Cyclic Prefixed CI/OFDM 시스템과 단일반송파 시스템의 ABR 비교 분석

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Achievable Bit Rate Comparison of Cyclic Prefixed CI/OFDM System and Single Carrier System

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ABSTRACT

Since OFDM system suffers from high peak-to average power ratio(PAPR) drawbacks, more energy has been converted to seek for a new substitutable system which can maintain OFDM system's inherent virtues while avoid its defects. Consequently, a new multicarrier system called as CI/OFDM system has been proposed which applied carrier interferometry(CI) code to OFDM system. Due to its low PAPR advantage and orthogonal property, it has received more and more attention. Simultaneously, an old technique called single carrier(SC) system has retaken its attractions for the same purposes. This paper analyzes two cyclic prefixed transmission schemes variants of OFDM system: 1.carrier interferometry-Orthogonal Frequency-Division Multiplexing (CI/OFDM); 2. Cyclic prefixed single carrie(CP-SC) with frequency domain equalization. We compare the achievable bit rate transmission of the two systems in terms of signal to noise ratio(SNR) by mathematical derivation. We demonstrated that CI/OFDM achieves a bit higher transmission bit rate to that of the CP-SC with frequency domain equalizer.

Key Words : carrier interferometry(CI) code; CI/OFDM; CP-SC; SNR; transmission bit rate.

I. Introduction

In wireless communication environment, the received signal is always the sum of the delayed and attenuated transmitted signals through the multipath fading channel if diversity is adopted. A key issue in wireless communication is to find a suitable strategy to combat with multipath fading. Orthogonal Frequency Division Multiplexing(OFDM) is one well known approach proposed more than 40 years ago. Briefly speaking, in OFDM, the incoming high-rate data stream which contains N symbols is converted to N low-rate data streams through serial to parallel conversion. The overall system transmission data rate and spectrum remained unchanged during this operation, however, the data rate per subcarrier is a factor of N of the original data rate. Thus the bandwidth of any given sub-carrier is (1/N)th of the total system bandwidth, as a result, the selected sub-carrier number bandwidth can produce a narrowband transmission and each subcarrier experiences a flat fading.

Carrier interferometry(CI) is a type of spread spectrum multiple access technique, typically employed in OFDM system. CI code which essentially is an orthogonal complex spreading sequence to allow spreading the OFDM symbols over the N subcarriers for

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※ 이 논문은 2009년도 정부(교육과학기술부)의 재원으로 한국연구재단의 지원을 받아 수행된 연구임.(No. 2009-0066336)
※ 본 연구는 지식경제부 및 정보통신산업진흥원의 대학 IT연구센터 지원사업의 연구결과로 수행되었음. (NIPA-2010-C1090-1011-0007)
접수일자 : 2009년 10월 20일, 수정완료일자 : 2009년 12월 30일, 최종게재확정일자 : 2010년 1월 5일

frequency diversity in [1]. And [1] also has pointed out that high PAPR, the inherent drawback of OFDM system, can be reduced by CI code by shaping the resulting superposition of subcarriers. The other drawback of OFDM is that it does not demonstrate robustness to narrowband interference. The performance analysis and simulation results of [2] have confirmed the anti-narrowband interference capability of CI/OFDM over additive white Gaussian noise channels(AWGN) and frequency-selective fading channels.

Single carrier(SC) model combined with frequency domain equalization(FDE) was viewed as an alternative approach to OFDM [3]. It deliveries similar performance to OFDM system and has advantages as low sensitive to carrier frequency offset, low complexity at transmitter and robustness to spectrum null and so on. Most importantly, it modulates all the data symbol to a single carrier instead of many typically used in OFDM, therefore the PAPR is much lower than that of OFDM system. Since amplifier is the most costly components in broadband wireless transceiver, SC system enables a low cost amplifier for low PAPR.

Until now, many comparison works have been done to OFDM system and SC system, such as SC system and conventional OFDM system in [4], adaptive OFDM and SC-FDE in [5], however few attention have been paid to CI/OFDM system although it is also an alternative approach to OFDM system.

In this paper, we focus on the achievable bit rate for a target bit error rate of CI/OFDM system and cyclic prefixed SC-FDE system. We mathematically derive the available bit rate by Shannon capacity theorem in terms of signal to noise ratio(SNR). Adaptive OFDM is applied as a benchmark. The simulation results in AWGN and frequency dispersive channel both show that in adaptive OFDM, CI/OFDM and SC-FDE systems, CI/OFDM can attain the highest data rate, and adaptive OFDM usually can achieve higher data rate than CPSC system.

This paper is organized as following: in the first sector, we analyze the CI/OFDM system and deduce the SNR equations of CI/OFDM system under equal gain combining(EGC) and minimum mean square error(MMSE) combining schemes. In the second section, SNR of SC system is given. Then we compare the achievable bit rate of these systems using the acquired SNR in the third section. Simulation results are given in the fourth part, and then the conclusion was given in the fifth section. Lastly, the derivation process of the MMSE combiner coefficients is added in the appendix.

Ⅱ. CI / OFDM SYSTEM MODEL

Fig.1 depicts the block diagram of conventional OFDM system. The stripped blocks are the difference parts of OFDM system, CI/OFDM system and SC system. For CI/OFDM system, these parts are especially drawn in Fig.2 (a) for the transmitter which introduces the CI spreading code and Fig.2 (b) for the receiver which introduces the CI dispreading code, identically.

