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Alternating Sunspot Area and Hilbert Transform Analysis

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We investigate the sunspot area data spanning from solar cycles 1 (March 1755) to 23 (December 2010) in time domain. For this purpose, we employ the Hilbert transform analysis method, which is used in the field of information theory. One of the most important advantages of this method is that it enables the simultaneous study of associations between the amplitude and the phase in various timescales. In this pilot study, we adopt the alternating sunspot area as a function of time, known as Bracewell transformation. We first calculate the instantaneous amplitude and the instantaneous phase. As a result, we confirm a ~22-year periodic behavior in the instantaneous amplitude. We also find that a behavior of the instantaneous amplitude with longer periodicities than the ~22-year periodicity can also be seen, though it is not as straightforward as the obvious ~22-year periodic behavior revealed by the method currently proposed. In addition to these, we note that the phase difference apparently correlates with the instantaneous amplitude. On the other hand, however, we cannot see any obvious association of the instantaneous frequency and the instantaneous amplitude. We conclude by briefly discussing the current status of development of an algorithm for the solar activity forecast based on the method presented, as this work is a part of that larger project.

Keywords: Sun, Sunspots, data analysis

1. INTRODUCTION

The Sun presents variability in several timescales, ranging from days to several hundreds of years. The periodic characteristic of solar activity has been studied in great detail (Chang 2007, 2008, 2009, 2011, Usoskin 2008). In particular, prediction of solar variation has become an extremely hot topic, in view of space weather forecasting, encompassing a very wide variety of prediction methods and many different timescales (Petrovay 2010). The methods are mainly categorized into three groups: Precursor methods, Extrapolation methods, and Model-based methods.

It is evident that the sunspot cycle is rather irregular. The mean length of the most well-known cycle is ~11 years (a half of Hale's ~22 year periodicity), with a standard deviation of ~1 year. The profile of sunspot cycles is asymmetrical, e.g., the rise is faster than the decay. That is, the solar activity maximum occurs 3 to 4 years after the minimum, while it takes another 7-8 years to reach the next minimum. It has been noticed that the length of the rise phase anticorrelates with the maximal value of solar activity. Historically, the relation was first formulated by Waldmeier (1935) as an inverse correlation between the rise time and the cycle amplitude. The observed anticorrelation between rise time and maximum cycle amplitude is approximately linear (Lantos 2000). A more significant anticorrelation between the cycle amplitude and the length of the 'previous' cycle was reported by Hathaway et al. (1994), who also showed a weaker anticorrelation between the decay time and the cycle amplitude. An anticorrelation between cycle length and amplitude can be understood as a characteristic of a class of stochastically forced nonlinear oscillators. Indeed, it can be reproduced

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by introducing a stochastic forcing in dynamo models (Charbonneau & Dikpati 2000).

In this paper, we investigate the sunspot area data spanning from solar cycles 1 to 23 by employing the Hilbert transform technique. The Hilbert transform analysis is used to analyze variations of phase and amplitude of a local wave field (Komm et al. 2001). Instead of measuring time intervals and amplitudes of the solar activity within a predetermined duration, such as a length of ~11 years, we attempt to analyze sunspot area data as a deformed oscillator using the proposed method. This is possible in the sense that one of the most important advantages of using Hilbert transform is that one can study associations between the amplitude and the phase over various timescales. Once we demonstrate its availability in this pilot study, we will develop the method to extend to a prediction algorithm, and discuss it elsewhere.

This paper begins with descriptions of data and a procedure by which sunspot area data is transformed in Section 2. We present and discuss results obtained with the Hilbert transform technique in Section 3. Finally, we discuss and conclude in Section 4.

2. SUNSPOT AREA AND BRACEWELL TRANSFORM

Despite its somewhat arbitrary construction, the time series of sunspot data is the longest homogeneous global indicator of solar activity. As shown in Fig. 1, for the present analysis we have used the monthly sunspot area data from solar cycles 1 (March 1755) to 23 (December 2010), taken from the NASA website[1].

