Channel Estimation and LDPC Code Puncturing Schemes Based on Incremental Pilots for OFDM

Sung-Yoon Jung and Sunghwan Kim

In this letter, we propose a channel estimation algorithm based on incremental pilots. These are pilots additionally inserted after puncturing the modulated orthogonal frequency division multiplexing (OFDM) symbols to enhance channel estimation performance without lowering bandwidth efficiency. A low-density parity-check code puncturing scheme is also proposed to prevent the performance degradation due to the codeword bit loss caused by punctured OFDM symbols.

Keywords: Channel estimation, low-density parity-check (LDPC) code, incremental pilots.

I. Introduction

For the channel estimation in coherent orthogonal frequency division multiplexing (OFDM) system, conventional pilot-aided schemes are widely used in practice because they decouple symbol detection from channel estimation. However, increasing the number of pilot tones for better channel estimation decreases the amount of data for transmission [1]. To overcome this problem, we propose a channel estimation algorithm based on *incremental pilots*. These are pilots additionally inserted after puncturing the modulated OFDM symbols for more accurate channel estimate without any loss in bandwidth efficiency. However, the additional punctured bits due to OFDM symbol puncturing for incremental pilots will affect bit error performance. Therefore, we propose a low-density parity-check (LDPC) code puncturing scheme. It determines the mapping between the new pilots in the OFDM symbols and the punctured parity bits in the structured LDPC codes in order to prohibit the performance degradation.

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II. System Description

The message bits $\mathbf{i}$ of size k are encoded into $\mathbf{c}$ of length n by an encoder of a systematic LDPC code. Then, the encoded block $\mathbf{c}$ is interleaved to form $\underline{\mathbf{c}}$. After passing through the M-QAM modulator, $\underline{\mathbf{c}}$ is mapped to form N_s encoded data symbols $\mathbf{s}$. Then, $\mathbf{s}$ and N_p pilots $\mathbf{p}$ are multiplexed to form $\mathbf{d}$ with $N(=N_s+N_p)$ elements. Note that the placement of pilots at the desired positions can be determined on the optimal pilot placement for channel estimation [2]. Then, we implement N-point inverse discrete Fourier transform (DFT) on $\mathbf{d}$ and insert the cyclic prefix (CP) of length N_{cp} to form $\mathbf{x}$ of length $(N+N_{cp})$. Following the parallel-to-serial conversion, an OFDM symbol is transmitted through the multipath channel.

The frequency-selective fading channel is modeled into the L length finite impulse response discrete-time filter. We assume that the channel variations are negligible over one OFDM block. Then, we define the channel tap vector $\mathbf{h}=[h(0), h(1),\cdots, h(L-1)]^T$, where $\mathbf{h} \sim CN(\mathbf{0};\mathbf{C}_h)$ and $\mathbf{C}_h=E\{\mathbf{h}\mathbf{h}^H\}$. In addition, let us define the channel frequency response of $\mathbf{h}$ as $\mathbf{h}_F=[h_F(0), h_F(1),\cdots, h_F(N-1)]^T$. Then, $\mathbf{h}_F$ can be represented as $\mathbf{h}_F=\sqrt{N}\,\mathbf{G}\mathbf{h}$, where $\mathbf{G}$ consists of the first L columns of an $N\times N$ DFT matrix $\mathbf{F}$ that has an (p, q) entry $(1/\sqrt{N})\times \exp(-j2\pi pq/N)$, where $p, q=0, 1,\cdots, N-1$.

After removing CP and taking the DFT operation by OFDM demodulation in the receiver, we can obtain the following input-output relationship:

$$\mathbf{y} = \mathbf{D}_N(\mathbf{h}_F)\mathbf{d} + \mathbf{w} = \sqrt{N}\mathbf{D}_N(\mathbf{d})\mathbf{G}\mathbf{h} + \mathbf{w}, \tag{1}$$

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where $\mathbf{D}_M(\mathbf{h}_F)$ and $\mathbf{D}_M(\mathbf{d})$ are diagonal matrices whose principal diagonal components are $\mathbf{h}_F$ and $\mathbf{d}$, respectively; and $\mathbf{w} := [w(0),\cdots, w(N-1)]$ denotes additive white Gaussian noise. Then, channel estimation is performed prior to decoding by using known N_p pilot symbols. Finally, demodulation and decoding are carried out after channel compensation.