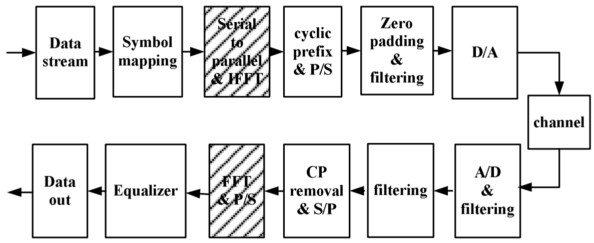

Fig 1. Block diagram of Conventional OFDM system

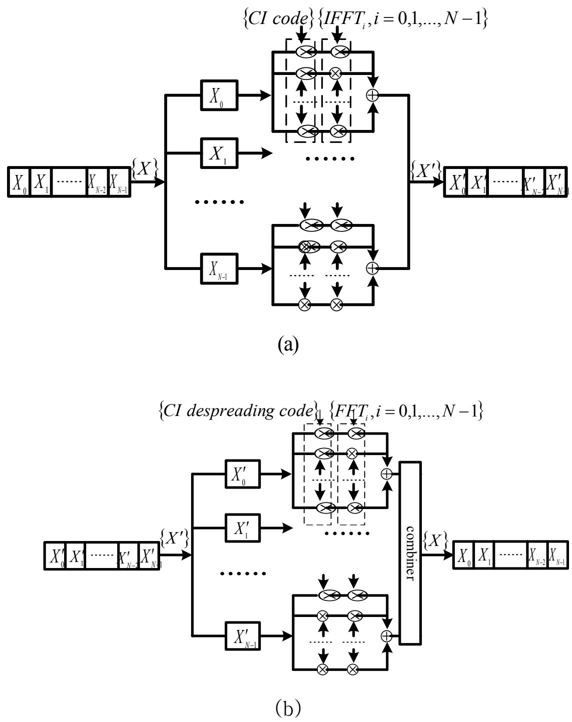

Fig 2. (a) Block diagram of system transmitter structure
(b) Block diagram of CI/OFDM system receiver structure

We consider using N subcarriers in these two systems that, at first both of them convert the incoming high-rate data stream with length N into N parallel low-rate data streams through serial to parallel conversion. The different point is that each low-rate symbol stream is modulated onto its own subcarrier and transmitted to the channel in OFDM while in CI/OFDM each symbol is simultaneously modulated onto all of the N subcarriers.As shown in Fig. 1 and Fig.2, the high-rate data stream $X=[X_0,X_1,\cdots,X_{N-1}]$ are mapped into low-rate data symbols $X_k, k=0,1,\ldots,N-1$ According to OFDM system structure, symbol X_k is modulated onto the K_{th} subcarrier. However, CI code sequence which can be expressed as $\{e^{jki\triangle\theta}, k=0,1,\ldots,N-1\}$[1], where i denotes the subcarrier numberof CI/OFDM system and $i\triangle\theta$ refers to the phase offset used to ensure the orthogonality between the K_{th} subcarrier and the other $(K-1)$ subcarriers. For each symbol, spreading the first CI code to the first subcarrier, the second CI code to the second subcarrier and so on, after this procedure, we define the resulted sequence as, which is corresponding to the original low-rate data. We describe this spreading operation as following:

$$S_k=\sum_{i=0}^{N-1}X_k e^{jk(i\triangle\theta)} \tag{1}$$

Where $\triangle\theta=\dfrac{2\pi}{N}$, $i\triangle\theta$ denotes the phase offset level.

Based on the signal processing knowledge, we know that IFFT operation realizes the signal transformation from frequency domain to time domain. We use an arbitrary symbol as an example:

$$q_k=\frac{1}{N}\sum_{i=0}^{N-1}Q_k e^{jk(\frac{2\pi i}{N})}\,(0\le K\le N-1) \tag{2}$$

Where Q_k is the frequency domain symbol, and q_k is its corresponding time domain expression.

Compare it to Equa. (1), we can see that and have common form except the coefficients, so we can rewrite signal S_k in terms of X_k IFFT operation:

$$S_k=N\bullet IFFT[X_k] \tag{3}$$

For the succeeding mathematical reasoning, we firstly illustrate the relationship between spreading CI code sequences and IFFT operation in the transmitter side (also FFT in receiver side).

$$X'=\sum_{k=0}^{N-1}\{X'_k\}=\sum_{k=0}^{N-1}\{IFFT[S_k]\}$$

$$=\sum_{k=0}^{N-1}\{IFFT[X_k e^{jk0},X_k e^{jk(\triangle\theta)},\cdots,X_k e^{jk[(N-1)\triangle\theta]}]\} \tag{4}$$

Where X' represents the signal sequence which will be transmitted to the channel after IFFT operation.

Due to IFFT's linearity, we can interchange the computation order and get

$$X'=IFFT[\sum_{k=0}^{N-1}X_k e^{jk0},\sum_{k=0}^{N-1}X_k e^{jk\triangle\theta},\cdots,\sum_{k=0}^{N-1}X_k e^{jk[(N-1)\triangle\theta]}] \tag{5}$$