There is an alternative way of plotting the sunspot series, in which alternating signs are given to successive ~11-year cycles (Bracewell 1953). In Fig. 2 we show the alternating sunspot area as a function of time, whose period is ~22 years rather than ~11 years. Instead of the 'raw' sunspot area series, many authors have found the alternating cycle convenient, but the idea has not yet been widely adopted. The basic idea for this alternation is based on Hale's well known polarity rule, implying that the period of the solar cycle is actually ~22 years rather than ~11 years. Alternating the 'raw' sunspot series is also well motivated from the physical point of view that attributes an alternating sign to even and odd Schwabe cycles (Bracewell 1953, 1986). Even though the time series of the sunspot area is a somewhat arbitrary construct, there may be an underlying physical quantity. The toroidal magnetic field strength B, or the magnetic energy, proportional to B^2, can be such a quantity, which is related to it in some nonlinear fashion (Passos & Lopes 2011). This representation is called the Bracewell transform, after Bracewell, who first suggested it (Bracewell 1953). Besides this physical reasoning, in this analysis this modification is a most helpful adaptation to the method of the proposed analysis.

[1]http://solarscience.msfc.nasa.gov/greenwch.shtml

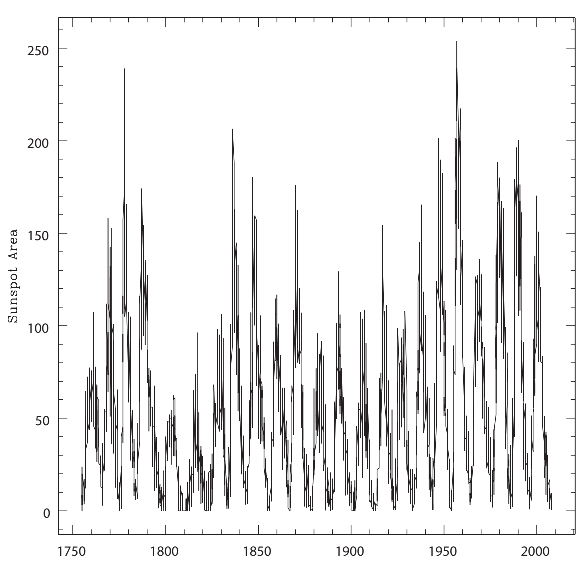

Fig. 1. The monthly sunspot area data from the solar cycles 1 (March 1755) to 23 (December 2010) as a function of time, in units of millionths of a hemisphere.

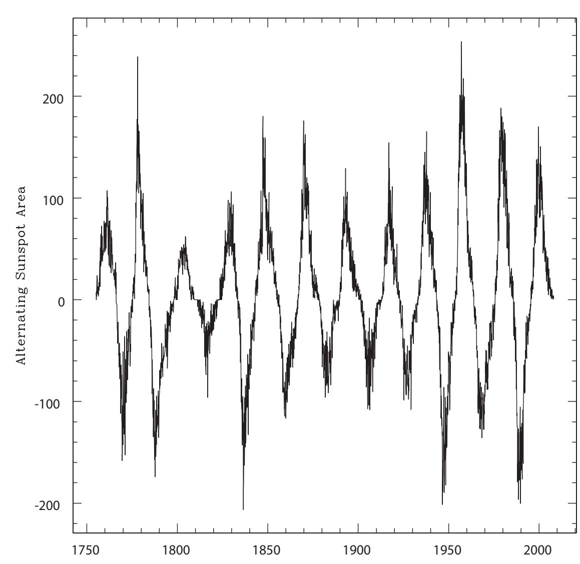

Fig. 2. The alternating sunspot area as a function of time, whose period is ~22 years rather than ~11 years.

3. HILBERT TRANSFORM ANALYSIS AND RESULTS

There is a well-known mathematical technique for extracting the instantaneous amplitude and instantaneous phase from oscillating data, which is well-defined and widely used in practice. This is the Hilbert transform. The underlying idea for this method is to represent a real oscillating quantity by a complex function whose real part varies with time.

