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Availability Analysis of a System Having Three Units : Super Priority, Priority and Ordinary Under Pre-empty Resume Repair Policy

V. V. Singh*
Department of Mathematics, AIMT, Gr. Noida, INDIA

S. B. Singh
Department of Mathematics, Statistics and Computer Science
G. B. Pant University of Agriculture and Technology, Pantnagar, INDIA

M. Ram
Department of Mathematics, Graphic Era University, Dehradun, INDIA

C. K. Goel
Department of Mathematics, Amity University, Noida, INDIA

Abstract. In the present paper we develop a mathematical model that facilitates the calculation of reliability of a complex repairable system having three units namely super priority, priority and ordinary. The system is analyzed with the application of Gumbel Hougaard copula when different types of repair possible at a particular state due to deliberate failure. Various reliability measures such as reliability, MTTF and profit function have been evaluated by using supplementary variable and Laplace transform techniques.

Key Words : *Complex System, Reliability, Pre-emptive resume repair, Deliberate failure, MTTF, Availability, Cost Analysis, Gumbel-Hougaard Family Copula.*

1. INTRODUCTION

Many researchers have proposed mathematical models for complex repairable systems under priority/ non priority extending the common assumption of one type of repair between two successive states. (Govil (1974), Gupta and Sharma (1993), Kumar and Singh (2008) and Cui and Li (2007)) It has also been observed that opposite to the priority units rather little attention has been devoted to repairing of less priority units. One of the interesting facts observed now a days are in some cases employees handling the

* Corresponding Author.
E-mail address: singh_vijayvir@yahoo.com,

complex system deliberately disturbs it because of their grievances/ annoyance with their employer may be due to one reason or the other. In general optimal values of parameters of preventive/ corrective maintenance depend on the underlying failure or repair time distribution. But when the failure is deliberate, one needs to select the distribution for repairing appropriate to the demand. When the situation is mixture of deliberate failure as well as less attention to ordinary unit the repair can follow two different type of repair between two transition states. Ram and Singh (2008) and Ram and Singh (2009) discuss availability and cost analysis of a parallel redundant complex system with two types of failure under different repair policies using Gumbel-Hougaard family copula in repair but they didn't analyze the system under super priority which can be a real life possibility. Further when a system consists of super priority unit, it may be the case that ordinary unit is getting less attentions. Copulas can be of great help to tackle this type of problem. (Nelsen (2006))

Present work has emerged from the need to find new and efficient ways to deal with above mentioned facts. The main aim of the present paper is to show how to evaluate the reliability of complex repairable system which consists of three unit namely super priority, priority and ordinary. Whenever super priority unit fails, priority unit starts functioning and failed unit goes for repairing. If the failed unit repaired before failure of priority unit then super priority unit starts functioning and priority unit goes in standby mode. Super priority unit will never be in standby mode. If priority unit fails before repairing of super priority unit then ordinary unit starts functioning and priority will have to wait for repair. The system will be in completely failed state whenever all units fail. The system can also be in completely failed state due to deliberate failure. It has been assumed that the combination of deliberate failure and less priority to ordinary unit lead to two different types of repair. Here different repair facilities are available between adjacent states S_8 and S_0 (where S_8 is completely failed state due to deliberate failure and S_0 is initial state where all units are in good and working condition). The failure and repair times of system follow exponential time distribution, however the repairs follow general time distribution except at state S_8. Gumbel-Hougaard family of Copula has been used in repairing at deliberate failure state. The system is studied by supplementary variable technique and Laplace transforms. The various measures of reliability have been discussed such as availability, state transition probabilities, asymptotic behavior of system and cost analysis. At last some particular cases are also taken to highlight the different possibilities. The paper is organized as follows: Sections 2, 3, 4 and 5 introduce state description, assumptions, state transition diagram of model and notations respectively. Section 6 discusses formulation of mathematical model. Sections 7, 8 and 9 contain main results of the paper.

2. STATE DESCRIPTION

State	State description.
S_0	All units are in good working condition.
S_1	The super priority unit has failed and is under repair.

S_2	Super priority unit is in operational mode and priority unit is in repairing.
S_3	Super priority unit is under repair, priority unit failed and ordinary unit in operational mode.
S_4	All units super priority, priority and ordinary failed.
S_5	Super priority unit failed, priority is operational mode.
S_6	Super priority is in operational mode priority is in standby mode ordinary unit is under repairing.
S_7	Super priority unit is in operational mode and priority unit is under repair.
S_8	System completely failed due to deliberate failure.