Substitute Equa. (1) to the above Equa. (5), we can get

$$X'=IFFT[S] \tag{6}$$

Where S is the symbol sequence of S_k,

$S=\{S_k\}_{k=0,1,\ldots,N-1}=[S_0,S_1,\ldots S_{N-1}]$

Considering relationship between S_k and X_k together, the output signal X' can be expressed by the input signal X in the following way:

$$X'=IFFT\{N\bullet IFFT[X]\} \tag{7}$$

Then, we can see that symbol X_k is the K_{th} symbol of the whole input symbol sequence $\{X=[X_o,X_1,\ldots,X_{N-1}]\}$ which is transmitted over all of the N subcarriers, and operated on the K_{th} column of the whole IFFT matrix. Accordingly, symbol X'_k is the K_{th} symbol of the whole output symbol sequence $X'=[X'_0,X'_1,\ldots,X'_{N-1}]$ We adopt the above conclusion and apply N point IFFT to each block to express its physical meaning, thus we rewrite the Equa. (7):

$$X'_k=(W_{1_N^k}\bullet W_{2_N^k})^H X_k \tag{8}$$

Where superscript H represents the complex-conjugate transpose, denotes the CI code matrix, and denotes the IFFT matrix, denotes the Hadamard product of columns of matrix and matrix, which is defined as. Both and are the N point FFT matrixes so that each of them can be written as follows:

$$W_{1_N}=\frac{1}{N}\begin{pmatrix} W^{0\times 0} & W^{0\times 1} & \cdots & W^{0\times(N-1)} \\ W^{1\times 0} & W^{1\times 1} & \cdots & W^{1\times(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ W^{(N-2)\times 0} & W^{(N-2)\times 1} & \cdots & W^{(N-2)\times(N-1)} \\ W^{(N-1)\times 0} & W^{(N-1)\times 1} & \cdots & W^{(N-1)\times(N-1)} \end{pmatrix} \tag{9}$$

Where

$$W_{1_N}=e^{jk\left(\frac{2\pi i}{N}\right)},(0\le i\le N-1, 0\le k\le N-1)$$

and $W_{1_N} \bullet W_{1_N}^H = W_{1_N}^H \bullet W_{1_N} = I_{N\times N}$.

Matrix W_{2_N} has the same expression and properties with W_{1_N}.

The salient difference of CI/OFDM system from OFDM system which we mentioned previously is that each of the low-rate incoming symbol streams is spread across all of the N subcarriers other than one subcarrier. Then the whole time domain symbol sequence is transferred by parallel to serial conversion to form X'.To be convenient, we introduce matrix Q to realize the spreading procedure and then describe the whole data stream to be transmitted as

$$X' = Q^H X \tag{10}$$

where

$$Q=\begin{pmatrix} W_{1_N^0} \bullet W_{2_N^0} & 0 & \cdots & 0 \\ 0 & W_{1_N^1} \bullet W_{2_N^1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_{1_N^{(N-1)}} \bullet W_{2_N^{(N-1)}} \end{pmatrix} \tag{11}$$

$$Q^H Q = QQ^H = I_{N\times N}$$

We can see that the above matrix Q is essentially composed of the whole columns of the two times IFFT hardmard product matrix on its diagonal.For mathematic derivation, it is equivalent to a $(IFFT \bullet IFFT)$ matrix, so we can rewrite it as:

$$X' = \left(W_{1_N} \bullet W_{2_N}\right)^H X \tag{12}$$

The whole time multiplexed symbol sequence is transmitted to channel after cyclic prefix(CP) insertion. CP duplicates the last part of IFFT output siganl and appends them to the beginning of X. To avoid ISI completely, the cyclic prefix length should be equal or larger than the delay. Therefore, the linear convolution of the channel matrix is converted into circulant and the channel matrix has the following composition:

$$C = \left(W_{1_N} \bullet W_{2_N}\right)^H \Omega \left(W_{1_N} \bullet W_{2_N}\right) \tag{13}$$

Where matrix Ω is a size $N \times N$matrix which is composed of the channel frequency domain impulse response at each subcarrier's frequency on its diagonal. Therefore channel matrix C is a circular matrix with the first column is given by the channel impulse response in frequency domain [3].

The receiver structure is illustrated in Fig. 2(b). The received signals can be transformed into time domain by the inverse operation as the transmitter. In the receiver side, we discard the signals according to the CP length. The received signal in time domain expression after CP removal is obtained:

$$\begin{aligned} R &= CX' + n \\ &= \left(W_{1_N} \bullet W_{2_N}\right)^H \Omega \left(W_{1_N} \bullet W_{2_N}\right)\left(W_{1_N} \bullet W_{2_N}\right)^H X + n \\ &= \left(W_{1_N} \bullet W_{2_N}\right)^H \Omega X + n \end{aligned} \tag{14}$$

Where n represents the noise characterized using the Gaussian random process.

Be similar to the transmitter's operation, we divide the long received symbol sequence into many short sequences and spread each of these short sequences to CI code modulated subcarriers. Then by FFT operation, we get the new symbol vector as:

$$\begin{aligned} R &= Qr = \left(W_{1_N} \bullet W_{2_N}\right)\left[\left(W_{1_N} \bullet W_{2_N}\right)^H \Omega X + n\right] \\ &= \Omega X + \left(W_{1_N} \bullet W_{2_N}\right) n \end{aligned} \tag{15}$$

Through the same process, as a reference, the received conventional OFDM symbol vector can be described as

$$R = \Omega X + W_N n \tag{16}$$

Where W_N is the N-point FFT operation ,matrix Ω has the same meaning as in CI/OFDM and X is the same input high-rate symbol stream as CI/OFDM system.

In principle, it is possible to use different modulation schemes on each subcarrier, due to multipath channel delay effects, the channel gains may differ between subcarriers, some subcarriers may transmit data at high rate and others at low rate. We describe one subcarrier's SNR in terms of its channel coefficients. Zero forcing(ZF) equalizer applies the inverse of the channel impulse response to the received signals, suppose there are no nulls located in the channel, so we can use the zero forcing equalizer, otherwise, if the channel has zero in the frequency response, the impulse response of the equalizer will grow to be infinite, after noise added to the channel, the overall signal to noise ratio will be totally meaningless. We applied the ZF equalizer to the conventional OFDM system and calculate SNR as the following:

$$SNR_{OFDM,i} = \frac{{\sigma_X}^2}{{\sigma_n}^2} |\Omega_i|^2 \tag{17}$$

Where σ_X^2 is the variance of the i_{th} symbol transmitted by subcarrier i, σ_n^2 is the variance of channel corrupted noise. Ω_i is the i_{th} subcarrier impulse response.

