

Spectrally-Efficient Decode-and-Forward Scheme Based on Constellation Rotation

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A spectrally-efficient scheme is proposed for orthogonal decode-and-forward relaying. By utilizing constellation rotation, the scheme can achieve twice the spectral efficiency as that of the conventional one, with low implementation complexity. It can offer a full diversity order as well, whereas the loss in coding gain is less than 1 dB for practical environments.

Keywords: Spectral efficiency, diversity order, decode-and-forward, constellation rotation.

I. Introduction

Cooperative communications [1] is a promising technology to improve the performance of wireless networks. Various cooperation protocols have been proposed [2], among which orthogonal decode-and-forward (ODF) has attracted much attention [3]. In this protocol, the relay decodes the received data from the source before forwarding it to the destination, and the source and relay transmit over orthogonal time intervals. The main drawback of this protocol is that the achievable spectral efficiency, in terms of the number of symbols transmitted per channel use, is decreased by half compared to that of the non-cooperation scheme, which limits its deployment in a high rate transmission scenario. In this letter, motivated by the idea of constellation rotation [4], we propose a novel ODF protocol which can achieve twice the spectral efficiency of the conventional one and full diversity gain as well. Compared with the conventional ODF, the complexity of

our scheme is just slightly increased, and the loss in coding gain is very small if the source-to-relay link has good channel quality. Our work can be viewed as a generalization of [4] from the point-to-point channels to relay channels.

II. System Model

We consider a basic cooperative system with a source (S), a destination (D), and a relay (R). Each node has a single antenna and operates in a half-duplex mode, that is, they cannot transmit and receive simultaneously. We assume that the transmitting nodes have no channel state information (CSI), and the receiving nodes have perfect CSI. In ODF protocol (either conventional or proposed), every frame is composed of two slots. In the first slot, S broadcasts its message to D and R. In the second slot, R retransmits the same message if it decodes correctly; otherwise, it remains inactive. At the end of the second slot, D makes decisions. The channel between any two nodes is assumed to be slow flat Rayleigh fading, for which the channel coefficient is constant within one frame and changes independently from frame to frame. The additive noise at each receiver is modeled as a zero-mean complex Gaussian random variable with variance N_0 . $\Re\{x\}$, $\Im\{x\}$, and x^* denote the real part, imaginary part, and the complex conjugate of x , respectively. Also, $j = \sqrt{-1}$ and E is the expectation operator.

III. Proposed Scheme

In our scheme, constellation rotation is applied to every symbol prior to transmission. Specifically, if u is the un-rotated complex constellation, taking values from an M-ary alphabet \mathcal{X} , then the transmitted symbol x is $e^{j\theta}u$, where θ is chosen

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in such a way that no two rotated constellation points have the same in-phase or quadrature component. That is to say, for any $i \neq k$,

$$\Re\{x^i\} \neq \Re\{x^k\}, \quad \Im\{x^i\} \neq \Im\{x^k\}, \quad \forall x^i, x^k \in e^{j\theta} \mathcal{X}. \quad (1)$$

For clarity of exposition, we consider a symbol-by-symbol transmission and describe the scheme as follows. In the first slot, the source jointly transmits two consecutive symbols by constructing a new symbol as

$$x = \Re\{x_1\} + j\Re\{x_2\}, \quad (2)$$

where x_1 and x_2 are two consecutive symbols. By denoting the channel between node a and b by h_{ab} , we have

$$y_{D1} = h_{SD}x + w_{D1}, \quad (3)$$

$$y_{R1} = h_{SR}x + w_{R1}, \quad (4)$$

where y_{mm} is the received signal at node m at the end of the n -th slot, and w_{mm} is the corresponding additive noise. The relay performs the following operations to construct two decision variables:

$$r_{R1} = \Re\left\{\frac{h_{SR}^*}{|h_{SR}|} y_{R1}\right\}, \quad r_{R2} = \Im\left\{\frac{h_{SR}^*}{|h_{SR}|} y_{R1}\right\}. \quad (5)$$