3. ASSUMPTIONS

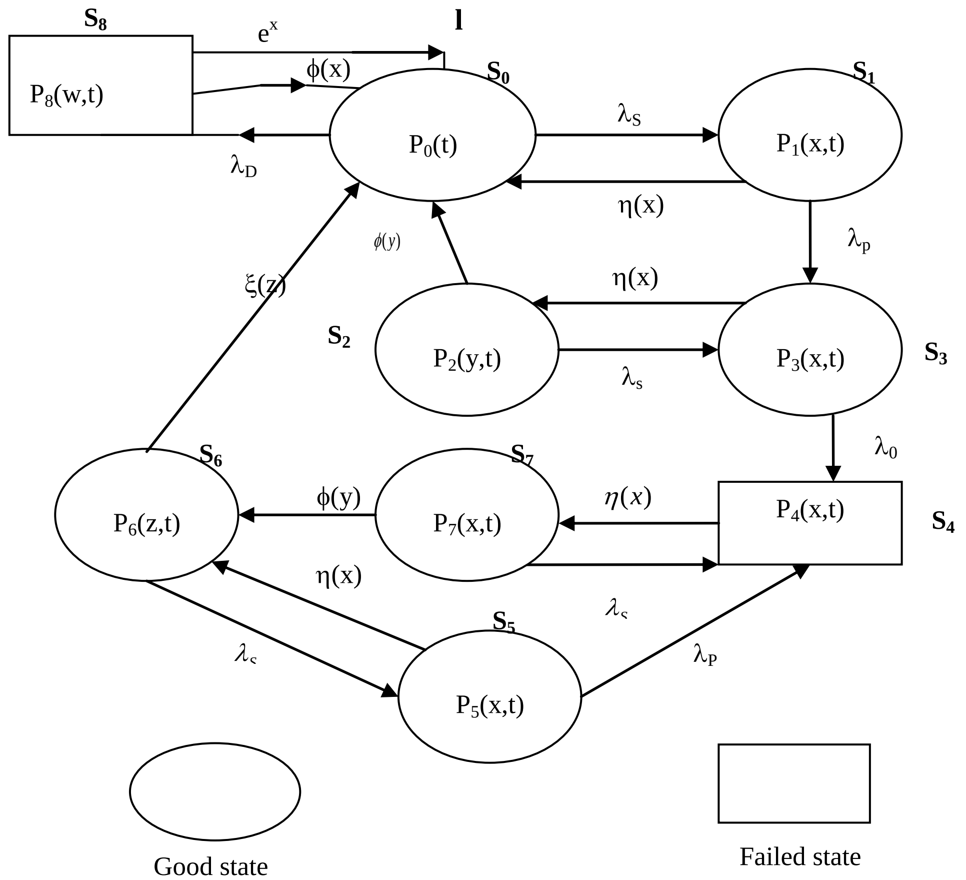
(1) Initially the system is in good and operational state.
(2) Failure of super priority, the priority unit starts functioning.
(3) All failure rates are constant and follow negative exponential distribution.
(4) Priority in repair is given in the order to super priority, priority and ordinary unit.
(5) After repair the sub system/unit works like a new, repair never damage anything.
(6) Repairs follow general time distribution except from S_8 to $S_{0.}$
(7) Completely failed state S_8 repaired by joint probability distribution.

4. NOTATIONS

$P_i(t)$	State transition probability *i*= 0, 1, 2, 3, 4, 5, 4, 6, 7, 8.
$\overline{P}(s)$	Laplace transformation of *P(t).*
μ_1, μ_2	Marginal distribution of random variables, where $\mu_1 = \phi(x)$ and $\mu_2 = e^x$.
$\lambda_s / \lambda_p / \lambda_0 / \lambda_D$	Failure rates of super priority / priority / ordinary unit/ deliberate failure.
$\eta(x) / \phi(y) / \xi(z)$	Repair rates for super priority/priority/ ordinary unit respectively.
$P_i(x, t)$	Probability that the system is in state S_i, *i* =1, 3, 4, 5, 7, super priority unit is under repair and elapse repair time is *x*, *t*.
$E_p(t)$	Expected profit during the interval [0, *t*].
K_1, K_2	Revenue per unit time and service cost per unit time

