over some set of t-primes P of R. We say that T is *t-flat* over R if $T_M = R_{M \cap R}$ for each t-maximal ideal M of T. Finally, recall from [6] that T is *t-linked* over R if for each finitely generated ideal I of R with $I^{-1} = R$ we have $(IT)^{-1} = T$. Following [5], we say that an integral domain R is *t-linkative* if every overring of R is t-linked over R.

For unexplained terminology and notation, we refer to [15, 18, 24].

2. On Prüfer v-multiplication domains

Let R be an integral domain and let $*$ be a star operation of finite character on R. An integral $*$-ideal I of R is said to be *$*$-coprimely packed* (resp., *$*$-compactly packed*) if for any set Λ of $*$-maximal (resp., $*$-prime) ideals of R with $I \subseteq \bigcup_{Q \in \Lambda} Q$, one has $I \subseteq P$ for some $P \in \Lambda$. A class $\mathscr{I}$ of integral $*$-ideals is said to be *$*$-coprimely packed* (resp., *$*$-compactly packed*) if every element of $\mathscr{I}$ is $*$-coprimely packed (resp., $*$-compactly packed). Finally, R is said to be *$*$-coprimely packed* (resp., *$*$-compactly packed*) if every $*$-ideal of R is $*$-coprimely packed (resp., $*$-compactly packed). Then it is clear that if R

is $*$-compactly packed, then it is $*$-coprimely packed, but we believe that in general the converse does not hold (we will provide a counterexample for the case $* = t$ after Corollary 3.3). It is obvious that $*$-compactly packedness and $*$-coprime packedness are the same in the case of $*$-$\dim(R) = 1$. If $* = d$, then the notions of d-compactly packed and d-coprimely packed are just those of compactly packed and coprimely packed introduced in [32] and [9] respectively. It was shown in [4, Proposition 3.1] that t-Spec(R) is t-compactly packed if and only if every prime t-ideal of R is the radical of a principal ideal.

If S is a multiplicatively closed subset of R, then $\mathscr{F}_S$ denotes the localizing system consisting of all ideals J such that $J \cap S \neq \emptyset$. As mentioned in the introduction, $\mathscr{T}_S := \mathscr{F}_S \cap t(R)$ is a t-localizing system consisting of all t-ideals I such that $I \cap S \neq \emptyset$. In particular, if P is a prime ideal of R, then $\mathscr{T}_P := \mathscr{T}_{R\setminus P} = \mathscr{F}(\{P\}) \cap t(R)$.

Theorem 2.1. *Let R be an integral domain. Then the following statements are equivalent.*

(1) *R is t-compactly packed.*
(2) t-Spec(R) *is t-compactly packed.*
(3) *Every prime t-ideal of R is the radical of a principal ideal.*
(4) *If a t-localizing system $\mathscr{F}$ is of the form $\mathscr{F}(\Lambda) \cap t(R)$ for some $\Lambda \subseteq t$-*Spec(R)*, then there exists a multiplicatively closed subset S of R such that $\mathscr{F} = \mathscr{T}_S$.*

Proof. (1) $\Rightarrow$ (2). It is obvious.

(2) $\Leftrightarrow$ (3). [4, Proposition 3.1].

(3) $\Rightarrow$ (1). Suppose that every prime t-ideal of R is the radical of a principal ideal and that $I \subseteq \bigcup_{P\in\Lambda} P$ for some t-ideal I of R and subset Λ of t-Spec(R). Let $S := R\setminus\bigcup_{P\in\Lambda} P$, which is a multiplicatively closed subset of R and expand I to a t-ideal Q, which is maximal with respect to avoiding S. Indeed, this follows from Zorn's Lemma and the fact that, for every directed family $\{I_\alpha\}$ of integral t-ideals, $\bigcup_\alpha I_\alpha$ is a t-ideal. Then such a Q is necessarily (t-)prime, so the radical of a principal ideal, say xR. Now $x \in P$ for some $P \in \Lambda$, whence $I \subseteq P$ as desired.

(1) $\Rightarrow$ (4). Let $\mathscr{F}$ be a t-localizing system of the form $\mathscr{F}(\Lambda) \cap t(R)$ for some $\Lambda \subseteq t$-Spec(R) and let $S = R \setminus \bigcup_{P\in\Lambda} P$. Then it is easy see that S is a multiplicatively closed subset of R and $\mathscr{T}_S \subseteq \mathscr{F} = \bigcap_{P\in\Lambda} \mathscr{T}_P$. Now we will show that $\mathscr{F} \subseteq \mathscr{T}_S$. If I is a t-ideal of R such that $I \notin \mathscr{T}_S$, then $I \cap S = \emptyset$, and so $I \subseteq \bigcup_{P\in\Lambda} P$; thus by hypothesis $I \subseteq P$ for some $P \in \Lambda$. This implies that $I \notin \mathscr{T}_P$, whence $I \notin \mathscr{F}$. Therefore $\mathscr{F} = \mathscr{T}_S$.

