# Polygonal Approximation of Digital Curves to Preserve Original Shapes

Daeho Lee and SeungGwan Lee

*In this letter, we propose a novel polygonal approximation of digital curves that preserve original shapes. The proposed method first detects break points, which have two different consecutive vectors, and sets an initial dominant point set. The approximation is then performed iteratively by deleting a dominant point using a novel distance, which can measure both the distance and the angle acuteness. The experimental results show that the proposed method can preserve original shapes and is appropriate for various shapes, including slab-sided shapes.*

*Keywords: Polygonal approximation, shape representation, dominant points, digital planar curves.*

## I. Introduction

For shape representation, the approximation of objects is very useful in many object recognition and tracking applications. The objects may be approximated by boundaries [1] or by region [2]. Boundary-based approximation is more efficient and preserves the complete shape [3]. Boundaries are approximated as simple curves by splines [4] or polygons by line segments [3], [5], [6] using the dominant points (DPs). DPs are considered to be representative features for object contours [7] and are used as the end points of straight line segments in a polygonal approximation.

For a polygonal approximation, curvatures [8], $k$ cosines [9] and break point suppression [3], [5], [6] may be applied. Curvatures and $k$ cosines are not appropriate for shape representation because they are not calculated by correlation among DPs.

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The polygonal approximation is performed by deleting some of the DPs. To eliminate minor DPs, the perpendicular distance between a point and a straight line is generally used [3], [5], [6]. This distance yields good performance, considering only the perpendicular distance (error) of all curve points from the approximated polygon. The DP with the smallest perpendicular distance, however, cannot be guaranteed to be the most minor DP, especially when the DP is near the straight line and the included angle is acute. To solve this drawback, we define a new distance term for eliminating the DPs. This distance is appropriate for arbitrarily-shaped object contours, including slab-sided shapes.

## II. Polygonal Approximation

Let $C$ be the boundary point set of the original shape:

$$C = \{p_i(x_i, y_i) \mid i = 0, 1, 2, \cdots, N-1\}, \quad (1)$$

where $N$ is the number of points, and the last point $p_{N-1}$ is followed by the first point $p_0$. If there is constant difference between two consecutive vectors ($\mathbf{a}_{i1} = (x_i - x_{i-1}, y_i - y_{i-1})$ and $\mathbf{b}_{i1} = (x_{i+1} - x_i, y_{i+1} - y_i)$) of $p_i$, the approximated line segment outside of $p_i$ is equal to the original boundary. We first detect break points which have two different consecutive vectors and set those as the initial DPs. Figure 1 shows a toothbrush shape and its boundary and break points.

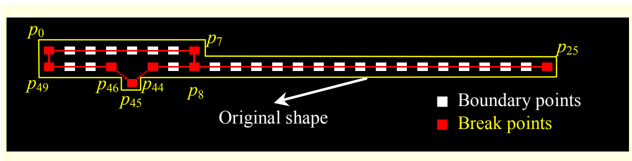


Fig. 1. Break points of the toothbrush shape.

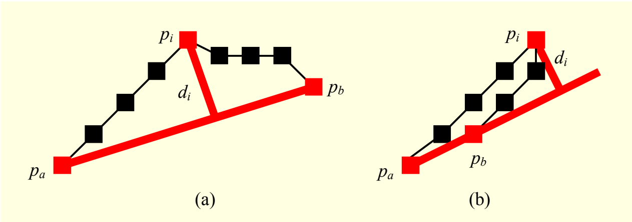


Fig. 2. Perpendicular distances.

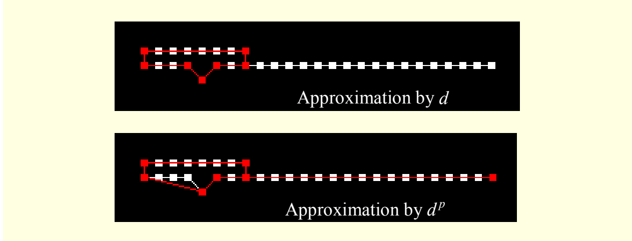


Fig. 3. The first DP elimination of the toothbrush shape.

To detect the most minor DP, most research [3], [5], [6] uses the perpendicular distance ($d$) as

$$d_i = \sqrt{\frac{((x_i - x_a)(y_b - y_a) - (y_i - y_a)(x_b - x_a))^2}{(x_a - x_b)^2 + (y_a - y_b)^2}}, \qquad (2)$$

where $d_i$ is the perpendicular distance of any point $p_i$ from the straight line connecting the points $p_a$ and $p_b$, and $p_a$ and $p_b$ are the neighbor DPs which are not eliminated with counterclockwise and clockwise turns, respectively. The magnitude of $d_i$ is used to determine the dominance of the DPs, so that the DP with the smallest $d_i$ is eliminated. When a DP is near to the straight line and the included angle with the two near DPs is acute, as shown in Fig. 2(b), $p_i$ is a minor DP because of its small $d_i$, so it may be eliminated.