	respectively.
$P_2(y, t)$	Probability that the system is in state S_2, the priority unit is under repair and elapse repair time is y, t.
$P_6(z, t)$	Probability that the system is in state S_6, the ordinary unit is under repair elapse repair time is z, t.
$P_8\ (w, t)$	Probability that the system is in state S_8, system is under repair and elapse repair time is w, t.
$S_\eta(x)$	$= \eta(x) e^{-\int_0^x \eta(x)dx}$
$\overline{S}_\eta(s)$	Laplace transform of $S_\eta(x) = \overline{S}_\eta(s) = \int_0^\infty \eta(x) e^{-sx - \int_0^x \eta(x)dx} dx$

5. STATE TRANSITION DIAGRAM OF MODEL



6. FORMULATION OF MATHEMATICAL MODEL

By probability of considerations and continuity arguments we can obtain the following set of difference differential equations governing the present mathematical model

$$\left[\frac{\partial}{\partial t}+\lambda_D+\lambda_S\right]P_0(t)=\int_0^\infty \eta(x)P_1(x,t)dx+\int_0^\infty \phi(y)P_2(y,t)dy+\int_0^\infty \xi(z)P_6(z,t)dz$$
$$+\int_0^\infty \exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}P_8(x,t)dx \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_p+\eta(x)\right]P_1(x,t)=0 \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_s+\phi(y)\right]P_2(y,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_0+\eta(x)\right]P_3(x,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\eta(x)\right]P_4(x,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_P+\eta(x)\right]P_5(x,t)=0 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_s+\xi(z)\right]P_6(z,t)=0 \quad (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_s+\phi(y)\right]P_7(y,t)=0 \quad (8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}\right]P_8(x,t)=0 \quad (9)$$

Boundary conditions

$$P_1(0,t)=\lambda_s P_0(t) \quad (10)$$

$$P_2(0,t)=\int_0^\infty \eta(x)P_3(x,t)dx \quad (11)$$

$$P_3(0,t)=\lambda_p\int_0^\infty P_1(x,t)dx+\lambda_s\int_0^\infty P_2(y,t)dy \quad (12)$$

$$P_4(0,t)=\lambda_p\int_0^\infty P_5(x,t)dx+\lambda_s\int_0^\infty P_7(y,t)dy+\lambda_0\int_0^\infty P_3(x,t)dx \quad (13)$$

$$P_5(0,t)=\lambda_s\int_0^\infty P_6(z,t)dz \quad (14)$$

$$P_6(0,t) = \int_0^\infty \phi(y) P_7(y,t)dy + \int_0^\infty \eta(x) P_5(x,t)dx \tag{15}$$

$$P_7(0,t) = \int_0^\infty \eta(x) P_4(x,t)dx \tag{16}$$

$$P_8(0,t) = \lambda_D P_0(t) \tag{17}$$

Initials condition

$P_0(0) = 1$ and other state probabilities are zero at $t = 0$ (18)

Solving the equations (1)-(9) with the help of the equations (10)-(18), one may get

$$\overline{P}_0(s) = \frac{1}{D(s)} \tag{19}$$

$$\overline{P}_1(s) = \frac{\lambda_s}{D(s)} \frac{(1 - S_\eta(s + \lambda_p))}{(s + \lambda_p)} \tag{20}$$

$$\overline{P}_2(s) = \frac{\lambda_p \lambda_s}{D(s)} \frac{(1 - S_\eta(s + \lambda_0))}{(s + \lambda_0)} \left\{ S_\eta(s + \lambda_0) \right\} \frac{(1 - S_\eta(s + \lambda_s))}{(s + \lambda_s)} \tag{21}$$

$$\overline{P}_3(s) = \frac{\lambda_s \lambda_p}{D(s)} \left\{ \frac{(1 - S_\eta(s + \lambda_0))}{(s + \lambda_0)} \right\}^2 \tag{22}$$

$$\overline{P}_4(s) = \frac{\lambda_0 \lambda_p \lambda_s}{D(s)H} \left\{ \frac{(1 - S_\eta(s + \lambda_0))}{(s + \lambda_0)} \right\}^2 \frac{(1 - S_\eta(s))}{s} \tag{23}$$

