Performance of Selective Decode-and-Forward Relay Networks with Partial Channel Information

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In this letter, closed-form approximations for outage probability and symbol error rate are presented for a selective decode-and-forward relay network with partial channel information. An independent but not identically distributed Rayleigh fading environment is considered. Numerical and simulated results demonstrate the validity of the analytical results.

Keywords: Symbol error rate, outage probability, decode-and-forward, relay selection.

I. Introduction

Relay selection is an attractive method to improve the bandwidth efficiency of cooperative networks [1]-[3]. Specifically, Bletsas and others [1] proposed a distributed relay selection based on the channel information of both the first and second hops for each relay, showing that the diversity-multiplexing tradeoff of relay selection is identical with that of distributed space-time coding.

Recently, amplify-and-forward (AF) with partial relay selection and no source-destination link was proposed by Krikidis and others [4]. The basic idea of relay selection with partial channel information is to select the relay with the best instantaneous signal-to-noise ratio (SNR) across a single hop (the source-relay link) rather than two hops. Since only the first-hop channel state information is used for selection, the network lifetime can be prolonged in resource-constrained wireless systems such as sensor networks. In [5], Suraweera and others analyzed the outage and error performance of semi-blind AF with partial relay selection. Kim and others presented a comparison of tightly power-constrained performance for opportunistic AF relaying with partial or full channel information in [6]. However, to the best of our knowledge, there has been no work published concerning the performance of decode-and-forward (DF) relay selection with partial channel information.

In this letter, a previous work regarding the AF protocol [4] is extended to the DF protocol, and the direct link (the source-destination link) is considered. The approximate expressions for the cumulative distribution function (CDF), moment-generating function (MGF) of the end-to-end SNR, outage probability, and symbol error rate (SER) are derived for DF cooperative systems with partial relay selection over independent but not identically distributed (INID) Rayleigh fading channels.

II. System and Channel Model

Consider a DF relay system with one source S, L relays $R_l\ (1 \le l \le L)$ and one destination D. Every node operates in half-duplex mode and is equipped with single transmit and receive antennas. Let h_{SD}, h_{Sl}, and h_{lD} denote the channel impulse responses of S-D, S-R_l, and R_l-D links, respectively. All the links are assumed to be INID Rayleigh distributed.

Let $\gamma_{SD} = \gamma_0 |h_{SD}|^2$, $\gamma_{Sl} = \gamma_0 |h_{Sl}|^2$, and $\gamma_{lD} = \gamma_0 |h_{lD}|^2$ denote instantaneous SNR at the S-D, S-R_l, and R_l-D links, respectively, where γ_0 is the average SNR. Then, they are independent exponential distributed random variables with parameters $\alpha_0 = 1/(\gamma_0 \Omega_{SD})$, $\alpha_l = 1/(\gamma_0 \Omega_{Sl})$, and $\beta_l = 1/(\gamma_0 \Omega_{lD})$, respectively. For partial relay selection, the relay $K = \arg\max_{0 \le l \le L}(\gamma_{Sl})$ which provides the best first-hop SNR γ_K is selected. According to order statistics, the CDF of γ_K can

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be given by $F_{\gamma_K}(\gamma)=\prod_{l=1}^{L} F_{\gamma_{Sl}}(\gamma)=\prod_{l=1}^{L}\left(1-e^{-\alpha_l \gamma}\right)$.

For a DF scheme, the end-to-end instantaneous SNR at the destination can be tightly approximated in the high SNR regime as in [7] as

$$\gamma=\gamma_{SD}+\gamma_{\mathrm{eq}}=\gamma_{SD}+\min(\gamma_K,\gamma_{KD}), \quad (1)$$

where γ_{KD} is the second-hop SNR, and its CDF equals $F_{\gamma_{KD}}(\gamma)=1-e^{-\beta\gamma}$. Then, the CDF of γ_{eq} can be written as

$$\begin{aligned} F_{\gamma_{\mathrm{eq}}}(\gamma) &= \Pr(\gamma_{\mathrm{eq}}<\gamma)=\Pr[\min(\gamma_K,\gamma_{KD})<\gamma] \\ &= 1-[1-F_{\gamma_K}(\gamma)][1-F_{\gamma_{KD}}(\gamma)] \\ &= 1-\left[1-\prod_{l=1}^{L}\left(1-e^{-\alpha_l\gamma}\right)\right]e^{-\beta\gamma}, \end{aligned} \quad (2)$$

where the product of L terms $\prod_{l=1}^{L}\left(1-e^{-\alpha_l\gamma}\right)$ can be expanded using the following formula:

$$\prod_{l=1}^{L}\left(1-e^{-\alpha_l\gamma}\right)=1+\sum_{l=1}^{L}(-1)^l\sum_{\lambda_1=1}^{L-l}\sum_{\lambda_2=\lambda_1+1}^{L-l+1}\cdots\sum_{\lambda_l=\lambda_{l-1}+1}^{L}e^{-\gamma\sum_{n=1}^{l}\alpha_{\lambda_n}}. \quad (3)$$