III. Channel Estimation with Incremental Pilots

We use only N_p pilot symbols, which are placed in the OFDM block at known positions for the original channel estimation. From (1), we can extract observations corresponding to known pilot positions as

$$\mathbf{y}_p = \mathbf{B}_p\mathbf{h} + \mathbf{w}_p, \qquad (2)$$

where $\mathbf{y}_p := [y(i_0),\cdots, y(i_{Np-1})]^T$, $\mathbf{B}_p := \sqrt{N}\ \mathrm{diag}[d(i_0),\cdots, d(i_{Np-1})]\mathbf{F}_p$, with $\mathbf{F}_p$ of size $(N_p\times L)$, denoting the first L columns and pilot position-related N_p rows of $\mathbf{F}$, and $\mathbf{w}_p := [w(i_0),\cdots, w(i_{Np-1})]$. Here, i_q for $q=0,\cdots, N_p-1$ denotes the index corresponding to the pilot positions. From (2), we estimate the channel via linear minimum mean square error (LMMSE) schemes [1]. However, the original channel estimation may not be good if the frequency selectivity of the channel is severe.

Therefore, we use more pilots made from punctured OFDM symbols (incremental pilots) instead of using just N_p pilot symbols for channel estimation as in (2). However, codeword bits loss, caused by puncturing OFDM symbols, should be prevented to avoid performance degradation. If $\mathbf{y}_i$ denotes observations corresponding to original pilots and the incremental pilot positions, we formulate

$$\mathbf{y}_i = \mathbf{B}_i\mathbf{h} + \mathbf{w}_i,$$

where $\mathbf{y}_i := [y(i_0),\cdots, y(i_{N_i-1})]^T$, $\mathbf{B}_i := \sqrt{N}\ \mathrm{diag}[d(i_0),\cdots, d(i_{N_i-1})]\mathbf{F}_i$, with $\mathbf{F}_i$ of size $(N_i\times L)$, denoting the first L columns and the original plus incremental pilot position-related N_i rows of $\mathbf{F}$. Then, the estimated channel with the LMMSE estimator can be expressed as

$$\hat{\mathbf{h}}_i^{\mathrm{LMMSE}} := \left(\sigma_w^2\mathbf{R}_h^{-1} + \mathbf{B}_i^{\mathrm{H}}\mathbf{B}_i\right)^{-1}\mathbf{B}_i^{\mathrm{H}}\mathbf{y}_i.$$

Since the channel compensator requires knowledge of all channel frequency responses on the FFT grid, the estimate of $D_N\left(\hat{\mathbf{h}}_{\mathrm{F}}^{i}\right)$ corresponding to $\hat{\mathbf{h}}_i^{\mathrm{LMMSE}}$ can be computed as

$$D_N\left(\hat{\mathbf{h}}_{\mathrm{F}}^{i}\right) = \sqrt{N}\cdot\mathrm{diag}\left[\mathbf{G}\hat{\mathbf{h}}_i^{\mathrm{LMMSE}}\right].$$

IV. LDPC Code Puncturing Scheme

Among the noteworthy channel codes recently studied, structured LDPC codes as a special case of quasi-cyclic LDPC codes [3] are considered. The $r\times n$ parity-check matrix H in the structured LDPC code is composed of an $r\times(n-r)$ systematic part H_s and an $r\times r$ parity part H_p with duo-diagonal structure. The parity check matrix H can be constructed from the $r_b\times n_b$ base matrix H_b and $z\times z$ circulant permutation matrices. For convenience, we assume that z is $N_p\log_2 M$. In this letter, it is assumed that N_p and z are a power of 2.