$$\overline{P}_5(s) = \frac{\lambda_s}{D(s)H} \left\{ \frac{(1 - S_\xi(s + \lambda_s))}{(s + \lambda_s)} \right\} \lambda_0 \lambda_p \lambda_s \left\{ \frac{(1 - S_\eta(s + \lambda_0))}{(s + \lambda_0)} \right\}^2$$
$$\times \left\{ \frac{S_\eta(s) S_\phi(s + \lambda_s)}{(1 - \lambda_s(1 - S_\xi(s + \lambda_s))/(s + \lambda_s)) S_\eta(s + \lambda_p)} \right\} \left(\frac{(1 - S_\eta(s + \lambda_p))}{(s + \lambda_p)} \right) \tag{24}$$

$$\overline{P}_6(s) = \frac{\lambda_0 \lambda_p \lambda_s}{D(s)H} \left\{ \frac{(1 - S_\eta(s + \lambda_0))}{(s + \lambda_0)} \right\}^2 \left\{ \frac{S_\eta(s) S_\phi(s + \lambda_s)}{(1 - \lambda_s(1 - S_\xi(s + \lambda_s))/(s + \lambda_s)) S_\eta(s + \lambda_p)} \right\} \frac{(1 - S_\xi(s + \lambda_s))}{(s + \lambda_s)} \tag{25}$$

$$\overline{P}_7(s) = \frac{\lambda_0 \lambda_p \lambda_s}{D(s)H} \left\{ \frac{(1 - S_\eta(s + \lambda_0))}{(s + \lambda_0)} \right\}^2 S_\eta(s) \frac{(1 - S_\phi(s + \lambda_s))}{(s + \lambda_s)} \tag{26}$$

$$\overline{P}_8(s) = \frac{\lambda_D}{D(s)} \frac{(1 - S_{\exp[x^\theta + \{\log \phi(x)\}^\theta]^{1/\theta}}(s))}{(s)} \tag{27}$$

Here,

$$H=\left\{1-\left(\lambda_p\lambda_s\left(\frac{S_\eta(s)S_\phi(s+\lambda_s)}{(1-\lambda_s(1-S_\xi(s+\lambda_s))/(s+\lambda_s))S_\eta(s+\lambda_p)}\right)\right)-\lambda_s S_\eta(s)\left(\frac{(1-S_\xi(s+\lambda_s))}{(s+\lambda_s)}\right)\right\}$$

$$D(s)=s+\lambda_s+\lambda_D-\{\lambda_s\bar{S}_\eta(s+\lambda_P)+\lambda_S\lambda_P\frac{1-S_\eta(s+\lambda_0)}{(s+\lambda_0)}S_\eta(s+\lambda_0)S_\phi(s+\lambda_s)+\frac{\lambda_0\lambda_p\lambda_s}{H}$$

$$\times\left\{\frac{(1-S_\eta(s+\lambda_0))}{(s+\lambda_0)}\right\}\left\{\frac{S_\eta(s)S_\phi(s+\lambda_s)}{(1-\lambda_s(1-S_\xi(s+\lambda_s))/(s+\lambda_s))S_\eta(s+\lambda_p)}\right\}+\lambda_D S_{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}(s)\}$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_5(s)+\bar{P}_6(s)+\bar{P}_7(s) \quad (28)$$

$$\bar{P}_{failed}(s)=\bar{P}_4(s)+\bar{P}_8(s) \quad (29)$$

Notice that,

$$P_{up}(s)+P_{failed}(s)=1/s \quad (30)$$

7. ASYMPTOTIC BEHAVIOR OF SYSTEM

In long run as time $s\rightarrow 0$ the steady state transition probability can be obtained by Abel's lemma. Using Abel's lemma in Laplace transformation, i. e.

$$Lt_{t\rightarrow\infty}F(t)=Lt_{S\rightarrow 0}\{sf(s)\}=F \quad (31)$$

provided limit of right hand side exists. Time independent probabilities of different states are given by

$$P_0=\frac{1}{D'(0)} \quad (32)$$

$$P_1=\frac{\lambda_s}{\eta+\lambda_p}\frac{1}{D'(0)} \quad (33)$$

$$P_2=\frac{\lambda_p\lambda_s\eta}{(\eta+\lambda_0)^2(\eta+\lambda_s)}\frac{1}{D'(0)} \quad (34)$$

$$P_3=\frac{\lambda_p\lambda_s}{(\eta+\lambda_0)^2}\frac{1}{D'(0)} \quad (35)$$

$$P_4=\frac{\lambda_p\lambda_s\lambda_0}{(\eta+\lambda_0)^2(\eta)H(0)}\frac{1}{D'(0)} \quad (36)$$