(4) $\Rightarrow$ (1). Let I be a t-ideal of R and $\{P_i\}_i \subseteq t$-Spec(R) such that $I \subseteq \bigcup_i P_i$ be given. Set $S := R \setminus \bigcup_i P_i$ and $\mathscr{F} := \bigcap_i \mathscr{T}_{P_i}$. Then by hypothesis $\mathscr{F} = \mathscr{T}_{S'}$ for some multiplicatively closed subset S' of R. It is easy to see that $P_i \notin \mathscr{F}$ for any i. So $P_i \cap S' = \emptyset$, according to the definition of $\mathscr{T}_{S'}$. Thus we have shown that $S' \subseteq S$, and hence $\mathscr{T}_{S'} \subseteq \mathscr{T}_S$. But for any i we have $P_i \cap S = \emptyset$, whence $\mathscr{T}_S \subseteq \mathscr{T}_{P_i}$ and so $\mathscr{T}_S \subseteq \bigcap_i \mathscr{T}_{P_i} = \mathscr{F} = \mathscr{T}_{S'}$. But $I \notin \mathscr{T}_{S'}$ and then

$I \notin \bigcap_i \mathscr{T}_{P_i}$, thus $I \notin \mathscr{T}_{P_i}$ for some i. Thus $I \subseteq P_i$. Therefore R is t-compactly packed. □

Note that the equivalence (1) and (4) of Theorem 2.1 is the t-theoretic analogue of [31, Théorème 2.1].

Proposition 2.2. *If an integral domain R is t-compactly packed, then every t-flat overring of R is a ring of quotients of R.*

Proof. Let T be a t-flat overring of R, and let Φ be the family of all t-prime ideals P of R such that $R_P \supseteq T$. Then $T = \bigcap_{P\in\Phi} R_P$. Now let $S := R \cap U(T)$, where $U(T)$ is the set of all units in T. Then S is a multiplicatively closed subset of R. Since R_S is also a (t-)flat overring of R, we have that $T \supseteq R_S = \bigcap_{Q\in\Psi} R_Q$ for the family Ψ of all t-prime ideals Q of R such that $R_Q \supseteq R_S$ and $\Phi \subseteq \Psi$. Assume that $\Phi \neq \Psi$ and let $Q \in \Psi \setminus \Phi$. Then $Q \nsubseteq \bigcup_{P\in\Phi} P$ by the t-compact packedness. Indeed, if $Q \subseteq \bigcup_{P\in\Phi} P$, then $Q \subseteq P$ for some $P \in \Phi$, and so $T \subseteq R_P \subseteq R_Q$, and hence also $Q \in \Phi$, a contradiction. Now there exists an $x \in Q \setminus \bigcup_{P\in\Phi} P$. Thus $x \in U(T) \cap R = S$ but x is not a unit in R_S, which is impossible. Hence $\Phi = \Psi$ and $T = R_S$. □

An integral domain R is said to have the *QR-property* if every overring of R is a ring of quotients. The tQR-property, which is the t-theoretic analogue of the QR-property, was introduced and studied in [7]: A domain R has the *tQR-property* (or is a *tQR-domain*) if each t-linked overring of R is a ring of quotients of R. It was shown in [7, Theorem 1.3] that for a PvMD R, R is a tQR-domain if and only if for each finitely generated ideal A of R, we have $A^n \subseteq bR \subseteq A_v$ for some $n \geq 1$ and some $b \in R$. Thus it is clear that GCD-domains are tQR-domain, since every finite type v-ideal of a GCD-domain is principal. It follows from [25, Proposition 2.10] that a tQR-domain is necessarily a PvMD (and hence integrally closed), and conversely, we have the following result.

Corollary 2.3. *If an integral domain R is a t-compactly packed PvMD, then R has the tQR-property.*

Example 2.4. (1) A simple and nontrivial example of a t-compactly packed PvMD (i.e., non-Krull) is a rank-one non-discrete valuation domain V.

(2) There exists an example of a t-compactly packed domain which is not a tQR-domain. Let K be an algebraically closed field and F a proper subfield such that $[K : F] = \infty$ and X an indeterminate. Then it is well-known that $R := F + XK[X]$ is a non-Noetherian one-dimensional domain with $\mathrm{Max}(R) = \{XK[X]\} \cup \{(1 - aX)R \mid a \in K \setminus \{0\}\}$. Clearly, R is a coprimely packed (compactly packed) t-linkative domain ([6, Corollary 2.7]) that is not semilocal. Since R is t-linkative, we have $d = w$ on R, and so R is w-coprimely packed (w-compactly packed). Since w-$\dim(R) = 1$ and w-$\mathrm{Max}(R) = t$-$\mathrm{Max}(R)$, we have w-$\mathrm{Spec}(R) = t$-$\mathrm{Spec}(R)$, and so by Theorem 2.1, R is also t-coprimely packed (t-compactly packed). Note that R is not a Krull domain (actually it is not a PvMD). Indeed, if not, then R is a one-dimensional Krull domain. Thus

by [18, (43.16) Theorem] R is a Dedekind domain. Hence R is Noetherian, a contradiction.

Recall that an integral domain R is called a *weakly factorial domain* (or *WFD* for short) if each nonzero nonunit of R is expressible as a product of primary elements of R. Also we recall that a domain R is a *generalized weakly factorial domain* (GWFD) if each nonzero prime ideal of R contains a principal primary ideal. It was shown in [4, Theorem 3.2] that a domain R is a GWFD if and only if R is t-compactly packed with t-$\dim(R) = 1$. The following result generalizes [26, Corollary 9]: Every flat overring of a WFD R is a quotient ring of R.