In the receiver side, the received signal is decomposed into N components, and then we apply the combiner to minimize the interference which is introduced by channel. Until now many combining techniques has been suggested according to different environment, such as EGC, maximum ratio combining (MRC) and MMSEC [6].Of all,EGC is the simplest way to combining all the signals which adds up all the received signals and doesn't require any channel information .So it is a prior selection in AWGN channel. While in frequency selective fading channel, MMSEC is mostly utilized[7].

Zero forcing(ZF) equalizer applies the inverse of the channel impulse response to the received signals, suppose there are no nulls located in the channel, so we can use the zero forcing equalizer, otherwise, if the channel has zero in the frequency response, the impulse response of the equalizer will grow to be infinite, after noise added to the channel, the overall signal to noise ratio will be totally meaningless.

1. CI/OFDM with equal gain combining(EGC)

We first begin our derivation assuming CI/OFDM over AWGN channel. Since all information bits are spread to all the subcarriers, the interference can be view as uniform distributed across all the information bits. In EGC scheme, summation of all the symbols is equivalent to multiply the combining coefficients $\beta_i = 1$to each block in Fig.3. For mathematic illustration, we introduce an identity matrix I_N to describe the combined signal of all blocks as:

$$\widetilde{R} = I_N R = I_N(\Omega X + (W_{1_N} \cdot W_{2_N})n)$$
$$= I_N \Omega X + I_N(W_{1_N} \cdot W_{2_N})n \tag{18}$$

In this case, we can derive all symbols' SNR which modulated onto one subcarrier as:

$$SNR_{CI/OFDM}^{EGC} = \sum_{i=1}^{N-1} \frac{\sigma_X^2}{\sigma_n^2} |\Omega_i|^2 \tag{19}$$

We can note the only difference between Equa. (17) and Equa.(19) is that the latter SNR adds up all subcarriers coefficients for one symbol other than one subcarrier coefficient.

2. CI/OFDM with minimum mean square error combining (MMSEC)

In the case of frequency-selective fading channel, the conventional OFDM system has the same performance as in the flat fading channel without extra benefit. In MMSE combiner, the combining enables a frequency diversity gain [1], therefore different gain coefficients should be added to each subcarrier of CI/OFDM system. MMSE combiner is specifically designed to combat the ISI, the combing coefficients is no longer 1 as in EGC, instead differs in terms of the mean square error (MSE), the corresponding proof procedure has been given in the appendix. Therefore for simplicity, we adopt the MMSE combiner coefficients $\Omega^H\left(\frac{\sigma_n^2}{\sigma_X^2} I + \Omega\Omega^H\right)^{-1}\Omega$ (referring to the appendix) to the received signal equation:

Then the combiner result can be shown as following:

$$\widetilde{R} = \Omega\left(\frac{\sigma_n^2}{\sigma_X^2} I_N + \Omega\Omega^H\right)^{-1} \Omega^H R$$
$$= \Omega\left(\frac{\sigma_n^2}{\sigma_X^2} I_N + \Omega\Omega^H\right)^{-1} \Omega^H\left[\Omega X + (W_{1_N} \cdot W_{2_N})n\right]$$
$$= X + \underbrace{\left[\Omega\left(\frac{\sigma_n^2}{\sigma_X^2} I_N + \Omega\Omega^H\right)^{-1} \Omega^H\Omega - I_N\right] X}_{second\ term(ISI)}$$
$$+ \underbrace{\Omega\left(\frac{\sigma_n^2}{\sigma_X^2} I_N + \Omega\Omega^H\right)^{-1} \Omega^H(W_{1_N} \cdot W_{2_N})n}_{third\ term(Gaussian\ noise)} \tag{20}$$

Of the above Equa. the second term represents the intersymbol interference(ISI) and the last term is the addictive white Gaussian noise. Therefore we can define the totally energy of ISI and noise as Y given by:

$$Y = \left(\left[\Omega\left(\frac{\sigma_n^2}{\sigma_x^2} I_N + \Omega\,\Omega^H\right)^{-1} \Omega^H\Omega - I_N\right] X\right)$$
$$\times \left(\left[\Omega\left(\frac{\sigma_n^2}{\sigma_x^2} I_N + \Omega\,\Omega^H\right)^{-1} \Omega^H\Omega - I_N\right] X\right)^H$$
$$+ \left[\Omega\left(\frac{\sigma_n^2}{\sigma_x^2} I_N + \Omega\,\Omega^H\right)^{-1} \Omega^H(W_{1_N} \cdot W_{2_N})n\right]$$
$$\times \left[\Omega\left(\frac{\sigma_n^2}{\sigma_x^2} I_N + \Omega\,\Omega^H\right)^{-1} \Omega^H(W_{1_N} \cdot W_{2_N})n\right]$$

$$= I_N - \left(\frac{\sigma_n^2}{\sigma_x^2} I_N + \Omega\,\Omega^H\right)^{-1}$$

$$= \sigma_n^2 I_N \left(\sigma_x^2 \Omega^H \Omega + \sigma_n^2 I_N\right)^{-1} \tag{21}$$