$$P_5 = \frac{\lambda_p \lambda_s \lambda_0}{(\eta+\lambda_0)^2(\xi+\lambda_s)H(0)} \times \frac{\eta\phi(\xi+\lambda_s)(\xi+\lambda_p)}{(\eta)(\phi+\lambda_s)\left[(\xi+\lambda_s)(\eta+\lambda_p)-\lambda_s\eta\right]} \frac{1}{D'(0)} \quad (37)$$

$$P_6 = \frac{\lambda_p \lambda_s \lambda_0}{(\eta+\lambda_0)^2 H(0)} \frac{\eta\phi(\xi+\lambda_s)(\xi+\lambda_p)}{(\eta)(\phi+\lambda_s)\left[(\xi+\lambda_s)(\eta+\lambda_p)-\lambda_s\eta\right]} \frac{1}{D'(0)} \quad (38)$$

$$P_7 = \frac{\lambda_p \lambda_s \lambda_0}{(\eta+\lambda_0)^2(\phi+\lambda_s)H(0)} \frac{1}{D'(0)} \quad (39)$$

$$P_8 = \frac{\lambda_D}{(\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta})} \frac{1}{D'(0)} \quad (40)$$

Here, $D'(0) = \frac{d}{ds} D(s), at, s \to 0$

8. PARTICULAR CASE

When repair time follows exponential distribution, setting

$\bar{S}_{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}$, $\bar{S}_\eta(s) = \frac{\eta}{s+\eta}$, $\bar{S}_\phi(s) = \frac{\phi}{s+\phi}$,

$\bar{S}_\xi(s) = \frac{\xi}{s+\xi}$ in equation (28).

8.1 Availability

Taking the values of different parameters as λ_0=0.4, λ_p=0.2, λ_D = 0.1, λ_S = 0.3, η = 1, ϕ = 1, ξ = 1, θ = 1 and x = 1 then taking inverse Laplace transform, one can obtain

Table 8.1. Time vs. Availability

Time(t)	$P_{up}(t)$
0	1.0000
1	0.9812
2	0.9668
3	0.9602
4	0.9510
5	0.9405
6	0.9294
7	0.9181
8	0.9068
9	0.8956

$$P_{up}(t) = 0.04194302431\, e^{(-2.841820454\, t)} - 0.08007794307\, e^{(-1.829129280\, t)} + 0.08944749140\, e^{(-1.687074711\, t)} - 0.06704740471\, e^{(-1.290474413\, t)} \cos(3279153506t) - 0.142078437\, e^{(-1.290474413\, t)} \sin(3279153506t) - 0.001177757004\, e^{(-1.214136926\, t)} + 0.03758082294 e^{(-1.204991743\, t)} \cos(0.5498088597t) + 0.04801054684 e^{(-1.204991743\, t)} \sin(0.5498088597t) + 0.02573091636 e^{(-0.6443708433\, t)} + 1.002707186 e^{(-0.01253547282\, t)} \quad (41)$$

8.2 Mean Time to Failure (MTTF)

Taking all repairs to zero for exponential distribution in equation (28) as *s* tends to zero one can obtain the MTTF as:

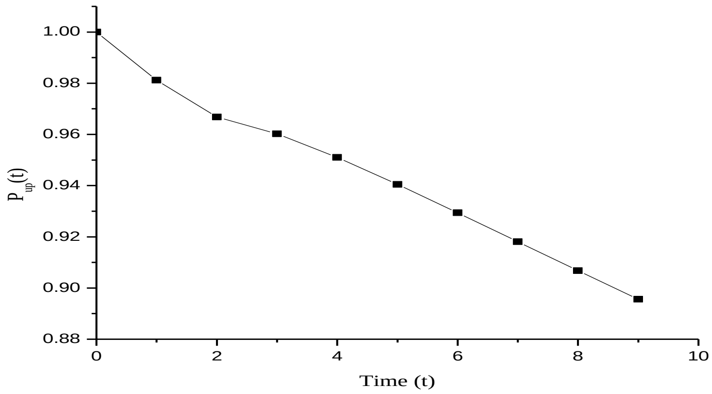


Figure 8.1. Time vs. Availability

$$MTTF = \lim_{s \to 0} \overline{P}_{up}(s) = \frac{1}{\lambda_D + \lambda_s}\left[1 + \frac{\lambda_s}{\lambda_p}\left(1 + \frac{\lambda_p^2 \lambda_s}{\lambda_0^2}\right)\right] \quad (42)$$