Corollary 2.5. *Every t-flat overring of a WFD R is a ring of quotients of R.*

Proof. This follows from [4, Proposition 3.1 and Theorem 3,2] and the fact that every WFD is a GWFD. □

Corollary 2.6. *If an integral domain R has only a finite number of t-prime ideals, then every t-flat overring of R is a ring of quotients of R.*

Proof. It follows from the Prime Avoidance Theorem that R is t-compactly packed. □

The $t\#$-property was introduced and studied in [16]: A domain R has the *$t\#$-property* (or is a *$t\#$-domain*) if $\bigcap_{M\in\mathscr{M}_1} R_M \neq \bigcap_{N\in\mathscr{M}_2} R_N$ for any two distinct subsets $\mathscr{M}_1, \mathscr{M}_2$ of the set of t-maximal ideals of R. It follows from [16, Corollary 1.3] and [36, Proposition 5.7] that every H-domain (and hence (strong) Mori domain) is a $t\#$-domain (A domain R is said to be an *H-domain* if for each ideal I of R with $I^{-1} = R$, there exists a finitely generated ideal $J \subseteq I$ such that $J^{-1} = R$). Let Δ denote the set of all t-maximal ideals of R, and let $\Delta_P = \Delta \setminus \{P\}$ for each $P \in \Delta$.

Lemma 2.7. *Let R be a PvMD. Then*

(1) *R has the property $t\#$ if and only if $R_Q \nsupseteq \bigcap_{P\in\Delta_Q} R_P$ for each $Q \in \Delta$.*
(2) *If R has the property $t\#$ and if t-$\dim R$=1, then each t-linked overring of R has the property $t\#$.*
(3) *If $Q \nsubseteq \bigcup_{P\in\Delta_Q} P$ for each $Q \in \Delta$, then R has the property $t\#$.*
(4) *If R has the tQR-property, then $P \subseteq \bigcup_{\alpha\in I} P_\alpha$ if and only if $\bigcap_{\alpha\in I} R_{P_\alpha} \subseteq R_P$ for each family $P \cup \{P_\alpha\}_{\alpha\in I}$ of t-prime ideals of R.*
(5) *Every t-linked overring of R has the property $t\#$ if and only if, for each t-prime ideal P of R, there exists a finitely generated ideal $J \subseteq P$ such that each t-maximal ideal of R containing J also contains P.*
(6) *R has the property $t\#$ if and only if there is a unique set $\{P_\alpha\}$ of incomparable t-primes such that $R = \bigcap R_{P_\alpha}$.*

Proof. (1) [16, Theorem 1.2]. (2) This follows from [16, Proposition 2.8] and [16, Theorem 1.2] (cf. [17, Corollary 2]). (3) This follows from (1) (cf. [17, Lemma 2]). (4) (cf. [17, Lemma 3]). (5) [16, Proposition 2.8: (7) $\Leftrightarrow$ (10)] (cf. [19, Theorem 3]). (6) [16, Theorem 1.8]. $\square$

Since Lemma 2.7(3) holds, t-compact packedness of R implies the property $t\#$. Moreover, from Lemma 2.7(5) and Theorem 2.1 it can be proved that if R is t-compactly packed, then each t-linked overring of R has the property $t\#$.

Theorem 2.8. *A PvMD R is t-compactly packed if and only if R has the tQR-property and every t-linked overring of R has the property $t\#$.*

Proof. Suppose that a PvMD R is t-compactly packed. Then the tQR-property follows from Corollary 2.3, while Lemma 2.7(5) implies that every t-linked overring of R has the property $t\#$.

Conversely, suppose that a PvMD R has the tQR-property and every t-linked overring of R has the property $t\#$. Let $P \subseteq \bigcup_{\alpha\in I} P_\alpha$, where P and P_α are t-prime ideals of R. Suppose that P is not contained in any P_α. Without loss of generality, we may assume that there are no containment relations among the P_α's. Since $S := \bigcap_{\alpha\in I} R_{P_\alpha}$ has the property $t\#$, it follows from Lemma 2.7(6) that $S' := S \cap R_P \neq S$. Since R has the tQR-property, we have that $S = R_M$ and $S' = R_N$ for some saturated multiplicatively closed subsets M, N of R. In this case both sets M and N must equal $R \setminus \bigcup_{\alpha\in I} P_\alpha$, so that $S = S'$, a contradiction. Thus P is contained in some P_α. $\square$

As a consequence, since Lemma 2.7(2) holds, we obtain the following generalization of Lemma 2.7(3).

Corollary 2.9. *Let R be a PvMD with t-$\dim(R) = 1$. Then $Q \nsubseteq \bigcup_{P\in\Delta_Q} P$ for each t-maximal ideal Q of R if and only if R has the tQR-property and the property $t\#$.*

Let R be an integral domain and $*$ be a star-operation on R. Then we set $N_* := \{f \in R[X] \mid C(f)_* = R\}$, where $C(f)$ is the content ideal of R generated by the coefficients of f. We call $R[X]_{N_*}$ the *$*$-Nagata ring* of R.

Lemma 2.10. (1) (*cf.* [18, (19.6) Theorem]) *Let R be an integral domain with quotient field K and let P be a prime t-ideal of R. Then there exists a t-linked valuation overring V with maximal ideal M such that $M \cap R = P$.*

(2) *Let R be a PvMD. Then any t-linked valuation overring of R is of the form R_P for some prime t-ideal P of R.*

Proof. (1) Let P be a prime t-ideal of R. Then $PR[X]_{N_t}$ is a prime ideal of the ring $R[X]_{N_t}$. Thus by [18, (19.6) Theorem], there exists a valuation overring V' of $R[X]_{N_t}$ with maximal ideal M' such that $M' \cap R[X]_{N_t} = PR[X]_{N_t}$. Set $V := V' \cap K$. Then by [3, Lemma 3.3], V is a t-linked valuation overring with maximal ideal $M := M' \cap K$. It is clear that $M \cap R = P$.