To overcome the disadvantage of $d$, we define a novel distance ($d^p$) between a point $p_i$ and the straight line connecting the points $p_a$ and $p_b$ as

$$d_i^p(p_i, \overline{p_a p_b}) = \sqrt{\left(\overline{p_i p_a} + \overline{p_i p_b}\right)^2 - \left(\overline{p_a p_b}\right)^2}, \qquad (3)$$

where $p_a$, $p_b$, and $p_i$ are not collinear, and $\overline{p_a p_b}$ is always less than the sum of other lines, so that $d^p$ is always greater than zero, and they can construct a triangle. $d^p$ is symmetric because $d_i^p(p_i, \overline{p_a p_b}) = d_i^p(\overline{p_a p_b}, p_i)$.

In (3), the squares are significant because the equivalence of $d^p$ is calculated using the law of cosines as

$$\begin{aligned} & d_i^p(p_i, \overline{p_a p_b}) \\ &= \sqrt{\left(\overline{p_i p_a} + \overline{p_i p_b}\right)^2 - \left(\overline{p_i p_a}\right)^2 - \left(\overline{p_i p_b}\right)^2 + 2\overline{p_i p_a}\,\overline{p_i p_b}\cos\theta} \qquad (4) \\ &= \sqrt{2\overline{p_i p_a}\,\overline{p_i p_b}(\cos\theta_i + 1)}, \end{aligned}$$

where $\theta_i$ is the included angle of two line segments ($\overline{p_i p_a}$ and $\overline{p_i p_b}$), so $d^p$ can measure both the distance and the angle acuteness.

Although $p_{25}$ of the toothbrush shape of Fig. 1 is one of the most dominant points, $p_{25}$ is eliminated first by $d$ (as shown in Fig. 3) because the distance from the line connecting the two neighbor DPs ($p_8$ and $p_{44}$) is zero. However, $p_{46}$ is eliminated first by $d^p$ because both the distance and the angle acuteness are considered.

The elimination is performed iteratively until the required

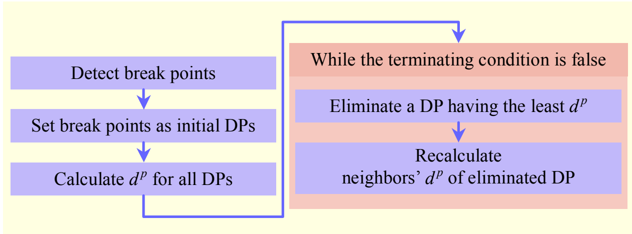


Fig. 4. Flow of the proposed method.

level of approximation is obtained. The terminating conditions used may be the desired number of DPs, the desired value of the root mean square error (RMSE), or the desired value of the mean of the absolute difference (MAD). Therefore, the proposed polygonal approximation is processed using the flow shown in Fig. 4.

## III. Experimental Results

To evaluate the performance of the proposed method, we use two measures defined by

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1}(d_i)^2}, \qquad (5)$$

$$\text{MAD} = \frac{1}{N}\sum I(x, y) - I_a(x, y), \qquad (6)$$

where $d_i$ is the perpendicular distance of all boundary points calculated from the straight line connecting the nearest right and left DPs, and $I(x, y)$ and $I_a(x, y)$ are the original image and the image approximated using region filling, respectively. The RMSE evaluates the boundary error, while the MAD evaluates the region error. Therefore, MAD is more appropriate for evaluating the shape representation.

To evaluate the performance of our proposed distance, we compared it with the approximation obtained using the perpendicular distance in the experiment. Figure 5 shows the RMSE and MAD of the toothbrush shape, where $n$ denotes the number of DPs. The RMSE and MAD obtained using the proposed method are less than those obtained using the approximation by $d$.

Figures 6 and 7 compare the results for the chromosome

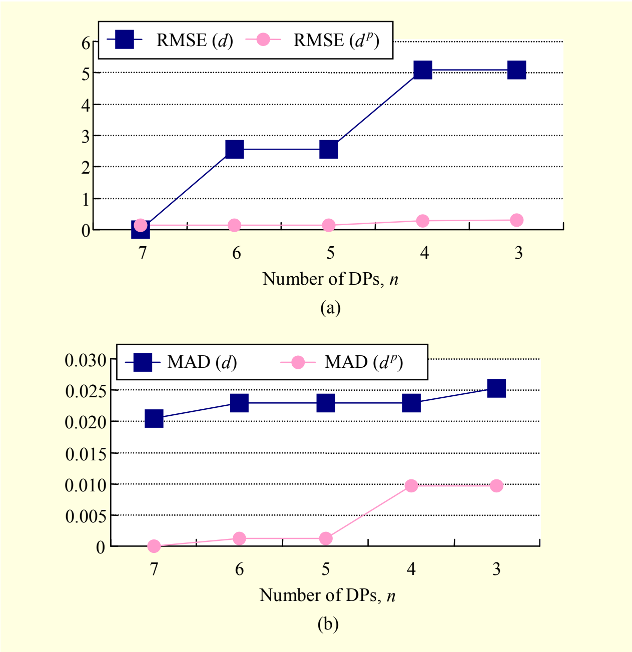


Fig. 5. (a) RMSE and (b) MAD of the toothbrush shape.