Setting $\lambda_D = 0.1,\ \lambda_0 = 0.4,\ \lambda_P = 0.2$ and varying λ_s as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 in (42), one may acquire the variation of MTTF with respect to λ_s. Setting $\lambda_s = 0.3, \lambda_0 = 0.4, \lambda_D = 0.1,$ and varying λ_P as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 in (42), one may get the variation of MTTF with respect to λ_P. Again setting $\lambda_s = 0.3, \lambda_P = 0.2, \lambda_D = 0.1,$ and varying λ_0 as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 in (42), which gives the variation of MTTF with respect to λ_0. Setting $\lambda_s = 0.3, \lambda_P = 0.2, \lambda_0 = 0.4$ and varying λ_D as 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90 in (42), one may obtain the variation of MTTF with respect to λ_D.

The variation of MTTF with respect to different failure rates is shown in Table 8.2.

Table 8.2. Failure Rates vs. MTTF

Failure Rate of $\lambda_s, \lambda_P, \lambda_0, \lambda_D$	MTTF with respect to failure rate			
	λ_s	λ_P	λ_0	λ_D
0.10	8.125	10.468	10.750	4.031
0.20	7.500	7.187	7.375	3.225
0.30	7.187	6.406	6.750	2.687
0.40	7.000	6.256	6.531	2.303
0.50	6.875	6.343	6.430	2.015
0.60	6.785	6.562	6.375	1.791
0.70	6.718	6.852	6.342	1.612
0.80	6.666	7.187	6.320	1.465
0.90	6.625	7.552	6.305	1.343

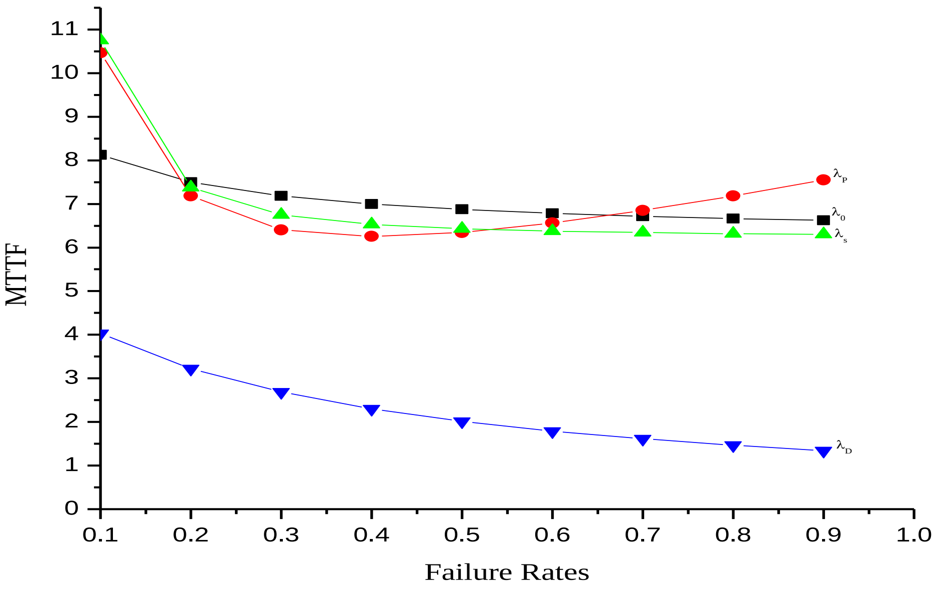


Figure 8.2. Failure Rates vs. MTTF

8.3 Expected Profit

Let the service facility be always available, then expected profit during the interval [0, t] is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \qquad (43)$$

Where K_1 is the revenue per unit time and K_2 is service cost per unit time.
Using (41), Expected Profit for same set of parameters is given by

$$E_p(t) = K_1[(-0.0147593074\ e^{(-2.2841800t)} + 0.04378\, e^{(-1.829100t)} - 0.053018196\ e^{(-1.687100t)} + 0.0750822\ e^{(-1.2909t)}\cos(.32792\,t) + 0.0910182219\ 6e^{(-1.290500t)}\sin(.32792\,t) - 0.0009701013\ 096e^{(-1.214100t)} - 0.0408603874\ 3e^{(-1.20500t)}\cos(0.549810\,t) - 0.0211992688\ e^{(-1.20500t)}\sin(0.549810\,t) + .0399320266\ 3e^{(-.6443700t)} - 79.99202234\ e^{(-0.01253500t)} + 79.943)] - K_2 t \tag{44}$$

Setting K_1=1 and K_2 =0.5, 0.25, 0.15, 0.10, 0.05, 0.02, 0.01 respectively, one get Table 8.3.