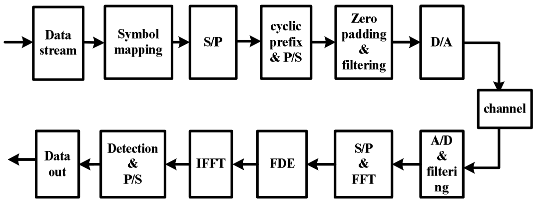

Fig 3. Block diagram of CP-SC system employing frequency domain equalizer

Then the received SNR of all the symbols in the block can be written as:

$$SNR^{MMSEC}_{\frac{CI}{OFDM}} = \frac{\left(N\sigma_x^2 - Y\right)}{Y}$$

$$= N\frac{\sigma_x^2}{\sigma_n^2 I_N \left(\sigma_x^2 \Omega^H \Omega + \sigma_n^2 I_N\right)^{-1}} - 1$$

$$= N\left(\sum_{i=0}^{N-1} \frac{1}{\sigma_x^2 \left(SNR_{OFDM,\,i} + 1\right)}\right)^{-1} - 1 \tag{22}$$

Ⅲ. CP-SC SYSTEM

Although OFDM has become the physical layer choice for broadband communication standards, such as IEEE 802.11, 802.16e, 802.22 and so on, it still has some drawbacks including relatively high PAPR which reduces the power efficiency of the RF amplifier, sensitive to frequency offset and phase noise. A promising alternative approach to mitigate ISI is the single carrier modulation combined with frequency domain equalizer(SC-FDE), in which IFFT operation has been inversed from the transmitter side to the receiver side. The first SC-FDE in digital communication was proposed in 1973[10], only several years later after OFDM system was firstly designed. Compared to OFDM, little attention has been paid to SC-FDE, although the system performance and complexity of SC-FDE is comparable to OFDM system, its notable advantage has attracted lots of research in the last decade.

The block diagram of a single SC employing frequency domain equalization is depicted in Fig.3. The cyclic prefix is added to the front of each block assume that the length of the cyclic prefix is longer than the channel response. Except IFFT locations, we have the same signal processing procedure with CI/OFDM system. So similarly we restrict to a single block of the transmitted SC system contains N symbols. The signals on the receiver' s side undergoes the frequency down conversion, analog to digital conversion and filtering, then the cyclic prefix is discarded in each block; therefore the received signa lcan be described as

$$r = CX + n \tag{23}$$

Where the channel matrix C has the similar decomposition as CI/OFDM system, W_N^H denotes the N-point IFFT operation and W_N is the size-N FFT operation.

$$C = W_N^H \Omega\, W_N \tag{24}$$

Applying a size-NFFT, time domain data block is converted to the frequency domain. The received signa lis obtained as:

$$R = W_N r = \Omega\, W_N X + W_N n \tag{25}$$

Actually the effect of multipath propagation can be mitigated by both time domain equalization and frequency domain equalization, where the latter one was investigated by Walzman and Schwartz in 1973[10]. But it is worth noting that FDEs usually require a low computation complexity than its TDEs counterparts. And SC systems employing FDE can enjoy a similar complexity as OFDM without the highly accurate frequency synchronization requirements.

In the following parts, a linear FDE is introduced to compensate for the channel distortion. The frequency domain zero-forcing equalizer can be operated by multiplying the corresponding inverse channel coefficients to each sample of the equalizer and then by taking the IFFT, the data block is brought back to time domain, the output provides the following result:

$$\widetilde{R} = W_N^H \Omega^{-1} R = X + W_N^H \Omega^{-1} W_N n \tag{26}$$

The covariance of noise n is

$$w = \sigma_n^2 W_n^H \Omega^{-1} \Omega^{-1,H} W_N$$

Where ω is a diagonal matrix, σ_n^2 is the Gaussian noise matrix and we can see that the noise is identical to every symbol in the block.

Finally, data decisions are made on symbol and send to the link layer after S/P conversion. Based on the matrix knowledge, the trace of the matrix $Tr\{W_N^H \Omega^{-1} \Omega^{-1,H} W_N\} = Tr\{\Omega^{-1} \Omega^{-1,H}\}$ and substitute it to the resulting SNR of all the symbols in one block.

$$SNR_{CP-SC}^{ZF} = \frac{\sigma_X^2 I_N}{\sigma_n^2 Tr\left(W_N^H \Omega^{-1} \Omega^{-1,H} W_N\right)} \quad (27)$$

Follow it, all the N symbols in one block can be computed in terms of OFDM SNR as

$$SNR_{CP-SC}^{ZF} = N\left(\sum_{i=0}^{N-1} \frac{1}{SNR_{OFDM,i}}\right)^{-1} \quad (28)$$

The derivation process to get the MMSE equalizer equalized signal is similar to ZF equalizer, by multiplying the MMSE coefficients $\left(\Omega^H \Omega + \sigma_n^2 I_N\right)^{-1} \Omega^H$ [3], thus the obtained SNR for all the symbols in the block is given by

$$SNR_{CP-SC}^{MMSE} = N \frac{\sigma_X^2}{\sigma_n^2 I_N \left(\Omega^H \Omega + \sigma_n^2 I_N\right)^{-1}} - 1$$

$$= N\left(\sum_{i=0}^{N-1} \frac{1}{\left(SNR_{OFDM,i} + 1\right)}\right)^{-1} - 1 \quad (29)$$

Ⅳ. Bit Rate Comparison

We know that, the transmitted data rate of any realizable system cannot approach to the channel theoretical capacity. In [8], SNR gap (Γ) which means the gap between the actual SNR and the theoretical SNR that can be provided by a given coding scheme and a given error probability of symbol error, is introduced because the number of bits that can be transmitted is less than the theoretical capacity which is denoted by the Shannon theorem. $\left(C = \log_2(1 + SNR)\right)$ The Shannon theorem considering the SNR gap can be applied to describe the actual channel capacity. Then the number of bits can be transmitted in one symbol with its corresponding channel SNR_i can be approximated as:

$$b_i = \epsilon \log_2\left(1 + \frac{SNR_i}{\tau}\right) \quad (30)$$

More powerful but implementable codes have smaller gaps as small as 1dB. Generally, PAM or QAM symbols both have a constant gap for b>0.5 are employed, when approaches 1(0dB) then the achievable data rate of the QAM system approaches channel capacity.

