Real-time Modeling and Rendering of Tidal in Qiantang Estuary

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Abstract − Tidal bore is a peculiar nature phenomenon which is caused by the lunar and solar gravitation. Based on the physical characters of tidal bores, in this paper we propose a novel method to model and render this phenomenon, especially the tidal waves in Qiantang estuary. According to Boltzmann equation for tidal waves, we solve it with the novel triangle mesh of Kinectic Flux Vector Splitting (KFVS) mode. Then a method combining a curve forecasting wave and particles model is proposed to render the dynamic scenes of overturning tidal waves. Finally, with some rendering technologies, various realistic tidal waves under diversified conditions is rendered in real time.

Keywords: Modeling, Rendering, Physically base method, Real-time

1. Introduction

Tidal bore is one of rare natural phenomena in the world. Tidal bore of Qiantang River is not only caused by lunar and solar gravitation but also relates to the horn-shaped entrance of Qiantang River. In August fifteen of lunar calendar every year, Qian Jiang pours tide is maximum, and the tide head may amount to several meters and the sound is just like ten thousand horses galloping ahead with thunder. Tidal bore is so charming that many people strive to see them. So realistic simulation of tidal bores will have much significance in science and will be found applications in many areas, such as meteorology, scientific visualization, virtual reality, special effects in film, TV and digital entertainment, tour propaganda, etc.

Due to the complex theory of tidal bore, realistic simulation of tidal waves is very difficult. So up to date, relatively few research works have been reported on the realistic simulation of tidal bores. Vasily [1] proposed a numerical method to model of tidal wave run-up based on solving characteristic form using the method of characteristics. The model used a splitting method to generate two 1+1 propagation problems, one in the onshore and the other in long-shore direction. Shi [2] established a coupled 2-D numerical model for hdrody-namic-sediment transport and applied to simulate the tidal currents and sediment movement in the southern Yellow Sea. Xu [3] used the finite element method to simulate the currents of Jiaozhou Bay and the near coast areas, and then established the model of the transport and diffusion of suspended particulate matter. Li [4] employed a three-dimensional

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Princeton Ocean Model (POM) to simulate the tide and current simultaneously for the first time in the Fushan Bay of Qingdao, China. Mao [5] researched the visualization of calculation of tidal current, but its results are only 2-D simulation.

When simulating the tidal bores, the tidal head is difficult to render. Lubin [6]et al. presented and discussed the results obtained from numerical simulating three-dimensional plunging breaking waves by solving the Navier-Stokes equations, in air and water, coupled with a dynamic sub-grid scale turbulence model. Thurey [7] presented a new method for enhancing shallow water simulations with the effect of overturning waves. They used two-way coupling method of rigid bodies with the fluid simulation, where steep wave fronts in the height field and these segments spawn sheets of fluid are represented by connected particles. Kkayyer [8] et al. used a corrected incompressible SPH method for accurate water-surface tracking in breaking waves. Losasso [9] et al. proposed a two-way coupled simulation framework that used the particle level set method to efficiently model dense liquid volumes and a smoothed particle hydrodynamics (SPH) method to simulate diffuse regions such as sprays. Based on multi-resolution grid and fractal noise surface, Pi [10] put forward a simulation method to simulate the wave and water surface near the seashore.

The above works either focused on numerically modeling tidal current and give simple visualization, or only rendered small-scale waves areas with the overturn breaking waves, failed to show the 3D dynamic process of the tidal fluctuation and the tidal bores under different conditions, which makes tidal magnificent and mythical.

Based on the physical theory of tidal bores, in this paper we propose a novel method to model and render this phenomenon, especially for the tidal waves in Qiantang

estuary. According to Boltzmann equation for two-dimensional shallow water, we build the triangle mesh of Kinectic Flux Vector Splitting (KFVS) mode, and then solve it. To simulate the dynamic effect of overturning tidal waves, we present a method combining a curve forecasting wave and particles model. We implement real-time rendering of various realistic tidal waves under diversified conditions.

The rest of this paper is organized as follows. In the next section, we discuss how to Model the tidal waves based on Boltzmann equation. A realistic rendering algorithm for a tidal bores scene is proposed in section 3. The simulation results of tidal bore scenes in real-time under different conditions are presented in Section 4. Conclusions and highlights for future work are presented in the last section.

2. Modeling of Tidal Waves

To accurately simulate the tidal fluctuation, we must first understand the physical characteristics of tidal waves. The main characteristics of tidal bore includes tidal height, tidal bore propagation velocity, bore flow rate, tidal patterns and the landscape, bore pressure, etc.