Let us assume that the column index i, $0\le i<r$, corresponding to the i-th variable node in a bipartite graph of the parity part, are represented as $i=l\times z+m$, where $0\le l<r_b$ and $0\le m<z$. The number of punctured bits will increase as the number of puncturing stages increase. We consider all punctured bits as 1-step recoverable nodes [4]. The punctured bits are used for new pilot symbols, and the new pilots are located with equal distance in the FFT grid for better channel estimation [2]. For these constraints, the number of punctured bits at the j-th stage is given as $(2^j-1)z$. Let $\mathbf{s}(t)$ be $[(t-1)z, (t-1)z+1,\cdots, (t-1)z+(z-1)]$. Then, at the j-th stage, the positions of punctured bits are given as

$$\left\{\mathbf{s}\left(\frac{r_b+1}{2^j}\right), \mathbf{s}\left(2\frac{r_b+1}{2^j}\right), \cdots, \mathbf{s}\left(\left(2^j-1\right)\frac{r_b+1}{2^j}\right)\right\}, \qquad (3)$$

where $0<j<\log_2(r_b+1)$. The 0th stage means the conventional channel estimation and LDPC code scheme without incremental pilots. The $(2^j-1)z$ parity bits of the LDPC codes

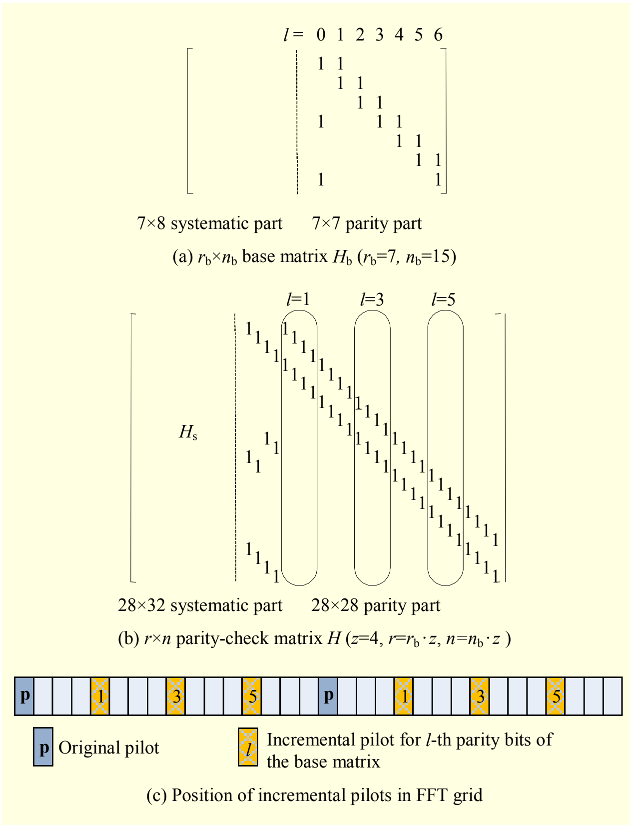

Fig. 1. Punctured bits and incremental pilots.

are punctured to be used for $(2^j-1)N_p$ incremental pilots.

Let $\mathbf{q}=[0, N/N_p, 2N/N_p, \cdots, (N_p-1)N/N_p]^T$ as the index vector of the initial N_p pilots in the FFT grid. Assume that $\mathbf{q}(f)$ is defined as $\mathbf{q}+[f, f, \cdots, f]^T$. At the j-th stage, the positions of the new incremental pilots in the FFT grid are given as

$$\left\{\mathbf{q}\left(\frac{N/N_p}{2^j}\right), \mathbf{q}\left(2\frac{N/N_p}{2^j}\right), \cdots, \mathbf{q}\left(\left(2^j-1\right)\frac{N/N_p}{2^j}\right)\right\}. \quad (4)$$

Let $\varphi_t := x \rightarrow y$ be a permutation function from t consecutive units in x to one unit in y. From (3) and (4), the mapping rule between the punctured bits and new pilot symbols at the j-th stage are given as

$$\varphi_{\log_2 M} : \mathbf{s}\left(g\frac{r_b+1}{2^j}\right) \rightarrow \mathbf{q}\left(g\frac{N/N_p}{2^j}\right),$$

where $1 \le g < 2^j$.