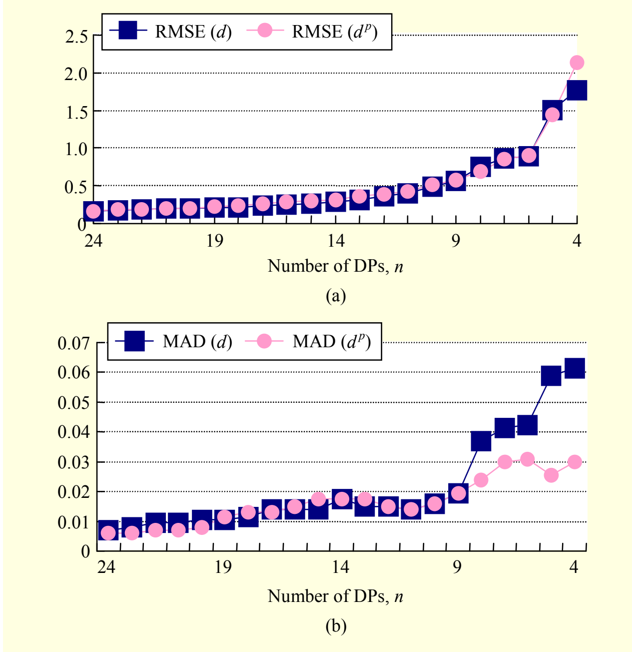


Fig. 7. (a) RMSE and (b) MAD of the chromosome shape.

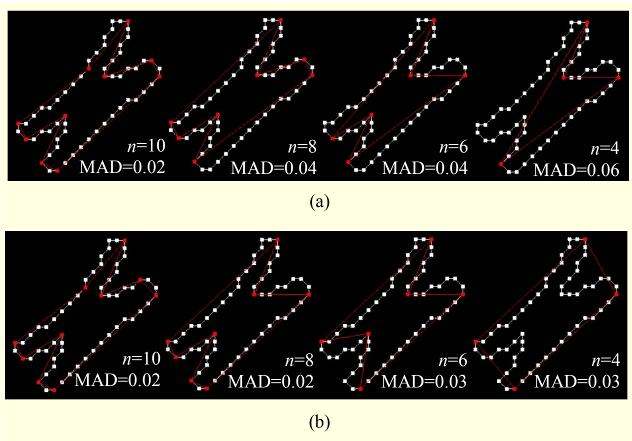


Fig. 6. Approximation of the chromosome shape using (a) $d$ and (b) $d^P$.

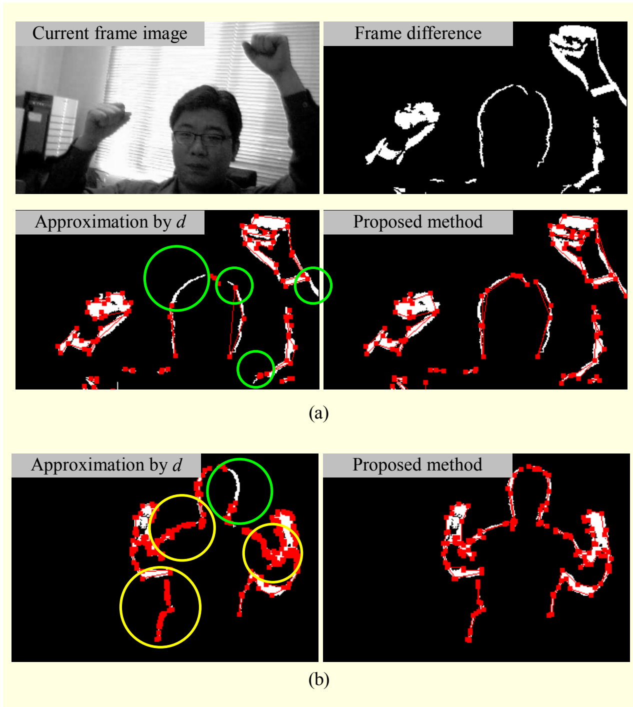


Fig. 8. Approximation of frame difference.

shape, which is commonly used in dominant point detection and polygonal approximation. As shown in Fig. 7, the two results are similar when $n > 8$, while the MADs of the proposed method when $n \leq 8$ are superior to those obtained using the approximation by $d$.

To show that the proposed method is more appropriate for arbitrary shapes including slab-sided shapes, we performed comparison tests of the frame difference images as shown in Fig. 8, where the curves approximated by $d$ do not preserve the original frame differences (marked with green circles), while the proposed method does preserve the original shapes. In addition, the approximation using $d$ may result in too many DPs with the same RMSE terminating condition as shown in Fig. 8(b) (marked with yellow circles).

## IV. Conclusion

A novel polygonal approximation was presented in this letter. The proposed method uses a novel factor that can measure both the distance and the angle acuteness, allowing it to more properly distinguish nearby DPs from far away DPs. The experimental results showed that this method can preserve original shapes even when they are arbitrary and slab-sided. Therefore, our method can be used for object recognition and tracking using matching with high accuracy. In addition, this method can be used for shape modeling, as in human body estimation in computer games.

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