According to (5), the relay estimates the real parts of x_1 and x_2 . Then, the complex symbols x_1 and x_2 can be determined because it is a one-to-one mapping between the rotated constellation point and its real (imaginary) part [4]. If there is no decoding error, the relay transmits the following symbol. (Otherwise, the relay remains idle.)

$$x' = \Im\{x_2\} + j\Re\{x_1\}, \quad (6)$$

and the received signal at the destination is

$$y_{D2} = h_{RD}x' + w_{D2}. \quad (7)$$

The destination then determines the symbols x_1 and x_2 separately from two decision variables as

$$r_{D1} = \Re\left\{\frac{h_{SD}^*}{|h_{SD}|} y_{D1}\right\} + j\Im\left\{\frac{h_{RD}^*}{|h_{RD}|} y_{D2}\right\}, \quad (8)$$

$$r_{D2} = \Im\left\{\frac{h_{SD}^*}{|h_{SD}|} y_{D1}\right\} + j\Re\left\{\frac{h_{RD}^*}{|h_{RD}|} y_{D2}\right\}. \quad (9)$$

Note that two symbols are transmitted over two time slots, so the spectral efficiency is doubled compared to that of the conventional ODF protocol. In addition, the two symbols can be estimated separately through single maximum likelihood (ML) decoding, yielding low implementation complexity.

IV. Symbol Error Probability Analysis

In this section, the upper bound for the symbol error probability (SEP) is derived to realize diversity analysis. For simplicity, we suppose a unit energy MPSK constellation and assume the relay can decode the source message correctly. The effect of decoding error will be analyzed in the next section. The variances of h_{SD} and h_{RD} are set to be one.

Consider the SEP of x_1 , that is, u_1 first. Assuming the transmitted symbol is x_1^i and letting $P\{x_1^i \rightarrow x_1^k\}$ be the pairwise error probability (PEP) of confusing x_1^i with x_1^k when x_1^i and x_1^k are the only two hypotheses, the conditional SEP of x_1 is upper bounded by

$$P\{\text{error} | x_1^i \text{ sent}\} \leq \sum_{x_1^k \in e^{j\theta} \mathcal{X}, k \neq i} P\{x_1^i \rightarrow x_1^k\}. \quad (10)$$

According to (8), it is easy to show that

$$P\{x_1^i \rightarrow x_1^k | h_{SD}, h_{RD}\} = Q\left(\sqrt{\frac{|h_{SD}|^2 d_{ik,1}^2 + |h_{RD}|^2 d_{ik,2}^2}{2N_0}}\right), \quad (11)$$

where Q is the Gaussian-Q function and

$$d_{ik,1} = \Re\{x_1^i\} - \Re\{x_1^k\}, \quad d_{ik,2} = \Im\{x_1^i\} - \Im\{x_1^k\}. \quad (12)$$

Using the Chernoff technique, the upper bound of (11) is given by

$$\begin{aligned} & P\{x_1^i \rightarrow x_1^k | h_{SD}, h_{RD}\} \\ & \leq \frac{1}{2} \exp\left(-\frac{|h_{SD}|^2 d_{ik,1}^2 + |h_{RD}|^2 d_{ik,2}^2}{4N_0}\right). \end{aligned} \quad (13)$$

Thus, the PEP can be calculated as

$$\begin{aligned} P\{x_1^i \rightarrow x_1^k\} &= E_{h_{SD}, h_{RD}} [P\{x_1^i \rightarrow x_1^k | h_{SD}, h_{RD}\}] \\ &\leq (1 + d_{ik,1}^2 / (4N_0))^{-1} (1 + d_{ik,2}^2 / (4N_0))^{-1}. \end{aligned} \quad (14)$$

In high signal-to-noise ratio (SNR) regime, (14) can be approximated by

$$P\{x_1^i \rightarrow x_1^k\} \leq 16(d_{ik,1} d_{ik,2} \text{SNR})^{-2}, \quad (15)$$

where $1/N_0$ is the average SNR. By substituting (15) into (10) and using the total probability formula, the upper bound for the average SEP of x_1 can finally be expressed as

$$P\{\text{error}\} \leq \frac{16}{M} \text{SNR}^{-2} \sum_{i \neq k} \frac{1}{d_{ik,1}^2 d_{ik,2}^2}. \quad (16)$$

In the same way, we can also obtain a similar expression for the SEP of x_2 , that is, u_2 . The above results demonstrate that the diversity order of 2 (a full diversity order) can be achieved, as long as (1) is satisfied.