For most parts of the Qiantang estuary area, the water is shallower than 30 meters. The two-dimensional shallow water equations is [11]:

$$\frac{\partial h}{\partial t}+\frac{\partial hU}{\partial x}+\frac{\partial hV}{\partial y}=0$$

$$\frac{\partial hU}{\partial t}+\frac{\partial}{\partial x}\left(hU^2+\frac{1}{2}gh^2\right)+\frac{\partial hUV}{\partial y}=gh(S_{0x}-S_{fx})$$

$$\frac{\partial hV}{\partial t}+\frac{\partial hUV}{\partial x}+\frac{\partial}{\partial y}\left(hV^2+\frac{1}{2}gh^2\right)=gh(S_{0y}-S_{fy}) \qquad (1)$$

Where f is the molecular distribution function, u, v are elements in the x,y directions of the molecular velocity, g is the gravity acceleration, S_{fx}, S_{fy} respectively is the resistances for x,y direction, S_{0x}, S_{0y} is the bottom slope of x,y direction, b is the base elevation, h is the water depth.

Generally, there are two ways to study the aerodynamic characteristics of the field. One is the Navier-Stokes differential equations. However, as the model is a nonlinear equation set, it is quite difficult to obtain an analytical solution in many circumstances. Another way is to employ the microscopic mechanism based on statistical physics. The Boltzmann equation conveys deeper physical meaning than the Navier-Stokes equation, which are grounded on the assumption of a continuous medium [2]. However, the Boltzmann equation is also difficult to solve, thus the idea of discrete kinetics is introduced here. Let time and space are fully discretized, fluid field is sampled at many grids, and the moving state at the grid nodes is described by distribution functions. Considering the special horn-shaped entrance of Qiantang River, we use the adaptive triangle discretization.

Assume that Ω_i is unit domain of the triangle for grid i, Γ is its borders. In Eq. (2), A_i is the area of triangle unit Ω_i, $(\cos\theta, \sin\theta)$ stands for the unit normal vector of Γ, dl is the infinitesimal element for line integral. Further, we apply the method of dispersing finite volume and before-subtraction for the derivative. So we do not solve the Eq. (1), instead of solving the Boltzmann equation of tidal wave.

$$E_i^{n+1}=E_i^n-\frac{\Delta t}{A_i}\sum_{j=1}^{3}F_{nj}\,l_j+\frac{\Delta t}{A_i}\oint\!\!\int_{\Omega_i}S_{oi}dxdy+\Delta tS_{fi} \qquad (2)$$

Where Δt stands for the time step, subscript j is j side of unit i, l_j stands for the length of triangle's sides, superscript n stands for the time step.

The key of solving Eq.(2) is the calculation of the normal numerical distribution function F_n, $F_n= F\cos\theta + G\sin\theta$, and calculation of S_{oi}, which is the process of source term of bottom slope[11].

Generally, considering the influence of gravity, we can use the BGK mode of flux [1]. But for the shallow water, especially tidal wave, the influence of second order kernels is very small. So here we use the KFVS model of Boltzmann equation, which can save 2/3 calculating time.

We introduce the local coordinal system x − t. x is the vertical to the surface, and u, v is the vertical and tangent velocity of these molecules. The molecules' distribution function at unit surface can be solved under the equilibrium condition:

$$f(X_{ij},t,u,v)=f_0(x_0,0,u_0,v) \qquad (3)$$

Where X_{ij} is the position of unit i and unit j, f_0 is the initial distribution function. Then through Tailer integral expanding, we can get the numerical distribution function of the quality and momentum through unit surface:

$$\begin{bmatrix} F_h(t) \\ F_{hU}(t) \\ F_{hV}(t) \end{bmatrix} = \int_{-\infty}^{+\infty}\!\!\int_{-\infty}^{+\infty} \begin{bmatrix} 1 \\ u \\ v \end{bmatrix} f_{ij}(0,t,u,v)dudy \qquad (4)$$

The special slope shape of Qianjiang River is the one of the main factors to form the tidal bores. In order to establish a well-balanced scheme, the source term effect is taken into account explicitly in the flux evaluation. On this basis, a special technique for dealing with source term due to bottom topography is adopted and the well-ballanced KFVS scheme [4] with triangular mesh possessing second order accuracy is established.