For a function *x(t)*, its Hilbert transform $\hat{x}(t)$ is defined by

$$\hat{x}(t) = \frac{1}{\pi} P \int_{-\infty}^{-\infty} \frac{x(\tau)}{t-\tau} d\tau, \quad (1)$$

where *P* is the Cauchy principal value of the singular integral. With the Hilbert transform $\hat{x}(t)$ of the function *x(t)*, we obtain the analytic complex function, *z(t)*, as

$$z(t) = x(t) + i\hat{x}(t) = a(t)e^{i\psi(t)}, \quad (2)$$

where $i = \sqrt{-1}$. Thus, the instantaneous amplitude *a(t)* and the instantaneous phase $\hat{x}(t)$ are given by

$$a(t) = \sqrt{x^2(t) + \hat{x}^2(t)}, \quad (3)$$

$$\psi(t) = \mathrm{Tan}^{-1} \frac{\hat{x}(t)}{x(t)}, \quad (4)$$

respectively. Subsequently, the instantaneous frequency $\omega(t)$ and the instantaneous period *T(t)* are simply

$$\omega(t) = \frac{d\psi(t)}{dt}, \quad (5)$$

$$T(t) = \frac{2\pi}{\omega(t)}, \quad (6)$$

respectively.

In Fig. 3, we show the smoothed alternating sunspot area in a solid curve and its Hilbert transform in a dotted curve. To get eliminate the arbitrariness of calendar years, we follow a standard practice, which is to adopt a moving-average using 11-year boxcar averages of the monthly averaged sunspot areas. Using Hilbert transform, one may immediately obtain the instantaneous amplitude *a(t)* and the instantaneous phase $\psi(t)$ with the equations given above. In Fig. 4 we show the instantaneous amplitude as defined by Eq. (3). The envelope of a sinusoidal function of ~22 year periodicity can be clearly seen. Other previously reported longer periodicities, such as, Gleissberg period (~70-100 years), can also be found, though it is not as straightforward as the obvious ~22-year periodic behavior. What is shown with the solid curve in Fig. 5 is the difference between $\psi(t)$ defined by Eq. (4) and the straight line representing a linear phase of 22-year periodicity over the observing duration. That is, the phase difference is given by $\psi(t) - \omega_0 t$, where ω_0 is a reference frequency that would be chosen as the angular frequency corresponding to the period of ~22 years. For comparison, we also show the instantaneous amplitude function in the dotted curve after scaling with an arbitrary factor. The phase difference appears to correlate with the

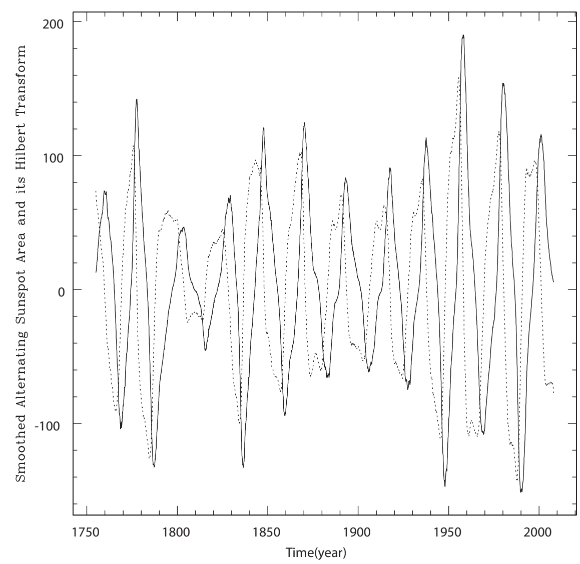

Fig. 3. The smoothed alternating sunspot area and its Hilbert transform. Solid and dotted curves are the former and the latter, respectively. We have adopted a moving-average using 11-year boxcar averages of the monthly averaged sunspot areas.

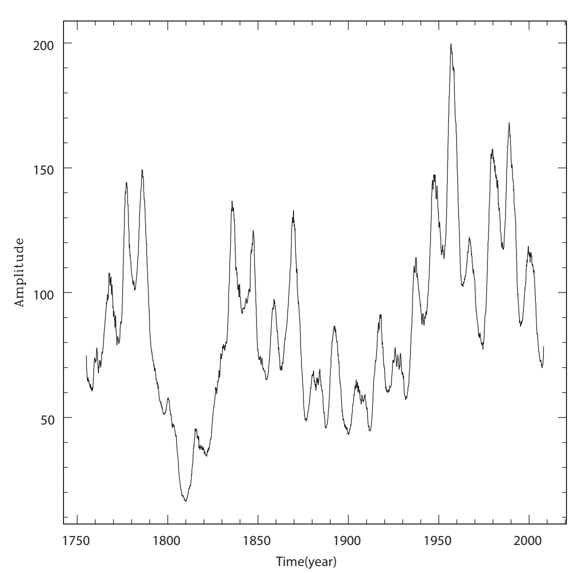

Fig. 4. The instantaneous amplitude defined by Eq. (3). The envelope of a sinusoidal function of ~22 year periodicity is clearly seen.

