JASS *Journal of Astronomy and Space Sciences*

Ricean Bias Correction in Linear Polarization Observation

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I developed an enhanced correction method for Ricean bias which occurs in linear polarization measurement. Two known methods for Ricean bias correction are reviewed. In low signal-to-noise area, the method based on the mode of the equation gives better representation of the fractional polarization. But a caution should be given that the accurate estimation of noise level, i.e. σ of the polarized flux, is important. The maximum likelihood method is better choice for high signal-to-noise area. I suggest a hybrid method which uses the mode of the equation at the low signal-to-noise area and takes the maximum likelihood method at the high signal-to-noise area. A modified correction coefficient for the mode solution is proposed. The impact on the depolarization measure analysis is discussed.

Keywords: polarization, data analysis, Ricean bias, radio loud active galactic nuclei

1. INTRODUCTION

Synchrotron radiation from radio galaxies is known to be linearly polarized. Even unresolved sources show measurable linear polarization. Using this linear polarization property, I can measure the fractional polarization and the polarization angle of the radiation after traversing the Faraday medium (e.g. a magnetized thermal plasma between the polarized source and us). First investigations of the galactic Faraday medium were made by Simard-Normandin et al. (1981) and others using the integrated polarization of extragalactic sources. With increased sensitivity and angular resolution of radio telescopes and telescope arrays, the investigation of the Faraday medium of radio galaxies has become one of the main interests of this field. Burn (1966) and Laing (1984) have shown that deciphering the geometry of the Faraday medium is very complex. In general, there are two basic geometric configurations.

- The Faraday medium is 'internal' to the source, and physically mixed with the emitting volume (e.g. a thermal cloud surrounded by a shell of relativistic electrons).

- A foreground Faraday medium (in front of the synchrotron source) has a fine structure that is unresolved by the telescope.

Decades of multi-frequency observations have shown that the Faraday rotation is highly linear in λ^2 when the spatial resolution is good enough. This linearity is the property of the resolved slab and the external (resolved) Faraday medium (Burn 1966). The difference between the internal slab geometry and the external medium becomes evident in the fractional polarization. In the internal case, I will see the secondary maxima of the fractional polarization in the low-frequency regime. In the external case, I will see no depolarization at low frequencies. The observational results seem to be 'in between.' The achievement of full frequency-sampled data is virtually impossible. While a depolarization trend is seen, it is not easy to distinguish between the internal slab and the unresolved external component.

The Faraday rotation of supernova remnants and spiral galaxies is an example of the first case. In this case, the Faraday rotation profile is non-linear in λ^2. On the other hand, the observational results of radio galaxies over the last decades, especially multi-frequency observations

with high angular resolution, have proven that Faraday rotation in radio galaxies does follow a λ^2 law. Therefore, it must be concluded that the Faraday-rotating media do not permeate the lobes of radio galaxies. Recently obtained sensitive X-ray images (e.g. with *ROSAT*, *Chandra*) exhibit cavities in the hot gas towards the lobes of radio galaxies. This confirms that the thermal electrons of the X-ray halos and the relativistic electrons of the radio lobes are not mixed. Physically, this could imply pressure balance between the thermal and the relativistic electrons. Alternatively, it could mean rapid acceleration of the thermal electrons in the radio lobes and rapid cooling of the relativistic electrons in the X-ray halo. In any case, the λ^2 behavior of the Faraday rotation of radio galaxies indicates that the differential depolarization (DP) and the rotation measure (RM) of radio galaxies are due to an 'external' Faraday medium. It also implies that the Faraday media are almost or even fully resolved. This should lead to a low differential de-polarization.

In the case of fully resolved Faraday cells and assuming that radio galaxies reside in the center of X-ray halos with King profiles, I expect an RM asymmetry instead of a depolarization asymmetry. Although the scale length of the Faraday cells is not definitely known yet, the λ^2 behavior suggests that I do resolve the Faraday screen. On the other hand, the asymmetry of the DP (i.e. the 'Laing-Garrington effect' Laing [1988], Garrington et al. [1988]) is known to be the best proof of the relativistic beaming effect of jets in radio galaxies. The Laing-Garrington effect means that a non-negligible amount of (magnetic) energy is contained in small-scale (unresolved) magnetic field structures. In that case, Faraday rotation will deviate from the λ^2 behavior and will be saturated at $\Delta\chi < \pi/3$ (Burn 1966).