Substituting (3) into (2), the CDF of γ_{eq} in (2) can be rewritten as

$$F_{\gamma_{\mathrm{eq}}}(\gamma)=1+\sum_{l=1}^{L}(-1)^l\sum_{\lambda_1=1}^{L-l}\sum_{\lambda_2=\lambda_1+1}^{L-l+1}\cdots\sum_{\lambda_l=\lambda_{l-1}+1}^{L}e^{-\gamma\delta}, \quad (4)$$

where $\delta=\beta+\sum_{n=1}^{l}\alpha_{\lambda_n}$ and, correspondingly, the probability density function (PDF) can be given by

$$f_{\gamma_{\mathrm{eq}}}(\gamma)=\sum_{l=1}^{L}(-1)^{l-1}\sum_{\lambda_1=1}^{L-l}\sum_{\lambda_2=\lambda_1+1}^{L-l+1}\cdots\sum_{\lambda_l=\lambda_{l-1}+1}^{L}\delta e^{-\gamma\delta}. \quad (5)$$

The MGF is defined as the Laplace transform of PDF. Then, the MGF of γ_{eq} can be given by

$$M_{\gamma_{\mathrm{eq}}}(s)=\sum_{l=1}^{L}(-1)^{l-1}\sum_{\lambda_1=1}^{L-l}\sum_{\lambda_2=\lambda_1+1}^{L-l+1}\cdots\sum_{\lambda_l=\lambda_{l-1}+1}^{L}\frac{\delta}{\delta+s}. \quad (6)$$

Since γ_{SD} and γ_{eq} in (1) are independent, we can get the MGF of $\gamma: M(s)=M_{\gamma_{\mathrm{eq}}}(s)M_{\gamma_{SD}}(s)$, where $M_{\gamma_{SD}}(s)=\alpha_0/(\alpha_0+s)$ denotes the MGF of γ_{SD}. After some manipulation and applying the inverse Laplace transform, a simple closed-form expression for the CDF of the end-to-end SNR γ in (1) can be obtained by

$$F(\gamma)=\sum_{l=1}^{L}(-1)^{l-1}\sum_{\lambda_1=1}^{L-l}\sum_{\lambda_2=\lambda_1+1}^{L-l+1}\cdots\sum_{\lambda_l=\lambda_{l-1}+1}^{L}\left(1-\frac{\delta e^{-\alpha_0\gamma}-\alpha_0 e^{-\delta\gamma}}{\delta-\alpha_0}\right). \quad (7)$$

III. Performance Analysis

In this section, the outage probability and SER of a DF cooperative system with partial relay selection over INID Rayleigh fading channels is investigated.

1. Outage Probability

The end-to-end mutual information of dual-hop DF relay selection with partial channel information can be expressed as in [1], [6] as

$$I=\frac{1}{2}\log_2(1+\gamma), \quad (8)$$

where the factor 1/2 denotes the rate loss due to the half-duplex operation.

Outage probability is an important performance for a wireless system and can be defined as the probability P_{out} that the instantaneous mutual information I in (8) is less than a target rate C. Therefore, a simple closed-form for outage probability can be given by

$$P_{\mathrm{out}}=\Pr(I<C)=\Pr(\gamma<2^{2C}-1)=F(2^{2C}-1). \quad (9)$$

2. Symbol Error Rate

In this subsection, a closed-form expression for the average SER of DF with partial relay selection is derived over INID Rayleigh fading. The average SER can be determined by

$$P_{\mathrm{e}}=E[aQ(\sqrt{2b\gamma})]=a\int_0^{+\infty}Q(\sqrt{2bx})f(\gamma)d\gamma, \quad (10)$$

where $Q(x)=(1/\sqrt{2\pi})\int_x^{+\infty}e^{-t^2/2}dt$ is the Gaussian Q-function, $f(\cdot)$ denotes the PDF of the output SNR, and a and b are the modulation-specific constants. For example, for binary phase-shift keying (BPSK) modulation, $a=1, b=1$; for M-ary pulse amplitude modulation, $a=2(M-1)/M$ and $b=3/(M^2-1)$. Our results also provide the approximate SER for M-ary PSK when $a=2$ and $b=\sin^2(\pi/M)$.