(2) Let V be a t-linked valuation overring of R with maximal ideal M. Let $P := M \cap R$. Then since V is a t-linked valuation overring of R, it follows from [25, Lemma 2.9] that P is a prime t-ideal of R. We show that $V = R_P$. The

inclusion $R_P \subseteq V$ is straightforward. Now since R is a PvMD, R_P is a valuation domain with maximal ideal PR_P. Since $P \subseteq M$, we have $PR_P \subseteq MV_M = M$. Thus $V \subseteq R_P$ by [18, (17.6) Theorem]. Hence $V = R_P$. □

The following result shows that for a t-compactly packed domain R, to determine if R is a tQR-domain, it suffices to consider all t-linked valuation overrings of R.

Theorem 2.11. *If a domain R is t-compactly packed and every t-linked valuation overring of R is a ring of quotients of R, then R has the tQR-property.*

Proof. Let T be a t-linked overring of R. Then $T = \bigcap T_M$, where the intersection is taken over all t-maximal ideals M of T. Then for such t-maximal ideal M of T, by Lemma 2.10(1), there exists a t-linked valuation overring (W, N) of R such that $T \subseteq W$ and $N \cap T = M$. But by hypothesis, W is a ring of quotients of R and hence is of the form R_P, where $P = N \cap R$. Therefore $R_P \subseteq T_M \subseteq W$ and $R_P = W$ implies that $R_P = T_M$. Thus $T = \bigcap R_{P_\lambda}$, where $\{P_\lambda\}$ is a collection of prime t-ideals of R. We then have $R \subseteq T' := R_S \subseteq T = \bigcap R_{P_\lambda}$, where $S = R \setminus \bigcup P_\lambda$. But as before, T' can be written $T' = \bigcap R_{P_\mu}$, where $\{P_\mu\}$ is a collection of prime t-ideals of R. Therefore $R_{P_\mu} \supseteq R_S$ implies that $P_\mu \subseteq \bigcup P_\lambda$. Since R is t-compactly packed, $P_\mu \subseteq P_\lambda$ for some λ, and so $R_{P_\mu} \supseteq R_{P_\lambda}$. Thus for each μ there exists a λ such that $R_{P_\lambda} \subseteq R_{P_\mu}$. So $T' = \bigcap R_{P_\mu} \supseteq \bigcap R_{P_\lambda} = T$. Therefore $T' = T$ and T is a ring of quotients of R. □

Remark 2.12. Using Lemma 2.10(2) and Theorem 2.11, we can give another proof of Corollary 2.3.

For the rest of this section, we fix the following notations. Let R be an integral domain with quotient field K, L be an algebraic extension of K, and D be the integral closure of R in L. If R is a QR-domain, then D need not be a QR-domain, even if L/K is finite-dimensional Galois ([20, Section 4]). As mentioned in [14, p. 46], H. Prüfer showed that the integral closure of a Prüfer domain (respectively, a PvMD) in an algebraic field extension is still a Prüfer domain (respectively, a PvMD). In [27], J. L. Mott established some sufficient conditions for D to be a QR-domain, for example, if R is a QR-domain and D is compactly packed, then D is a QR-domain ([27, Corollary 2.2]). Now we generalize this t-theoretically.

Theorem 2.13. *Let R be a PvMD (resp., Prüfer domain). If D is t-compactly packed, then D is a tQR-domain (resp., QR-domain).*

Proof. Let R be a PvMD. Then it follows from [14, Corollary 4.2] that D is a PvMD. Thus by Corollary 2.3, D is a tQR-domain.

Now let R be a Prüfer domain. Then R is a PvMD. By the PvMD case, D is a tQR-domain. It is well known that if R is a Prüfer domain, then D is a Prüfer domain ([18, (22.3) Theorem]). Thus $d = t$ on D. Therefore D is a QR-domain. □

Corollary 2.14. *Let R be a tQR-domain (resp., QR-domain). If D is t-compactly packed, then D is a tQR-domain (resp., QR-domain).*

3. On strong Mori domains

Recall from [15] that a domain R is said to be *almost factorial* if R is a Krull domain with torsion divisor class group.

Theorem 3.1. *If R is an SM domain, then R has the tQR-property if and only if R is almost factorial.*

Proof. If R is a Krull domain with torsion divisor class group, then R has the tQR-property by [7, Theorem 1.3]. Conversely, suppose that R is an SM domain with the tQR-property. Then R is a Krull domain (and hence $v = t = w$) since R is an integrally closed SM domain [37, Theorem 2.8]. If P is any prime t-ideal of R, then by Lemma 2.7(2), Theorem 2.8, and Theorem 2.1, $P = \sqrt{xR}$ for some $x \in R$. But since P is a t-maximal ideal of R, this implies that P is the only prime t-ideal containing x, so xR has a prime t-representation of the form $xR = (P^n)_t$ for some nonnegative integer n. Since every t-ideal can be written as a t-product of prime t-ideals, it then follows that the divisor class group of R is a torsion group. □

Proposition 3.2. *Let R be an SM domain in which every w-maximal ideal is the radical of a principal ideal. Then w-$\dim(R) = 1$.*

Proof. By [2, Corollary 2.3], the minimal prime ideals of a non-zero nonunit principal ideal aR have height one. Hence the nonunits of R are covered by prime ideals of height one. In particular, every w-maximal ideal M is contained in $\bigcup P_\alpha$, where the P_α are prime ideals of height one. But then $M = \sqrt{xR} \subseteq \bigcup P_\alpha$ and so it follows from $x \in P_\beta$ that $M \subseteq P_\beta$. Thus every w-maximal ideal of R has height one. Therefore w-$\dim(R) = 1$. □

Let $X^{(1)}(R)$ (or just $X^{(1)}$) denote the prime ideals of height one in R. We recall from [26] that an integral domain R is called an *infra-Krull domain* if $R = \bigcap_{P \in X^{(1)}} R_P$ where the intersection is locally finite and for each P in $X^{(1)}(R)$, R_P is a Noetherian domain. Then it is easily seen that R is an infra-Krull domain if and only if R is an SM domain with w-$\dim(R) = 1$.