3. Real-time Rendering of Tidal Wave

The rendering of tidal head is the focus difficulty for simulating tidal bores. Here we propose a method combining geometrical and particles method to model the overturning of tidal head. The decreased height of the water causes the breaking influence to become stronger [7]. When tidal wave moves forward, the wave overturns at some points where the wave steepness is greater than a threshold, as shown in Fig. 1.

According to this phenomenon, we can forecast the positions of wave points in the area which possibly overturns. As shown in Fig. 1, the positions of A and C will move to the positions of B and D at next time step. Here we consider that the area from wave peak to the steepest gradient on the wave slope is potential overturning area for the tidal bores. We can set a series of control points in this area and gain the overturning shape of tidal head through forecasting movement of wave.

The wave peak is easily calculated. To calculate the steepest point along the direction of the wave after peak, we can use its gradient direction $\overrightarrow{grad}\ f(x,z)=\dfrac{\partial f}{\partial x}\vec{i}+\dfrac{\partial f}{\partial z}\vec{j}$ and the partial derivatives of the two horizontal axes x to and z along the wave propagation direction. Here the axis y is the wave height. Then in accordance with propagation direction and *x*-axis angle, we can make out the maximum value of $\left(\dfrac{\partial f}{\partial x}\right)^2+\left(\dfrac{\partial f}{\partial z}\right)^2$. This can determine the fastest declining point in the direction of waves, that is, the last control point of overturning area.

As shown in Figure 1, the control points A and E are the beginning and ending points of overturning area. The control point B maybe are determined by the location of control point A after $2*\Delta t$. As the ocean wave marching on, the position of control point B is gradually near control point E, This will generate a real surging effect with crimp vertex downward. Control point D is between points A and E. Then we use it to determine other control points by similar way. The control point E is determined on a section along the direction of the wave moving. Using these control points, we can render the overturning of tidal head. We can fractionize the control mesh to render the tidal wave more realistically.
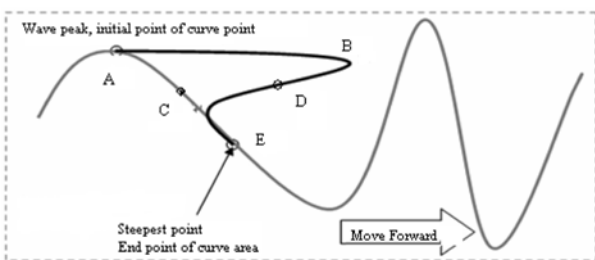
Along with the wave move forward, the overturning wave will disappear, and then formed, and then fade away again. Accordingly, it requires irregular changes of the control points at the height direction. When the height of the control points change to be the same as grid points, the overturning waves disappear and then go into another round. All the effects mentioned above can be performed through shift of the control points' position, which is namely, the increase or decrease of parameters in the control point expression.

By observing real tidal waves it can be seen that tidal head move so quick with many spray and foams. Then we add many particles along with the overturning wave. Moreover, particles are spawned along the tip of the wave patch. Here, the drag of the air causes disturbances of the fluid sheet, resulting in the formation of drops. We furthermore use a small scale bump map to distort the reflective river water surface, which gives the impression of smaller surface waves.

When overturning waves are successfully structured by the above method, more complex ocean scene and surrounding environment could be further mapped to make conspicuous by contrast of the reality of ocean waves. Some useful technology for the further mapping include sky environment mapping, light blanking, texture mapping and so on.

4. Results

Using the methods above, we have achieved realistic simulation of different tidal bores scenes on computer with Intel(R) Pentium(R) CPU 3.20 GHz, 4 GB memory and the graphic card is GeForce 9600. The average rendering speed reaches 26fps.

Fig. 2 is the calculation results by our modeling method. The red curve is the simulation results of tidal height at different time in the place of Yanguang of Zhejiang Province in Aug. 15 of lunar calendar, 2006, and the small black circles are the observation values. Fig. 3~4 are some simulation results of tidal waves from different viewports. These simulation results are quite satisfactory.

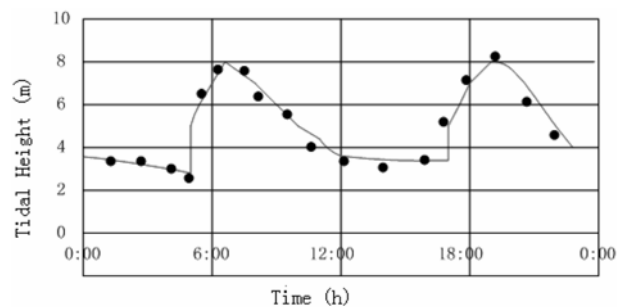**Fig. 1.** Schematic diagram of overturning waves.