Based on equation (32) in [8], the average SER expression in (10) can be rewritten as

$$P_{\mathrm{e}}=\frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^{+\infty}\frac{e^{-b\gamma}}{\sqrt{\gamma}}F(\gamma)d\gamma. \quad (11)$$

Substituting (7) into (11), we can obtain the following closed-form expression for average SER:

$$\begin{aligned} P_{\mathrm{e}}=&\sum_{l=1}^{L}(-1)^{l-1}\sum_{\lambda_1=1}^{L-l}\sum_{\lambda_2=\lambda_1+1}^{L-l+1}\cdots \\ &\sum_{\lambda_l=\lambda_{l-1}+1}^{L}\left[\frac{a}{2}-\frac{a\sqrt{b}}{2}\frac{\delta e^{-\alpha_0\gamma}(b+\alpha_0)^{-0.5}-\alpha_0 e^{-\delta\gamma}(b+\delta)^{-0.5}}{\delta-\alpha_0}\right]. \end{aligned} \quad (12)$$

IV. Numerical Results and Conclusion

In this section, some numerical examples are provided to illustrate and validate the analytical results derived in the previous sections. For simplicity, we assume that the number of relays L=2, and the second-hop parameter $\Omega = \Omega_{lD}$. Figure 1 plots outage probability in (9) when C=1 bit/s/Hz and $\Omega_{SD} = 1$. From this figure, we can observe that the SER performance is more sensitive to the second-hop parameter Ω when Ω_{SD} is fixed, and the SER increases when the value of Ω decreases.

Figure 2 shows the average SER with distinct fading parameters and BPSK modulation as a function of γ_0 when $\Omega_{S1} = \Omega_{S2} = 1$. For comparison, we also provide the results for relay selection without direct link ($\Omega_{SD} = 0$). As expected, the appearance of the direct link improves the system performance. In Figs. 1 and 2, it can be seen that the analytical results are in good agreement with results obtained from simulations.

In this letter, a DF cooperative network with relay selection based on partial channel information operating under INID Rayleigh fading channels was considered, and simple closed-form expressions for outage probability and average SER were derived to present the performance analysis conveniently.

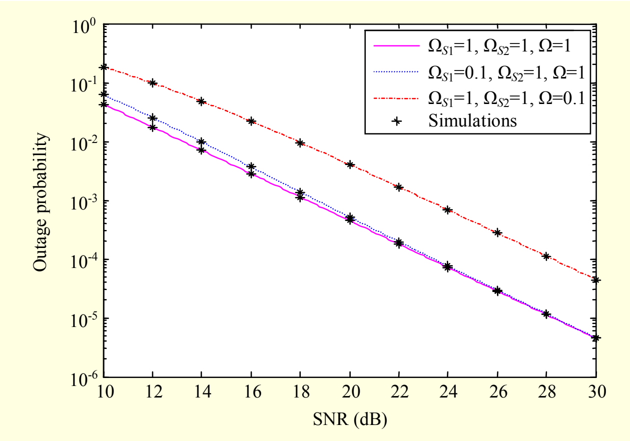

Fig. 1. Outage probability versus SNR.

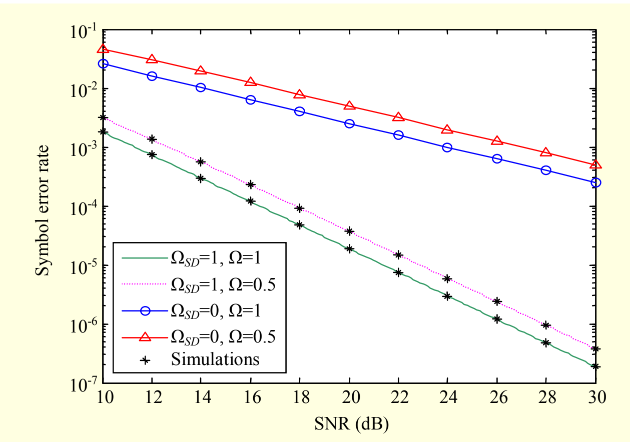

Fig. 2. Average SER of BPSK versus SNR.

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