Corollary 3.3. *If an SM domain R is t-compactly packed, then R is an infra-Krull domain.*

Now we can provide an example of t-coprimely packed domain R which is not t-compactly packed: Take R to be any SM domain with t-dimension at least 2 in which there are finitely many t-maximal ideals (cf., [22]).

Lemma 3.4. *Consider the following statements for a height-one prime ideal P.*

(1) *P is the radical of a principal ideal.*

(2) *There exists a non-zero* $f \in P$ *such that* fR *is a primary ideal of* R.
(3) *There exists a non-zero* $f \in P$ *such that* $f^n R$ *is a primary ideal of* R *for each* $n \in \mathbb{N}$.
(4) $P \nsubseteq \bigcup_{Q \in \Delta_P} Q$, *where* $\Delta_P := X^{(1)}(R) \setminus \{P\}$.

Then (3) $\Rightarrow$ (2) $\Rightarrow$ (1) $\Rightarrow$ (4). *Moreover, if* R *is an infra-Krull domain, then* (4) $\Rightarrow$ (3), *and hence all statements are equivalent.*

Proof. (3) $\Rightarrow$ (2) $\Rightarrow$ (1) $\Rightarrow$ (4). [29, Proposition 3]. (4) $\Rightarrow$ (3). Assume that $P \nsubseteq \bigcup_{Q \in \Delta_P} Q$. Then choose $f \in P \setminus \bigcup_{Q \in \Delta_P} Q$. Since R is an SM domain, it follows from [2, Corollary 2,3] that for each $n \in \mathbb{N}$, $f^n R = Q_{n,1} \cap \cdots \cap Q_{n,m_n}$ with each $Q_{n,i}$ primary. We also have that each $\sqrt{Q_{n,i}} \in X^{(1)}(R)$ since w-dim$(R) \leq 1$. Thus $P = \sqrt{f^n R} = \sqrt{Q_{n,1}} \cap \cdots \cap \sqrt{Q_{n,m_n}}$. Hence $P = \sqrt{Q_{n,i}}$ for all i, and so we conclude that P is the only prime divisor of $f^n R$, that is, $f^n R$ is a P-primary ideal of R. □

From [29, Proposition 1] and the fact that $(\sqrt{I})_w = \sqrt{I_w}$ for each ideal I of R ([36, Proposition 2.4]) one immediately derives the following result.

Lemma 3.5. *Let* R *be an integral domain and let* I *be a* w*-ideal of* R *with* $\sqrt{I} = P_1 \cap \cdots \cap P_n$ ($P_i \in w$-Spec(R)). *If each* P_i *is the radical of a principal ideal of* R, *then* $\sqrt{I}$ *is the radical of a principal ideal of* R.

It was shown in [4] that an integral domain R with t-dim$(R) = 1$ is a GWFD if and only if R is t-compactly packed.

Theorem 3.6. *Let* R *be an infra-Krull domain. Then the following are equivalent.*

(1) R *is* w*-compactly packed.*
(2) R *is* t*-compactly packed.*
(3) *Every* t*-ideal* I *of* R *is radically principal, i.e.,* $\sqrt{I} = \sqrt{(x)}$ *for some* $x \in R$.
(4) *Every* w*-ideal of* R *is radically principal.*
(5) *Every* (t-)*subintersection* $\bigcap_{P \in \Delta} R_P$ ($\Delta \subseteq t$-Spec(R)) *is a ring of quotients of* R.

Proof. (4) $\Rightarrow$ (3). This follows from the fact that every t-ideal of R is a w-ideal.

(3) $\Rightarrow$ (2). This follows from Theorem 2.1.

(2) $\Leftrightarrow$ (1). This follows from the fact that w-dim$(R) = 1$ and t-Max(R)=w-Max(R).

(1) $\Rightarrow$ (5). Let Δ be a subset of $X^{(1)}(R)$ and let $\bigcap_{P \in \Delta} R_P$ be a t-subintersection of R. By Theorem 2.1, for every $P \in X^{(1)}(R) \setminus \Delta$, there exists $f_P \in R$ such that $P = \sqrt{f_P R}$. Let S be the set of elements which are expressible as finite products of f_P's for $P \in X^{(1)}(R) \setminus \Delta$. Then we will show that $\bigcap_{P \in \Delta} R_P = R_S$. If $P \in \Delta$, then $P \cap S = \emptyset$. Indeed, suppose that $P \cap S \neq \emptyset$. Then $f_Q \in P$ for some $Q \in X^{(1)}(R) \setminus \Delta$. Since $Q = \sqrt{f_Q R} \subseteq P$, we have that $P = Q \in X^{(1)}(R) \setminus \Delta$, a contradiction. Hence $\Delta = \{Q' \cap R \mid Q' \in X^{(1)}(R_S)\}$.