The fractional polarization of a radio source is determined by the pitch angle distribution of the relativistic electrons, the geometry of the magnetic field, and by the energy distribution function of the relativistic electrons

$$N(E) \propto E^{-p}, \text{ with } \alpha_{inj} = (p-1)/2 \qquad (1)$$

The general power-law distribution is known from radio observations, cosmic-ray experiments, and from theory. The theoretical fractional polarization of the radio intensity is (almost) independent of frequency. On the other hand, the measured fractional polarization of the flux density is additionally dependent on the depolarization due to differential Faraday rotation, i.e. on the distribution of thermal electrons along the line-of-sight, as well as the line-of-sight component of the magnetic field. This way, the polarized component of the radiation under-

going Faraday rotation becomes frequency dependent.

For decades, one-sided jets of active galactic nuclei have been observed. The most successful explanation of this phenomenon is Doppler boosting caused by the relativistic bulk motion of the jet material. Since the radio lobes are connected to, and fed by, the jets, large efforts have been made in order to look for independent projection effects which do or do not support the interpretation in terms of projection effects of the relativistic jets. As far as the radio lobes are concerned, the depolarization asymmetry, i.e. the Laing-Garrington effect, is the strongest support of this view. The other asymmetries of the radio lobes, the lobe length and the spectral index asymmetry, do not show any clear connection to the projection effect of the relativistic jets. To conclude, the Laing-Garrington effect is interpreted in terms of DP. In case of such an *internal* DP, the foreground medium should also have magnetic field reversals along the line-of-sight.

2. RICEAN BIAS IN LINEAR POLARIZATION

In this Section, I discuss the Ricean bias in the polarization intensity and suggest a new correction method. The linearly polarized intensity is obtained from Stokes parameters U and Q according to $I_{P_{obs}} = \sqrt{U^2 + Q^2}$. It follows a Ricean distribution (Vinokur 1965):

$$f\left(I_{P_{obs}}; I_P, \sigma_{I_P}^2\right) = I_{P_{obs}} / \sigma_{I_P}^2 B_0\left(I_{P_{obs}} I_P / \sigma_{I_P}^2\right) e^{-0.5\left(I_P^2 + I_{P_{obs}}^2\right)/\sigma_{I_P}^2} \qquad (2)$$

where $I_{P_{obs}}$ is the observed polarized intensity, I_P is the intrinsic polarized intensity, B_0 is the modified Bessel function of zero order and σ_{I_P} is the noise level. For $I_P \gg \sigma_{I_P}$ and $I_P \sim \sigma_{I_P}$, the Rice distribution is approximated by the well known statistical forms. In the case of $I_P \gg \sigma$, the Rice distribution of $I_{P_{obs}}$ asymptotically approaches a Gaussian distribution (Fig. 1). Since the Gaussian is a symmetric distribution, the most probable value and the peak value, $df / dI_P = 0$ are the same. Therefore, the most probable $I_{P_{obs}}$ is the real I_P in this case (Fig. 1).
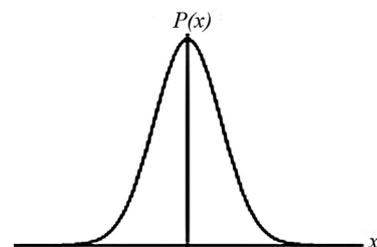

Fig. 1. A Gaussian distribution.

In the other case, $I_P \sim \sigma_{I_P}$, the Rice distribution of $I_{P_{obs}}$ is a Rayleigh distribution (Fig. 2). In this case, the most probable value is larger than the peak value, $df / dI_P = 0$, $I_{P_{obs}} = \sqrt{\pi/2}\sigma(= I_P)$. The reason is that the Rayleigh distribution is an asymmetric distribution. This has become known as the Ricean bias in the literature, and is caused by the positivity of the noise in $I_P = \sqrt{U^2 + Q^2}$. Different from the polarized intensity, the distribution function of the polarization angles remains symmetric, and no Ricean bias correction is needed (Wardle & Kronberg 1974), since there is no positivity problem in $\chi = \tan^{-1}(U/Q)$.