Fig. 2. Comparison of the tidal height between our calculation results and observation values at different time in the place of Yanguang of Zhejiang Province.
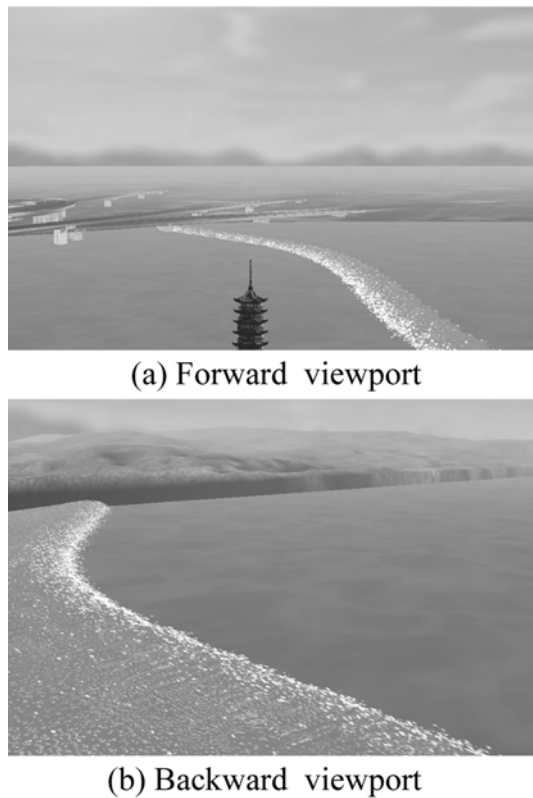
(a) Forward viewport

(b) Backward viewport

Fig. 3. Simulation results of tidal wave at far viewport.
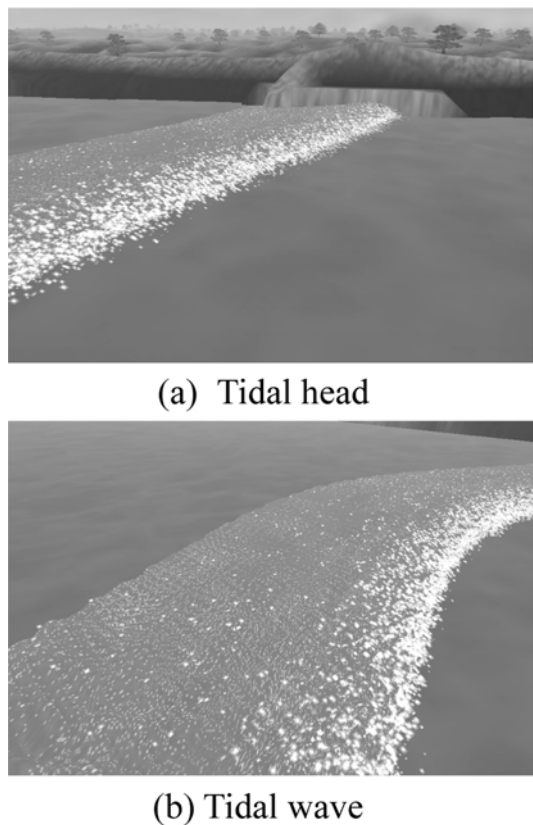

(a) Tidal head

(b) Tidal wave

Fig. 4. Simulation results of tidal wave at close viewport.

5. Conclusion

According to the characters of tidal bores in Qiantang River, this paper realistically modeling and rendering the tidal scene in real time at first time, including the fluctuate, movement, and overturning of tidal wave and surrounding environment. Different from the traditional methods, the physically based method proposed by this paper has the following contributions:

A physically based tidal wave model is first proposed in this paper. It is extended from Boltzmann equation, and solved by the novel elf-adaptive triangle mesh of KFVS mode. So it can simulate the tide movement more realistic.

A method combining a curve forecasting wave and particles model is proposed to render the dynamic overturning tidal waves. It achieves satisfying rendering results with smaller calculation amount.

With some rendering technologies, various realistic tidal waves of Qiantang River under diversified conditions are rendered in real time.

Future work includes: farther advancing the tidal model to simulate the tidal wave of random shallow water, accelerating the rendering speed to build the real-time disaster preventing system, etc.

Acknowledgements

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