instantaneous amplitude. The correlation is not perfect, but general trends seem noticeable. That is, when a particular solar cycle is strong, the value of the phase difference becomes large, and vice versa. What this means is that in the strong cycle the phase advances ahead, and vice versa, as if the solar cycle remembers a 'correct' phase (Udny Yule 1927, Dicke 1978). This finding also supports the previously claimed anticorrelation between

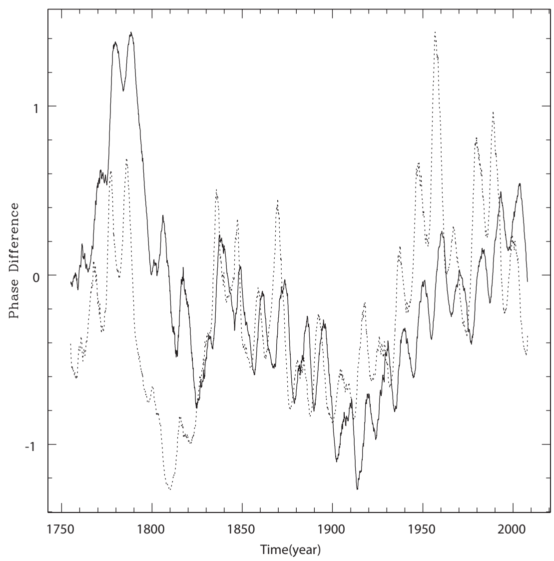

Fig. 5. The difference between *ψ(t)* defined by Eq. (4) and the straight line representing a linear phase of 22-year periodicity over the observing duration (solid curve). For comparison, we also show the instantaneous amplitude function in the dotted curve.

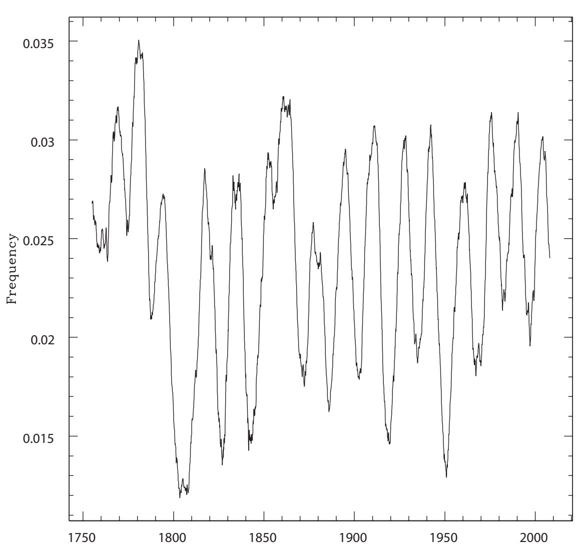

Fig. 6. The derivative of the instantaneous phase, which is a reciprocal of the period.

the cycle length and the maximum value of the solar cycle (Waldmeier 1935). In Fig. 6, we show the derivative of the instantaneous phase, which is the reciprocal of the period. As seen in Fig. 6, on the other hand, the association of the instantaneous frequency given by Eq. (5) with the instantaneous amplitude is far less obvious. In the first half of the total duration (1750-~1870) it apparently behaves similarly to the instantaneous amplitude function. But in the second half (~1870-2010) no resemblance can be found. This is somewhat unexpected, since, according to the Waldmeier effect, one may expect some kinds of correlated behavior between the instantaneous frequency and the instantaneous amplitude, at least in a timescale of ~11 years. We are currently investigating our findings and previous claims. This unexpected discrepancy should be explained in a further analysis of the currently proposed method.