2.1 The Two Known Solutions

In practice, two solutions of the Ricean bias correction are widely used.

2.1.1 The solution of Wardle & Kronberg

Wardle & Kronberg (1974) have suggested a solution based on the mode of the equation, namely, $\frac{df}{dI_{P_{obs}}} = 0$.

$$\left(1 - I_{P_{obs}}^2 / \sigma_{I_P}^2\right) B_0\left(I_{P_{obs}} I_P / \sigma_{I_P}^2\right) + I_{P_{obs}} I_P / \sigma_{I_P}^2 I_1\left(I_{P_{obs}} I_P / \sigma_{I_P}^2\right) = 0 \quad (3)$$

A good approximation of this solution is

$$I_P \sim \sqrt{I_{P_{obs}}^2 - \sigma_{I_P}^2} \quad (4)$$

Conventionally, $I_P \sim \sqrt{I_{P_{obs}}^2 - c_1\sigma_{I_{obs}}^2}$ with $c_1 = 1.2$. As seen above, $I_{P_{obs}} = \sigma_{I_P}\sqrt{\pi/2} \sim 1.25\sigma_{I_P}$ is the Rayleigh distribution.

2.1.2 The solution of Killeen et al. (1986)

The other solution which is based on a more solid statistical reasoning, viz. the maximum likelihood method, $\frac{df}{dI_P} = 0$, was suggested by Killeen et al. (1986). With increasing signal-to-noise, the Maximum Likelihood estimate approaches the value of I_P more rapidly:

$$I_{P_{obs}} I_P / \sigma_{I_P}^2 B_0\left(I_{P_{obs}} I_P / \sigma_{I_P}^2\right) - I_{P_{obs}}^2 / \sigma_{I_P}^2 I_1\left(I_{P_{obs}} I_P / \sigma_{I_P}^2\right) = 0 \quad (5)$$

A good approximation of this solution is

$$I_P \sim I_{P_{obs}} - 0.5\sigma_{I_{P_{obs}}}^2 / I_{P_{obs}} \quad (6)$$
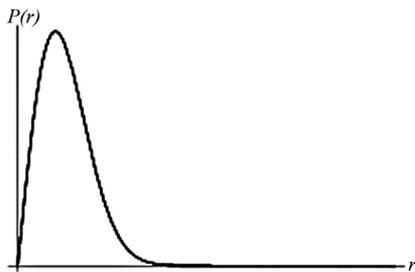
Fig. 2. A Rayleigh distribution.

which is implemented in the AIPS task POLCO as such.