Suppose error probability (p_e) is constant when all subchannels use the same kind of codes with the constant gap that a single subcarrier performance characterizes a multi-channel transmission system. And also different subchannels in the multitone are assumed to have the same signal power. For multicarrier system, the total bits transported in one symbol conveyed in one subcarrier, and the multi-channel SNR is approximately the geometric mean of the SNR on each of the subchannel.

So we can get that

$$B_{MC} = \sum_{i=0}^{N-1} b_i = \frac{\varepsilon}{N} log_2\left[\prod_{i=0}^{N-1}\left(1 + \frac{SNR_i}{\tau}\right)\right] \quad (31)$$

Replace the calculated OFDM SNR result in above Equa. (17), the total achieved bit rate per transmitted symbol for adaptive OFDM can be obtained as

$$B_{OFDM} = \varepsilon \log_2\left[\prod_{i=0}^{N-1}\left(1 + \frac{\sigma_X^2 |\Omega_i|^2}{\tau \sigma_n^2}\right)\right]$$

$$= \varepsilon \log_2\left[\prod_{i=0}^{N-1}\left(1 + \frac{SNR_{OFDM,i}}{\tau}\right)\right] \quad (32)$$

Similarly, bit rates per symbol with CI/OFDM system and two different combining schemes yield

$$B_{\frac{CI}{OFDM}}^{EGC} = \varepsilon \log_2\left[\prod_{i=0}^{N-1}\left(1 + \frac{\sum_{i=0}^{N-1} SNR_{OFDM,i}}{\tau}\right)\right] \quad (33)$$

$$B_{CI/OFDM}^{MMSEC} = \varepsilon \log_2\left\{1 + \frac{1}{\tau}\left[N\left(\sum_{i=0}^{N-1} \frac{1}{\sigma_X^2\left(SNR_{OFDM,i} + 1\right)}\right)^{-1} - 1\right]\right\} \quad (34)$$

In the case of single carrier system, the channel capacity has the following relationship with SNR:

$$B_{CP-SC} = \varepsilon \log_2\left(1 + \frac{SNR}{\tau}\right) = \varepsilon \log_2\left(1 + \frac{\sum_{i=0}^{N-1} SNR_{OFDM,i}}{\tau}\right) \quad (35)$$

SNR of all symbols is identical to each other in the same block, we can calculate the bit rate of N symbols in CPSC system with ZF equalizer and MMSE equalizer in different channel environment respectively as:

$$B_{CP-SC}^{ZF} = \varepsilon \log_2\left(1 + \frac{N}{\tau}\left(\sum_{i=0}^{N-1}\frac{1}{SNR_{OFDM,i}}\right)^{-1}\right) \quad (36)$$

$$B_{CP-SC}^{MMSE} = \varepsilon \log_2\left(1 + \frac{1}{\tau}\left[N\left(\sum_{i=0}^{N-1}\frac{1}{(SNR_{OFDM,i}+1)}\right)^{-1} - 1\right]\right) \quad (37)$$

For comparing the data rate in different channel scenarios in the next section, we divide all the results into two groups. Since we compare the data rate in the same symbol period of all systems, the totally transmitted bits can be considered as the achieved data rate. Results of CI/OFDM with ate combining in Equa.riod and CP-SC system with ZF equalizer in Equa.ri6) belong to the first group. CI/OFDM with MMSE combining in Equa.ri4d and CP-SC system with MMSE equalizer is formed into group 2. OFDM system' s data rate in Equa.ri2d will be intninuced to each group as a benchmark. It appears that $B_{CI/OFDM}^{EGC} > B_{OFDM}$ and $B_{CI/OFDM}^{MMSEC} > B_{CP-SC}^{MMSE}$. We can see that $B_{CI/OFDM}^{EGC}$ and B_{OFDM} are characterized by the geometric SNR while $B_{CI/OFDM}^{MMSEC}$, B_{CP-SC}^{ZF} and B_{CP-SC}^{MMSE} are denoted by the harmonic mean of SNR. Bhard on numerical analysis knowledge, we know that the same value of geometric SNR is always larger than its harmonic mean of SNR. Hence, we can get that is larger than B_{CP-SC}^{ZF}. So finally the data rate relationship of them can be obtained.

V . SIMULATION RESULTS

All of the three systems, that is, adaptive OFDM, CI/OFDM and CP-SC systems can be applied to fixed wireless access systems in AWGN channel or frequency selective fading channel. Based on the parameters which have been summarized in Table 1, we define the Eb/N0 equals to 25, equals to 0.5; SNR In our simulations, perfect channel knowledge was assumed for all systems.