Thus $(R_P)_S = R_P$ for all $P \in \Delta$, and hence $\bigcap_{P\in\Delta} R_P = \bigcap_{P\in\Delta}(R_P)_S \supseteq R_S$. For the reverse inclusion, if $x \in \bigcap_{P\in\Delta} R_P$, then $R :_R x$ is a w-ideal of R. Since R is an SM domain, by [2, Corollary 2.3] $R :_R x = Q_1 \cap \cdots \cap Q_n$ with each Q_i a primary ideal. Now since w-dim$(R) = 1$, we have that $\sqrt{Q_i} \in X^{(1)}(R) \setminus \Delta$ for all $i = 1, \ldots, n$. Hence there exists $f \in S \cap (R :_R x)$, which shows that $x \in R_S$. Thus $\bigcap_{P\in\Delta} R_P \subseteq R_S$. Therefore $\bigcap_{P\in\Delta} R_P = R_S$.

(5) $\Rightarrow$ (4). We first show that every prime w-ideal of R is the radical of a principal ideal of R. Let P be a prime w-ideal. Since w-dim$(R) = 1$ and w-Max(R)=t-Max(R), P is a prime t-ideal. Let $\Delta_P := t\text{-Spec}(R) \setminus \{P\}$. By Lemma 3.4 it is sufficient to show that $P \nsubseteq \bigcup_{Q\in\Delta_P} Q$. Then by hypothesis, we would have $\bigcap_{Q\in\Delta_P} R_Q = R_S$ for some multiplicatively closed subset S of R. Assume that $P \subseteq \bigcup_{Q\in\Delta_P} Q$. Then since $Q \cap S = \emptyset$ for all $Q \in \Delta_P$, we have that $P \cap S = \emptyset$. Thus $(R_{Q'})_S = R_{Q'}$ for all $Q' \in t\text{-Spec}(R) = X^{(1)}(R)$, and hence $R_S = \bigcap_{Q\in\Delta_P} R_Q = \bigcap_{Q\in\Delta_P}(R_Q)_S = \bigcap_{Q\in X^{(1)}}(R_Q)_S = \bigcap_{Q\in X^{(1)}} R_Q = R$. Thus $R_S = R$. By [36, Theorem 4.3], we can write $P = (x_1, \ldots, x_n)_w$. Since $P \nsubseteq Q$ for any $Q \in \Delta_P$, we have that $R_{x_i} \subseteq \bigcap\{R_Q \mid x_i \notin Q$ and $Q \in \Delta_P\}$, and so $\bigcap_i R_{x_i} \subseteq \bigcap_{Q\in\Delta_P} R_Q = R_S = R$. Hence by [1, Lemma 2.1], $P_v = ((x_1, \ldots, x_n)_w)_v = (x_1, \ldots, x_n)_v = R$, and so $P_w = R$, which contradicts P is a prime w-ideal. Therefore $P \nsubseteq \bigcup_{Q\in\Delta_P} Q$. Hence by Lemma 3.4, every prime w-ideal of R is the radical of a principal ideal of R. Now let I be a w-ideal of R. Then $\sqrt{I}$ is also a w-ideal of R ([36, Proposition 2.4]). Since R is an SM domain, by [2, Corollary 2.3] $\sqrt{I} = P_1 \cap \cdots \cap P_n$ for some $P_i \in w\text{-Spec}(R)$. Thus by Lemma 3.5 every w-ideal I of R is radically principal. □

Corollary 3.7. *Let R be a Krull domain. Then every subintersection $\bigcap_{P\in\Delta} R_P$ $(\Delta \subseteq X^{(1)}(R))$ of R is a ring of quotients of R if and only if every height-one prime ideal of R is the radical of a principal ideal.*

4. t-coprime packedness

Recall that a t-ideal I of an integral domain R is said to be *t-coprimely packed* if for every set Λ of t-maximal ideals of R with $I \subseteq \bigcup_{Q\in\Lambda} Q$, one has $I \subseteq P$ for some $P \in \Lambda$, and if this holds for each t-ideal of R, then R is said to be *t-coprimely packed*. Then we have the following t-theoretic analogue of [13, Lemma 2].

Proposition 4.1. *The following statements are equivalent.*

(1) *If I is a t-ideal of R and if Λ is a set of t-maximal ideals of R with $I \subseteq \bigcup_{M\in\Lambda} M$, then $I \subseteq M$ for some $M \in \Lambda$.*
(2) *If I is a t-ideal of R and if Λ is a set of prime t-ideals of R with $I \subseteq \bigcup_{Q\in\Lambda} Q$, then $(I + P)_t \neq R$ for some $P \in \Lambda$.*
(3) *If I is a t-ideal of R and if $\Lambda(I) = \{M \mid M$ is a t-maximal ideal of R with $I \nsubseteq M\}$, then $I \nsubseteq \bigcup_{Q\in\Lambda(I)} Q$.*

(4) *If P is a t-prime ideal of R and if Λ is a set of t-maximal ideals of R with $P \subseteq \bigcup_{M\in\Lambda} M$, then $P \subseteq M$ for some $M \in \Lambda$.*

Recall that t-Max(R) is *t-coprimly packed* if for a t-maximal ideal M of R and for any set $\{M_\lambda\}_{\lambda\in\Lambda}$ of t-maximal ideals of R, if $M \subseteq \bigcup_{\lambda\in\Lambda} M_\lambda$, then $M = M_\lambda$ for some $\lambda \in \Lambda$. Equivalently, if no t-maximal ideal of R is contained in the union of the other t-maximal ideals.