Table 8.3. Time vs. Expected Profits

	Expected Profits						
Time(t)	K_2 =0.50	K_2=0.25	K_2=0.15	K_2=0.10	K_2=0.05	K_2=0.02	K_2=0.01
0	0	0	0	0	0	0	0
1	0.478	0.728	0.828	0.878	0.928	0.958	0.968
2	0.947	1.447	1.647	1.746	1.847	1.906	1.927
3	1.410	2.161	2.460	2.610	2.761	2.850	2.881
4	1.866	2.866	3.266	3.466	3.667	3.786	3.826
5	2.312	3.562	4.062	4.312	4.562	4.712	4.762
6	2.747	4.247	4.847	5.145	5.447	5.627	5.687
7	3.171	4.921	5.621	5.971	6.321	6.531	6.601
8	3.583	5.583	6.383	6.783	7.183	7.424	7.503
9	3.985	6.235	7.135	7.585	8.034	8.305	8.394

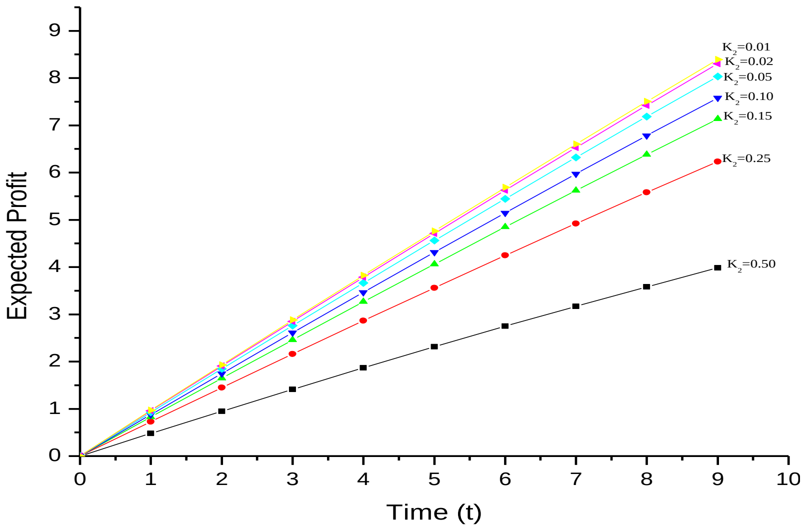


Figure 8.3. Time vs. Expected Profit

9. CONCLUSIONS

Table 8.1 and Figure 8.1 provide information, how availability of the complex repairable system changes with respect to time when failure rates are fixed at different values. One can observe that when failure rates are fixed at lower values λ_0= 0.4, λ_D = 0.1, λ_P = 0.2 and λ_s= 0.3 the availability of the system decreases and probability of failure increase, with passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence one can safely depict the future behavior of complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model.

Table 8.2 yields the mean-time-to-failure (MTTF) of the system with respect to variation in λ_s, λ_p λ_0, and λ_D respectively when other parameters have been kept constant. One can observe that with the increase in failure rates MTTF continuously decreases in all the cases except the increment in λ_P. It is interesting to note that in this case initially it decreases rapidly but after certain interval it starts increasing, which indicates that occurrence of failure λ_P in the range (0.3-0.5) is more frequent in comparison to other values of λ_P.

When revenue cost per unit time K_1 fixed at 1, service cost K_2 = 0.50, 0.25, 0.15, 0.10, 0.05, 0.02, 0.01 profit has been calculated and results are demonstrated by graphs. The values show that with the decrease in service cost expected profit increases with time.

APPENDIX

To find the transition state probabilities of the considered system find the Laplace transforms of equations (1)-(18). Then solve the Laplace transformed equations so obtained for (1) – (9) with the help of the boundary conditions and initial condition to find the transition state probabilities as given in the equations (19) - 27). Of course analytical mathematical concepts are to be used during the solution.

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