Table 1. Simualtion Parameters

System types	OFDM	CI/OFDM	CP/SC
Carrier freq.	2.0G Hz		
Error probability	0.000001		
modulation	adaptive	4QAM	4QAM
equalizer	--	--	ZF & MMSE
combining	--	EGC &MMSEC	--
channel	AWGN & frequency selective fading channel		

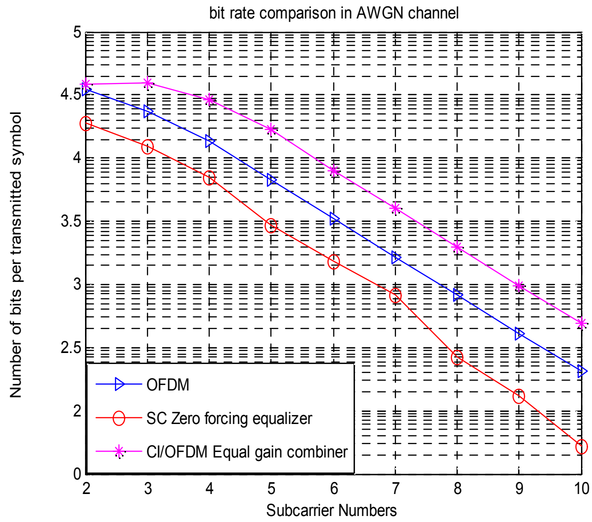

Fig 4. Bit rate per transmitted symbol when subcarrier number is $2^i\,(i = 2,3,\ldots,10)$ in AWGN channel

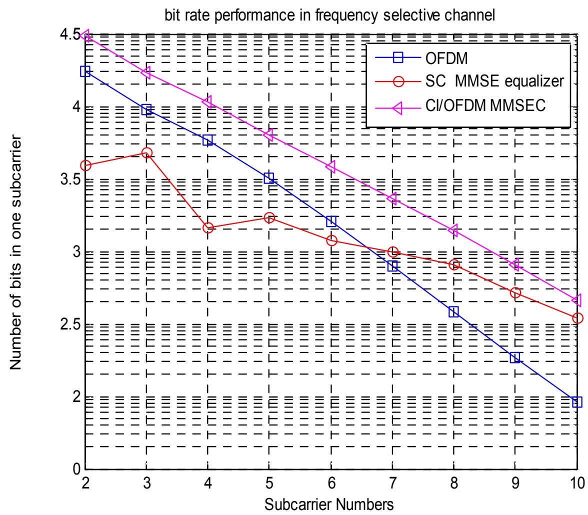

Fig 5. Bit rate per transmitted symbol when subcarrier number is $2^i\,(i = 2,3,\ldots,10)$ in frequency selective fading channel

In two channel scenarios, AWGN and frequency selective fading channel, we can see from Fig. 4 and Fig.5, and mainly from Equa. (26) and Equa.(29) in the AWGN channel and Equa. (27) and Equa.(30) in frequency selective fading channel, the achievable bit rate of CI/OFDM in each subcarrier is always higher than OFDM system and CP- SC system. Specifically, when the subcarrier number goes to $4 = 2^2\,(N = 2^i, i = 2)$, in the first channel scenario(AWGN channel), CI/OFDM with EGC can transmit about 4.5bits one symbol while 4.2 bits per symbol in SC-ZF system. As more subcarriers are used asiequals from 3 to 10, this bit rate transmission gap enlarged to 2 bits gap at most.In the second channel scenario(

frequency selective channel), the largest transmission data rate difference of 0.9bits can be observed when we use $2^i(i=10)$ subcarriers, in contrast, the bit rate difference nearly goes to smallest which is only 0.3bits when subcarriers are used. Between the two dramatic points, we also can see that CI/OFDM system with MMSE combiner always keeps a high transmission data rate, while the rate gap becomes smaller as subcarrier number increases except the fluctuation in the first period. These simulation results are consistent with the two groups' derivation results in the previous section.

Under AWGN channel environment, adaptive OFDM system, as a reference system, shows an intermediate performance of SC system and CI/OFDM system, 0.5 bits when subcarrier number is $2^i(i=3,4,5,6,7)$, 0.7 bits when subcarrier is 2^i $(i=8,9,10)$, higher than SC system and 0.6 bits lower than CI/OFDM system. While in frequency selective channel, there is a little difference, SC system can transmit more bits when subcarriers are applied when is more than 6, due to the dispersive channel characteristic and a high SNR assumption. As [10] points out that, when adaptive modulation was considered for OFDM, SC system with frequency domain linear equalizer has a degraded performance than OFDM system.

Ⅵ. CONCLUSION

Due to high PAPR and sensitive phase offset problems, we are trying to find a fungible system of OFDM, but with better performance. During choosing a suitable alternative system, many factors are needed to be considered, such as system complexity, transmit efficiency and cost etc. Until now, CI/OFDM and SC-FDE are the most promising selections. By our research, we can demonstrate that CI/OFDM system outperforms than conventional OFDM system and SC-FDE system, it can be consider as an alternative for SC system over AWGN and frequency selective channel in terms of the achievable transmit data rate.

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Appendix

In this section, we focus on the derivation process of obtaining MMSE combiner. Let us consider a common communication system as Fig.6 shown, the impulse response of the transmitted filter is $p(k)$ which is used to suppress the intersymbol interference, accordingly, $q[k]$ denotes the impulse response of the receive filter which can reduce the noise interference and maximize the signal to noise ratio in the receiver side. MMSE diversity combining is specifically designed to combat ISI [11]. Aim to find out the combiner' s coefficients we assume that the input signals $X[k]$ are totally uncorrelated with white random processed noise $n[k]$. We also assume that the variance σ_X^2 and σ_n^2 It means that