An integral domain R is an *almost Krull domain* if R_M is a rank-one discrete valuation domain for each t-maximal ideal M of R. It was shown in [8, Lemma 1.16] that R is an almost Krull domain if and only if R is a strongly discrete PvMD of t-dimension one.

Theorem 4.2. *Let R be an almost Krull domain. Then t-Max(R) is t-coprimely packed if and only if R is almost factorial.*

Proof. The "if" part is clear since R is of t-dimension one, so we need only prove the "only if" part. Suppose that t-Max(R) is coprimely packed. Since R is of t-dimension one, t-Max(R) is t-compactly packed. Thus by Theorem 2.8 R is an almost Krull domain satisfying the $t\#$-property, and hence by [8, Theorem 1.17] R is a Krull domain. Now the rest of the proof is the same as in Theorem 3.1. □

Combining Theorem 2.8 and Theorem 4.2, we can recover a classical result of Storch ([15, Proposition 6.8]) in the following.

Corollary 4.3. *Let R be a Krull domain. Then R is almost factorial if and only if every subintersection of R is a ring of quotients.*

Proposition 4.4. *Let R be a PvMD. Then the following statements are equivalent.*

(1) t-Max(R) *is t-coprimely packed.*
(2) *Each t-maximal ideal M of R contains a principal ideal xR such that $\sqrt{xR}$ is a prime t-ideal of R contained in M but no other t-maximal ideal.*

Proof. (1) ⇒ (2). Suppose that t-Max(R) is t-coprimely packed. Let M be a t-maximal ideal of R and let x be an element in M which is not contained in any other t-maximal ideal of R. Since $\sqrt{xR}$ is the intersection of all the prime t-ideals of R containing x, it follows that any prime t-ideal of R containing x is contained in M but no other t-maximal ideal. But since R is a PvMD, the set of prime t-ideals of R contained in M is linearly ordered ([28, Proposition 4.4]), and so $\sqrt{xR}$ is a prime t-ideal of R and M is the unique t-maximal ideal of R containing $\sqrt{xR}$.

(2) ⇒ (1). Suppose (2) holds. Let Λ be a nonempty subset of t-Max(R) and $M \in t$-Max$(R) \setminus \Lambda$. Then by assumption M contains a prime t-ideal of the form $\sqrt{xR}$ and $(\sqrt{xR} + N)_t = R$ for all $N \in \Lambda$. Thus it follows that $\sqrt{xR} \nsubseteq \bigcup_{N\in\Lambda} N$, and so $M \nsubseteq \bigcup_{N\in\Lambda} N$. Therefore t-Max(R) is t-coprimely packed. □

Theorem 4.5. *Let R be a tQR-domain with t-$\dim(R) < \infty$. Then t-$\mathrm{Max}(R)$ is t-coprimely packed if and only if every t-maximal ideal of R is the radical of a principal ideal.*

Proof. Suppose that t-$\mathrm{Max}(R)$ is t-coprimely packed. Let M be any t-maximal ideal of R and $x \in M \setminus \bigcup_{N \in \Delta_M} N$. Then by Proposition 4.4, $\sqrt{xR}$ is a prime t-ideal of R contained only in M. If $M = \sqrt{xR}$, then we are done. If not, then there exists $y \in M \setminus \sqrt{xR}$. Since R is a tQR-domain, by [7, Theorem 1.3] we have that $\sqrt{(xR + yR)_t} = \sqrt{zR}$ for some $z \in M$. Clearly $\sqrt{xR} \subsetneq \sqrt{zR}$. Again by Proposition 4.4, we have $\sqrt{zR}$ is a prime t-ideal of R contained in M which is not contained in any other t-maximal ideal of R. Continuing in this way (after a finite number of steps) we can find $m \in M \setminus \bigcup_{N \in \Delta_M} N$ such that $M = \sqrt{mR}$. The proof of the converse is easy and will be omitted. □

Corollary 4.6. *Let R be a GCD-domain with t-$\dim(R) < \infty$. Then t-$\mathrm{Max}(R)$ is t-coprimely packed if and only if every t-maximal ideal of R is the radical of a principal ideal.*

Proposition 4.7. *Let R be a tQR-domain with t-$\dim(R) < \infty$ in which every t-ideal of R is contained in only finitely many t-maximal ideals. Then t-$\mathrm{Max}(R)$ is t-coprimely packed.*

Proof. Let M be a t-maximal ideal of R and $0 \neq x \in M$. Let $M_1, M_2, \ldots, M_n$ be the other t-maximal ideals of R containing x and let $y \in M \setminus \bigcup_{i=1}^n M_i$. Then $(xR + yR)_t \subseteq M$ but $(xR + yR)_t \nsubseteq \bigcup_{i=1}^n M_i$. Since R is a tQR-domain, $\sqrt{(xR + yR)_t} = \sqrt{zR}$ for some $z \in M$ and $\sqrt{zR}$ is a prime t-ideal of R contained in M but no other t-maximal ideal. Then $M = \sqrt{mR}$ for some $m \in M$ follows from the proof of Theorem 4.5. Therefore t-$\mathrm{Max}(R)$ is t-coprimely packed. □

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References

[1] D. D. Anderson and D. F. Anderson, *Locally factorial integral domains*, J. Algebra **90** (1984), no. 1, 265–283.

[2] G. W. Chang, *Strong Mori domains and the ring $D[X]_{N_v}$*, J. Pure Appl. Algebra **197** (2005), no. 1-3, 293–304.