4. DISCUSSION AND CONCLUSIONS

We have attempted to analyze the observed sunspot data spanning from solar cycles 1 (March 1755) to 23 (December 2010) by employing the Hilbert transform analysis method, which of the three main categories of prediction methods is considered an extrapolation method. One of the most important advantages in doing so is that the study of associations between the amplitude and the phase in various timescales is possible. In this pilot study, we adopt the alternating sunspot area as a function of time, as introduced by Bracewell (1953). This representation has an advantage in the sense that it is physically motivated and is suitable to the Hilbert transform analysis method suggested in this paper.

We have demonstrated that the instantaneous amplitude function clearly has a ~22-year periodicity (Hale's periodicity). Other longer periodicities, such as the Gleissberg period (~70-100 years), can also be seen. However, a shorter periodicity cannot be seen unless one utilizes another step for further Investigations. We also show that the phase difference apparently correlates with the instantaneous amplitude. On the other hand, however, we cannot find any obvious association of the instantaneous frequency and the instantaneous amplitude.

For forecasting space weather and understanding the solar magnetic field, studying solar activity is of great importance. Our ultimate goal in this study is to develop an algorithm that can be used to predict the solar activity in various timescales at the same time. Rather than examining possible associations of the amplitude with other properties of solar activity over different timescales separately, as is commonly done, we are utilizing the Hilbert transform method as a consistent tool that can be adjusted in the timescale of interest. We are going to use phase information to estimate the amplitude of solar activity over various timescales. We have succeeded in simultaneously extracting the instantaneous amplitude and instantaneous phase information from the solar activity data. We are currently carrying out further analyses of the method to thoroughly understand it; for example, to determine why the unexpected discrepancy we discussed above occurred. Having sufficiently understood the method, we may further refine the suggested method in developing an algorithm for solar activity prediction.

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REFERENCES

Bracewell RN, The sunspot number series, Natur, 171, 649-650 (1953). http://dx.doi.org/10.1038/171649a0

Bracewell RN, Simulating the sunspot cycle, Natur, 323, 516-519 (1986). http://dx.doi.org/10.1038/323516a0

Chang H-Y, A new method for North-South asymmetry of sunspot area, JASS, 24, 261-268 (2007). http://dx.doi.org/10.5140/JASS.2007.24.4.261

Chang H-Y, Stochastic properties in North-South asymmetry of sunspot area, NewA, 13, 195-201 (2008). http://dx.doi.org/10.1016/j.newast.2007.08.007

Chang H-Y, Periodicity of North-South asymmetry of sunspot area revisited: cepstrum analysis, NewA, 14, 133-138 (2009). http://dx.doi.org/10.1016/j.newast.2008.07.001

Chang H-Y, Correlation of parameters characterizing the latitudinal distribution of sunspots, NewA, 16, 456-460 (2011). http://dx.doi.org/10.1016/j.newast.2011.04.003

Charbonneau P, Dikpati M, Stochastic fluctuations in a babcock-leighton model of the solar cycle, ApJ, 543, 1027-1043 (2000). http://dx.doi.org/10.1086/317142

Dicke RH, Is there a chronometer hidden deep in the sun, Natur, 276, 676-680 (1978). http://dx.doi.org/10.1038/276676b0

Hathaway DH, Wilson RM, Reichmann EJ, The shape of the sunspot cycle, SoPh, 151, 177-190 (1994). http://dx.doi.org/10.1007/BF00654090

Komm RW, Hill F, Howe R, Empirical mode decomposition and Hilbert analysis applied to rotation residuals of the solar convection zone, ApJ, 558, 428-441 (2001). http://dx.doi.org/10.1086/322464

Lantos P, Prediction of the maximum amplitude of solar cycles using the ascending inflexion point, SoPh, 196, 221-225 (2000). http://dx.doi.org/10.1023/A:1005219818200

Passos D, Lopes I, Grand minima under the light of low order dynamo model, JASTP, 73, 191-197 (2011). http://dx.doi.org/10.1016/j.jastp.2009.12.019

Petrovay K, Solar cycle prediction, LRSP, 7, 6-59 (2010).

Udny Yule G, On a method of investigating periodicities in disturbed series with special reference to Wolfer's sunspot numbers, RSPTA, 226, 267-298 (1927).

Usoskin IG, A history of solar activity over millennia, LRSP, 5, 3-88 (2008).

Waldmeier M, Neue eigenschaften der sonnenfleckenkurve, Astron. Mitt. Zurich, 14, 105-130 (1935).