2.1.3 Pros and cons of the solutions

The performance of the maximum likelihood estimation is better at high signal-to-noise, $I_P / \sigma_{I_P} > 5$, but does not yield any significant effect, since the difference to the solution of Wardle & Kronberg (1974) is much less than 1% (Figs. 3 and 4). This is because the probability distribution of $I_{P_{obs}}$ becomes a (symmetric) Gaussian at high
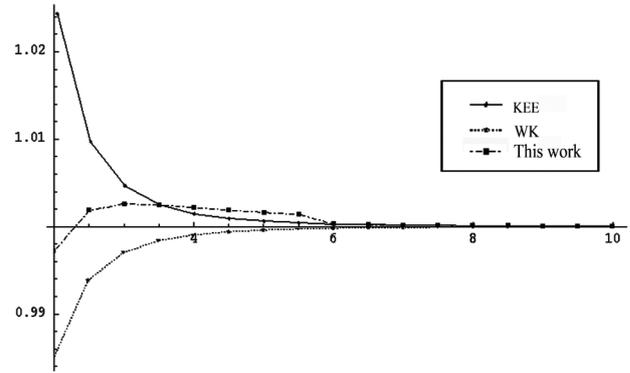
Fig. 3. The simulation shows the case $\sigma = \sigma_{I_{P_{obs}}}$, an ideal case. The abscissa is $I_{P_{obs}} / \sigma_{I_{P_{obs}}}$ and the ordinate is $I_{P_{obs}} / I_P$. 'WK' denotes the solution of $I_{P_{obs}}$ following Wardle & Kronberg (1974), 'KBE' that of Killeen et al. (1986), and 'this work' that of this work. In practice, one should bear in mind that $I_{P_{obs}}$ will NOT be the mathematical solution of the former section, but rather the distribution around this mean value. Therefore, fluctuations of differential depolarization and FPOL if obtained from low polarized intensities are largely purely statistical but not physically meaningful. The new solution shows an enhanced performance at low signal-to-noise. Between $4 < I_P / \sigma_{I_P} < 6$, the old solutions are better, but the difference is less than 0.1 percent.
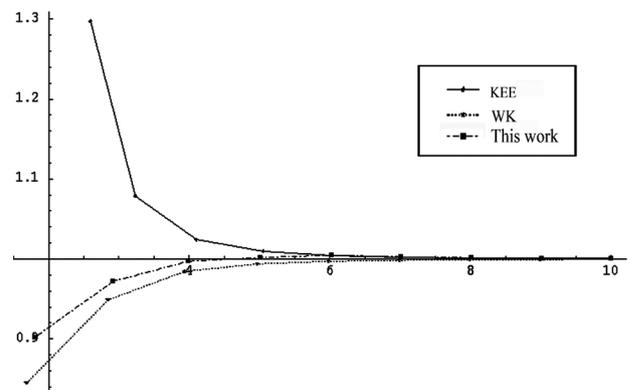
Fig. 4. The simulation shows the case $\sigma_{I_P} = 2\sigma_{I_{P_{obs}}}$. A underestimate of σ by a factor of 2 of σ_{I_P} could be possible, due to the blanked values (AIPS cookbook [National Radio Astronomical Observatory 2011], see also AIPS HELP). In view of the underestimate of σ_{I_P}, our new solution is the best one. The solution implemented into AIPS task POLCO Killeen et al. (1986) strongly overestimates the polarization intensity, while the Wardle & Kronberg (1974) solution underestimates it. The difference is much larger than that to our solution visible in Fig. 3.

signal-to-noise. On the contrary, the difference at low signal-to-noise, i.e. the overestimate of the maximum likelihood solution, is significant. As mentioned in the help tool to AIPS task POLCO, one should assume an underestimate of up to about a factor of 2 of the noise level, due to the so-called 'magic blanking'. Therefore, if one wants to go down to a $3 - \sigma_{I_P}$ level, the choices of the solution and of c_1 become critical.

In the Wardle & Kronberg (1974) solution, $I_P \sim \sqrt{I_{P_{obs}}^2 - c_1 \sigma_{I_{P_{obs}}}^2}$, c_1 should be carefully selected. As I am not interested in values of I_P as low as $I_{P_{obs}} \sim \sigma_{I_P}$, c_1 should be lower than $\sqrt{\pi / 2}$ for a better performance in the range $3 < I_{P_{obs}} / \sigma_{I_P} < 6 \sim 10$. Above $6\sigma_{I_P} \cdots 10\sigma_{I_P}$, I can expect the best performance using the ML solution. From our experiments, I conclude that $c_1 = 0.9$ is a reasonable choice (Figs. 3 and 4).

3. THE 'HYBRID' SOLUTION

Our goal is to improve the performance of the Ricean correction such that $m' = \frac{I_P}{I}$ should be reliable down to $3\sigma_{I_P}$ and $3\sigma_I$. In the next subsection, I will show the importance of the areal mean m' and the position information of DP. Our scheme is simple. I make a hybrid solution of the two solutions explained in the former subsection. At high signal-to-noise, I will still use the scheme of Killeen et al. (1986). At low signal-to-noise, the solution of Wardle & Kronberg (1974) is better, but with $c_1 = 0.9$.