[3] ———, *Prüfer $*$-multiplication domains, Nagata rings, and Kronecker function rings*, J. Algebra **319** (2008), no. 1, 309–319.

[4] G. W. Chang and C. J. Hwang, *Covering and intersection conditions for prime ideals*, Korean J. Math. **17** (2009), 15–23.

[5] D. E. Dobbs, E. G. Houston, T. G. Lucas, M. Roitman, and M. Zafrullah, *On t-linked overrings*, Comm. Algebra **20** (1992), no. 5, 1463–1488.

[6] D. E. Dobbs, E. G. Houston, T. G. Lucas, and M. Zafrullah, *t-linked overrings and Prüfer v-multiplication domains*, Comm. Algebra **17** (1989), no. 11, 2835–2852.

[7] ———, *t-linked overrings as intersections of localizations*, Proc. Amer. Math. Soc. **109** (1990), no. 3, 637–646.

[8] S. El Baghdadi and S. Gabelli, *Ring-theoretic properties of PvMDs*, Comm. Algebra **35** (2007), no. 5, 1607–1625.
[9] V. Erdoğdu, *Coprimely packed rings*, J. Number Theory **28** (1988), no. 1, 1–5.
[10] ———, *The prime avoidance of maximal ideals in commutative rings*, Comm. Algebra **23** (1995), no. 3, 863–868.
[11] ———, *Three notes on coprime packedness*, J. Pure Appl. Algebra **148** (2000), no. 2, 165–170.
[12] ———, *Coprime packedness and set theoretic complete intersections of ideals in polynomial rings*, Proc. Amer. Math. Soc. **132** (2004), no. 12, 3467–3471.
[13] V. Erdoğdu and S. McAdam, *Coprimely packed Noetherian polynomial rings*, Comm. Algebra **22** (1994), no. 15, 6459–6470.
[14] M. Fontana, P. Jara, and E. Santos, *Prüfer $\star$-multiplication domains and semistar operations*, J. Algebra Appl. **2** (2003), no. 1, 21–50.
[15] R. Fossum, *The Divisor Class Group of a Krull Domain*, Springer-Verlag, New York-Heidelberg, 1973.
[16] S. Gabelli, E. G. Houston, and T. G. Lucas, *The t#-property for integral domains*, J. Pure Appl. Algebra **194** (2004), no. 3, 281–298.
[17] R. Gilmer, *Overrings of Prüfer domains*, J. Algebra **4** (1966), 331–340.
[18] ———, *Multiplicative Ideal Theory*, Queen's Papers in Pure and Applied Mathematics, **90**, Queen's University, Kingston, Ontario, 1992.
[19] R. Gilmer and W. J. Heinzer, *Overrings of Prüfer domains. II*, J. Algebra **7** (1967), 281–302.
[20] R. Gilmer and J. Ohm, *Integral domains with quotient overrings*, Math. Ann. **153** (1964), 97–103.
[21] E. G. Houston, *Prime t-ideals in $R[X]$*, Commutative ring theory (Fes, 1992), 163–170, Lecture Notes in Pure and Appl. Math., 153, Dekker, New York, 1994.
[22] E. G. Houston and A. Mimouni, *On the divisorial spectrum of a Noetherian domain*, J. Pure Appl. Algebra **214** (2010), no. 1, 47–52.
[23] B. G. Kang, *Prüfer v-multiplication domains and the ring $R[X]_{N_v}$*, J. Algebra **123** (1989), no. 1, 151–170.
[24] I. Kaplansky, *Commutative Rings*, Revised edition. The University of Chicago Press, Chicago, Ill.-London, 1974.
[25] D. J. Kwak and Y. S. Park, *On t-flat overrings*, Chinese J. Math. **23** (1995), no. 1, 17–24.
[26] M. B. Martin and M. Zafrullah, *t-linked overrings of Noetherian weakly factorial domains*, Proc. Amer. Math. Soc. **115** (1992), 601–604.
[27] J. L. Mott, *Integral domains with quotient overrings*, Math. Ann. **166** (1966), no. 229–232.
[28] J. L. Mott and M. Zafrullah, *On Prüfer v-multiplication domains*, Manuscripta Math. **35** (1981), no. 1-2, 1–26.
[29] S. Oda, *Radically principal and almost factorial*, Bull. Fac. Sci. Ibaraki Univ. Ser. A No. **26** (1994), 17–24.
[30] J. V. Pakala and T. S. Shores, *On compactly packed rings*, Pacific J. Math. **97** (1981), no. 1, 197–201.
[31] N. Popescu, *Sur les C. P.-anneaux*, C. R. Acad. Sci. Paris Ser. A-B **272** (1971), A1493–A1496.
[32] C. M. Reis and T. M. Viswanathan, *A compactness property for prime ideals in Noetherian rings*, Proc. Amer. Math. Soc. **25** (1970), 353–356.
[33] D. E. Rush and L. J. Wallace, *Noetherian maximal spectrum and coprimely packed localizations of polynomial rings*, Houston J. Math. **28** (2002), no. 3, 437–448.
[34] W. Smith, *A covering condition for prime ideals*, Proc. Amer. Math. Soc. **30** (1971), 451–452.

[35] B. Wajnryb and A. Zaks, *On the flat overrings of an integral domain*, Glasgow Math. J. **12** (1971), 162–165.
[36] F. Wang and R. L. McCasland, *On w-modules over strong Mori domains*, Comm. Algebra **25** (1997), no. 4, 1285–1306.
[37] ———, *On strong Mori domains*, J. Pure Appl. Algebra **135** (1999), no. 2, 155–165.

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