V. Simulation Results and Discussion

In this section, simulation results are presented. QPSK modulation is employed in the simulations. By exhaustive search, the optimal rotation angle which minimizes (16) is computed to be $\theta^* \approx 15.37^\circ$. This value is optimal in the sense of minimizing the upper bound of SEP, and it is sufficient (but not necessary) for maintaining the full diversity order. For fairness, we restrict the total energy used for each symbol so that it is the same for both schemes and normalize it to be one. Furthermore, it is supposed that a perfect error checking can be performed at R. Figure 1 depicts the results for ideal inter-user channel, that is, there is no detection error at the relay. Figure 1 shows that the proposed scheme can provide the same diversity order as the conventional one, while the loss in coding gain is less than 0.5 dB, which is almost negligible in many applications. This loss is due to the fact that the minimum distance for the rotated constellation along the real or the imaginary axis is smaller than that for the full complex constellation. In Fig. 2, we present the results for a non-ideal

inter-user channel. Here we assume that the mean value of source-relay channel gain is 10 dB higher than that of source-destination and relay-destination links. Note that this is a reasonable assumption because in a practical network, such as cellular network or ad hoc network, there are many nodes, and it is easy to select one with good channel quality between it and the source as the relay. Similar observations can be obtained from Fig. 2 except that the loss in coding gain is slightly larger, about 1 dB. The performance loss incurred by our scheme can be compensated by using several techniques such as channel coding.

VI. Conclusion

A novel scheme was presented to improve the spectral efficiency for ODF systems. The proposed scheme can achieve full diversity with low implementation complexity. Further work will include investigating the performance of this method for M-QAM modulations.

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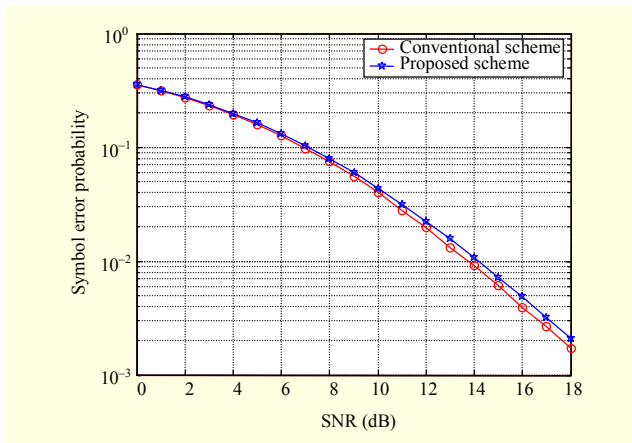


Fig. 1. Symbol error probabilities for ideal inter-user channel.

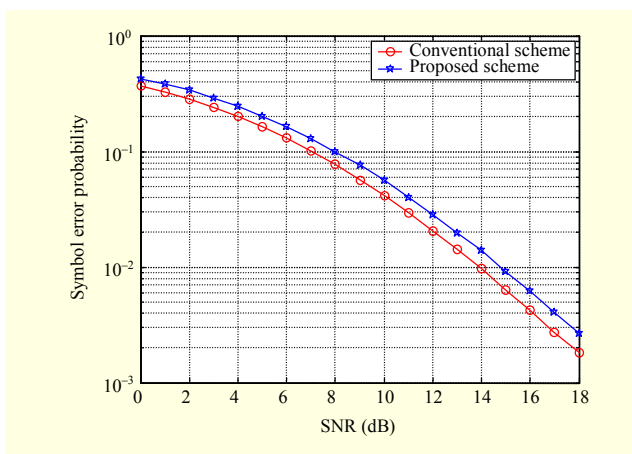


Fig. 2. Symbol error probabilities for non-ideal inter-user channel.