$$E[X[m]X^*[n]] = \begin{cases} \sigma_X^2 & m \neq n \\ 0 & m = n \end{cases}$$

$$E[n[k]n^*[l]] = \begin{cases} \sigma_n^2 & k = l \\ 0 & k \neq l \end{cases}$$

$$E[X[m]n^*[k]] = 0$$

In the practical implementation, we need consider about the filter taps, here we restraint it to so that $p[k]=0$, $q[k]=0$ for $k \leq 0$ and $k \geq L_f$ also the finite length L_c of the channel taps $h[k]$, then it can delay the input signals from $X[k]$ to $X[k+L_c]$.And for derivation simplicity, we combine the transmit filter pulse coefficients and channel complex impulse response into a 'combined channel impulse response' as

$$H[k] = p[k] \cdot h[k]$$

The combined system impulse response is

$$G[k] = (H[k] * q[k])w[k]$$

The complex received data symbol can be described as:

$$\widetilde{X} = X + \sum_{k=1} GX + n$$

Where the first term is the transmitted signal, the second and the third term denotes the intersymbol interference and the additive noise, respectively.

We already know that it is hard for the combiner to compensate the delay which is caused by the causal channel with finite length. If the channel delay $\triangle$ is precisely delayed, then we can measure that. $\widetilde{X}(k) = X(k \pm \triangle)$. While in the presence of noise, we would never expect such a perfect output. Thus we measure the received symbol $\widetilde{X}(k)$ in terms of minimizing the mean-square error (MSE) :

$$\begin{aligned}
\epsilon &= E\left[|\widetilde{X}(k) - X(k)|^2\right] = E\left[\left|\sum_k g_k X_k\right|^2 + |n_k|^2\right] \\
&= E\left\{[q^T(HX+n) - X][q^T(HX+n) - X]^*\right\} \\
&= E\{[q^T n + (q^T H - \delta^T)X][q^T n + (q^T H - \delta^T)X)]^*\} \\
&= E\{[q^T n + (q^T H - \delta^T)X][n^H q^* + X^H(H^H q^* - \delta)]\} \\
&= E q^T n n^H q^* + E q^T n X^H (H^H q^* - \delta) \\
&\quad + E(q^T H - \delta^T) X n^H q^* + E(q^T H - \delta^T) X X^H (H^H q^* - \delta) \\
&= q^T E\{nn^H\} q^* + q^T E\{nX^H\}(H^H q^* - \delta) \\
&\quad + (q^T H - \delta^T) E\{Xn^H\} q^* + (q^T H - \delta^T) E\{XX^H\}(H^H q^* - \delta)
\end{aligned}$$

Using the statistical properties of random vectors X and n, which are constructed uncorrelated with each other, then we can notice that

$$E\{XX^H\} = \sigma_X^2 I \text{ and } E\{nn^H\} = \sigma_X^2 I$$

$E\{nX^H\} = 0$ a matrix whose (m,k) entry produces $E\{n[k]X[m]^*\} = 0$

Similarly, we can get $E\{X^H n\} = 0$.

Applying these relationships, we can simplify the above formula to

$$\begin{aligned}
\epsilon &= \sigma_n^2 q^T q^* + \sigma_X^2 (q^T H - \delta^T)(H^H q^* - \delta) \\
&= q^T \Big(\underbrace{\sigma_n^2 I + \sigma_X^2 H H^H}_{BB^H}\Big) q^* - q^T \underbrace{H\delta\sigma_X^2}_{A} - \underbrace{\sigma_X^2 \delta^T H^H}_{A^H} q^* + \sigma_X^2
\end{aligned}$$

For simplicity we use matrix symbol A and B to implement the original computation,

$$\begin{aligned}
\epsilon &= q^T B B^H q^* - q^T A - A^H q^* + \sigma_X^2 \\
&= q^T B B^H q^* - q^T B B^{-1} A - A^H B^{-H} B^H q^* + \sigma_X^2 \\
&= \underbrace{(q^T B - A^H B^{-H})(B^H q^* - B^{-1} A)}_{\geq 0} + \sigma_X^2 - A^H (B^H B)^{-1} A
\end{aligned}$$

Usually we try to minimize the first term to zero and thus get the minimize value of the MSE

$$\begin{aligned}
\epsilon_{\min} &= \sigma_X^2 - A^H (B^H B)^{-1} A \\
&= \sigma_X^2 - \sigma_X^2 \delta^T H^H (\sigma_n^2 I + \sigma_X^2 H H^H)^{-1} H \delta \sigma_x^2 \\
&= \sigma_X^2 \Big[1 - \underbrace{\delta^T H^H (\sigma_n^2 I + \sigma_X^2 H H^H)^{-1} H \delta \sigma_x^2}_{\triangle}\Big]
\end{aligned}$$

Thus as $\triangle$ increases, $\epsilon_{\min}$ will decrease, thus the MSE-minimizing $\triangle$ refers to the maximum diagonal element matrix of the matrix $H^H(\sigma_n^2 I + \sigma_X^2 H H^H)^{-1} H \sigma_X^2$

Applied the previously channel expression in Equa. (13), we can rewrite diagonal matrix as $\Omega^H \left(\frac{\sigma_n^2}{\sigma_X^2} I + \Omega \Omega^H\right)^{-1} \Omega$

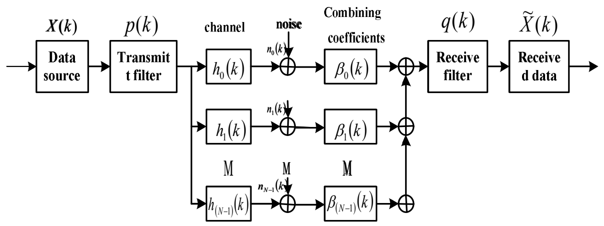

Fig 6. Block diagram of a common communication system

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