Figure 1 depicts an example of puncturing parity bits and incremental pilots with r_b=7, z=4, and M=4. The 28×60 parity check matrix in Fig. 1(b) is constructed from its 7×15 base matrix in Fig. 1(a) and z=4. Six new incremental pilots can be obtained in Fig. 1(c) by puncturing 12 parity bits.

V. Simulation Results

We evaluate the MSE of channel estimation and BER to investigate the performance of the proposed scheme. We used 40 MHz bandwidth, QPSK modulation, and the IEEE 802.11n channel model [5] with the frequency-selective channels of order L=15. The total number of subcarriers N is 1,024, and the number of equally-spaced pilot tones N_p is 16. The structured irregular LDPC codes [3] with 1024×2016 parity check matrix are used for LDPC codes.

Figure 2 illustrates the average channel estimation MSE defined as $E\left[\|\mathbf{h}-\hat{\mathbf{h}}\|^2\right]$ versus E_b/N_0, where $\hat{\mathbf{h}}$ denotes the

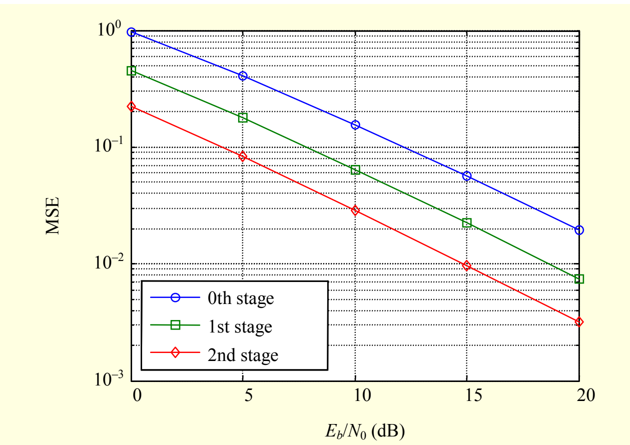

Fig. 2. Average channel estimation MSE.

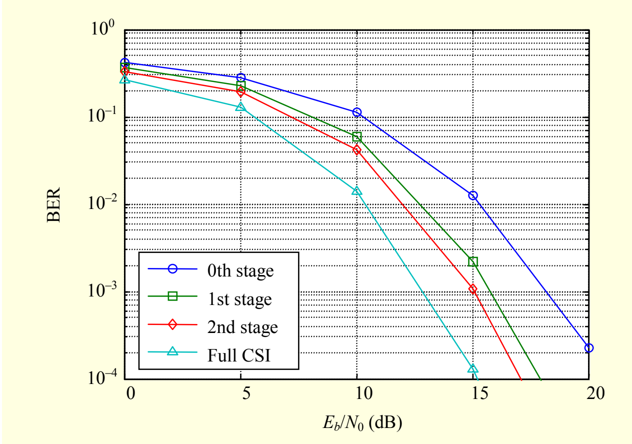

Fig. 3. BER performance when number of iteration is 10.

estimated channel. In Fig. 2, the MSEs of the proposed scheme are plotted at the 0th stage, the 1st stage, and the 2nd stage. This figure shows the MSE of the channel estimation is decreased by introducing incremental pilots.

Figure 3 plots BERs versus E_b/N_0 when the number of iterations for LDPC code decoding is 10. The ideal case, one with perfectly known full channel state information (CSI), is also depicted. Figure 3 shows that BER performance improves as the number of incremental pilots increase. This verifies our claim that the codeword bits loss caused by the OFDM symbol puncture for incremental pilots can be counterbalanced by our LDPC code puncturing scheme.

VI. Conclusion

In this letter, we proposed a channel estimation scheme based on incremental pilots and the corresponding LDPC code puncturing scheme. Through long-term channel simulations, we verified the improved channel estimation and BER performance without any loss in bandwidth efficiency. Note that if we want to consider the variation of the channel environment, it will be possible to introduce the adaptive decision of puncturing stage base on proper feedback information such as CSI.

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