The selection of c_1 has been done numerically, considering the fact that one should take account of the underestimation of the noise by up to factor of 2 in the intensity (National Radio Astronomical Observatory 2011), due to the blanked values. Further minor estimation errors could be caused, for example, through a wrong selection of the noise estimation area, which could further underestimate σ_{I_P}. I combine the two solutions (Wardle & Kronberg 1974, Killeen et al. 1986). Our solution accepts the Maximum Likelihood over $I_{P_{obs}} \sim 6\sigma_{I_P}$; below this, it uses $I_P \sim \sqrt{I_{P_{obs}}^2 - c_1 \sigma_{I_P}^2}$ with $c_1 = 0.9$, which is a good choice for $\sigma_{I_P} / I_{P_{obs}}$ between 1.5 to 6. Using this new fractional polarization, I can estimate DP with the positional information and can study any correlation between RM and DP.

4. DISCUSSION

4.1 Integrated Fractional Polarization

Before working out a new method to estimate the frac-

tion of polarization, I discuss the integrated fractional polarization as published in the literature, motivating this new correction method. There are two ways to estimate the fractional polarization of a region of interest. One can first integrate the polarized and the total intensity and divide them:

$$m' = \frac{\sum I_P}{\sum I} \qquad (7)$$

There are pros and cons to this integrated estimate. The pro is that the Ricean bias problems are largely removed by the integration. The obtained m' reflects the fractional polarization of bright source structures, such as cores, hot spots and jets. The con is the loss of the positional information. Conventionally, DP maps are presented after one of the Ricean bias corrections, i.e. (Wardle & Kronberg 1974, Killeen et al. 1986), down to $3\sigma_{I_P}$ of the total intensity. This will lead to significant over- and underestimates of the $low - \sigma_{I,P}$ regions, such as the lobes. Therefore, in order to estimate the fractional polarization including the positional information, an estimate $m' = \sum \left(\frac{I_P}{I} \right)$ is desirable.

In polarization studies, other important informations are the polarization angle and the RM. Since they are independent of the brightness, the use of DP as obtained from $m' = \frac{\sum I_P}{\sum I}$ could lead to a wrong interpretation.

4.2 Underestimates of σ_{I_P} in the Literature

In this subsection, I discuss the propagation of the systematic error as due to the insufficient Ricean bias correction in published DP maps. The discussion is based on the solution of Killeen et al. (1986). For the solution of Wardle & Kronberg (1974), this should be interpreted in an opposite sense. In that case, the systematic error in regions with low polarization intensity is less important, unless the signal-to-noise is too low in the absence of any σ_{I_P} cut-off.

I consider the problem of underestimating σ_{I_P} for two cases, namely for well resolved and for unresolved sources. I assume a power-law distribution for I_v. When the source and the Faraday medium are resolved, the overestimate caused by the $3\sigma_{I_P}$ cut is serious at the highest frequencies. In this case, the fractional polarization of regions with low I_p could be overestimated. The other case is that of low angular resolution. Because of the Faraday rotation in the foreground medium, depolarization will be significant. In the most depolarized regions, a cut at $3\sigma_{I_P}$ is serious at the lowest frequencies. Then the fractional polarization of such regions is overestimated at the lowest compared to the highest frequencies. In effect, the depolarization trend will thus be reduced.

In depolarization studies, the integrated DP is rather independent of the polarized and total intensity, but rather depends on $\int m'(S) \cdot S \, dS$, where S is the projected surface. The first case (resolved source) overestimates DP, while the second (unresolved source) underestimates it. Let us discuss this in the light of the Laing-Garrington effect. If one can assume that the jet side is brighter and more polarized, the two arguments will be more important for the counter-jet lobe. In the first case, the Laing-Garrington effect will be emphasized through the overestimate of DP, whereas it will be reduced in the second case because of the under-estimation.

One more word of caution seems to be necessary regarding DP structures. Patchy distributions of DP in regions with low I_p which are frequently seen should be interpreted with utter care. One can mis-interpret such a patchy structure as real turbulence or fluctuations. Unless such patchy structures are confirmed independently, e.g. also in RM maps, they could be solely statistical.

ACKNOWLEDGEMENTS

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