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An Optimization-based Computational Method for Surface Fitting to Update the Geometric Information of An Existing B-Rep CAD Model

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Abstract – For several years, researchers have focused on improving the integration of the CAD, CAM and Analysis through a better communication between the various analysis tools. This tendency to integrate the CAD/Analysis and automation of the corresponding processes requires data sharing between the various tasks using an integrated product model. We are interested in this research orientation to CAD/CAM/Analysis integration by rebuilding the CAD model (BREP), starting from the Analysis results (deformed mesh). Because this problem is complex, it requires to be split into several complementary parts. This paper presents an original interoperability process between the CAD and CAE. This approach is based on a new technique of rebuilding the CAD surface model (Nurbs, Bezier, etc.) starting from triangulation (meshed surface) as a main step of the BREP solid model. In our work, the advantages of this approach are identified using a centrifugal pump example.

Keywords **: Integration CAD/Analysis, Finite Element, Mesh, Nurbs surface, Energetic method.**

1. Introduction

Throughout the life cycle of mechanical products, design is seen as a set of interdependent tasks (geometric modelling tasks, analysis tasks, manufacturing tasks, assembly tasks, etc.). The mechanical analysis is one of the most important tasks associated with the design activity. It spreads out over the life cycle of the product and allows the prediction of its mechanical behaviour [1, 2].

In the mechanical design, two types of analysis can be considered: a dimensioning analysis, which consists in obtaining the dimensions according to applied loads, and a validation analysis (based on criteria) which consists in verifying that the chosen dimensions will be able to resist to the applied load. In order to achieve this task, the deployment of the tools and analysis methods are, in the view of the designers of a great industrial topicality [3, 4].

In the context of simultaneous engineering, the design support systems have to permit the integration of the mechanical analysis into the design process. This integration must be applied from design to analysis and vice-versa, allowing the minimisation of the classical back-and-forth between design and analysis and therefore reduce the design cost efficiently [1].

At present, the data exchange process between CAD and analysis tools (CAE) is possible only in one way (data is transferred from CAD to analysis). It is done throughout the neutral formats which can generate a loss of data.

Our research work has been developed to improve and automate this transfer. Several iterations between CAD and Analysis are required to validate the design model. Thus, the responsibility of the designer to return the analysis results (curves, values tables, ISO values, etc.) and take them into account in the design model (CAD model). In order to solve these problems, many research projects have been carried out to improve the integration between the mechanical model (CAD) and the Analysis one (CAE).

In the last few years, various solutions have been proposed to automate the transfer of data and support the automatic modification of the analysis model when there is a modification in the CAD model [5, 6]. The inverse transfer of data (the return of the deformed geometric model as a result of the CAD model) is the main objective of this work. The reconstruction of the deformed CAD model as a result of the Finite Element Analysis makes it possible to emphasize the process of design towards simulations. In fact, the validation and the verification of the design criteria will be well performed by simulation in an integrated CAD/Analysis environment [7, 8].

In this article, the objective is to reintegrate the CAD model, which has been reconstructed from the finite element results (deformed mesh) into the design tool (CAD). The validation of the design proposal is based on these results (Fig. 1).

This work is part of a research project that consists in proposing a unique CAD/CAM/Analysis model. The

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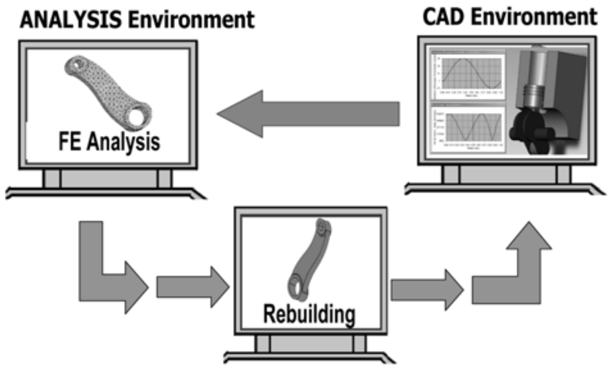

Fig. 1. CAD- Analysis interoperability.

objective of this research is the elaboration of an integrated dynamic model which allows an efficient interoperability between the CAD, CAM and the Analysis tools.

2. Proposed algorithm to rebuild CAD model using CAE results

2.1 Usefulness of the deformed CAD model reconstruction

The reconstruction of the CAD model from the Finite Element analysis results [9, 10, 11] has become more and more important in the integrated design, notably by real time simulation of the design process and/or the manufacture of a mechanical product (forming process simulation, for example). When we simulate an elastic-plastic behaviour of mechanical parts or a contact between them, the permanent deformations inherent in these studies bring about a deformation of the mesh which can become invalid, or of a poor quality. The convergence of the numeric solution is then blocked because the data are no longer valid. To finish the study, the mesh must be recalculated between selected simulation stages. Around the permanent deformations, not only the mesh must be recalculated but the boundary conditions must also be repositioned in the new form of the model. Within the context of the integrated design, the boundary conditions are captured on the CAD model, and for this reason, the deformation of the finite element model must be transmitted to the CAD model in order to respect the new boundary conditions.

Currently, the design of a mechanical product is based on a numerical simulation of the non-deformed model of the parts (rigid body). The reconstruction of the CAD models from the analysis results tends to raise this limitation and make it possible to visualise and simulate the behaviour of mechanical assemblies in their deformed configuration (normal operating state) and detect possible interference effects which are undetectable in the non-deformed state.

2.2 General algorithm

Figure 2 shows the different steps of our proposed

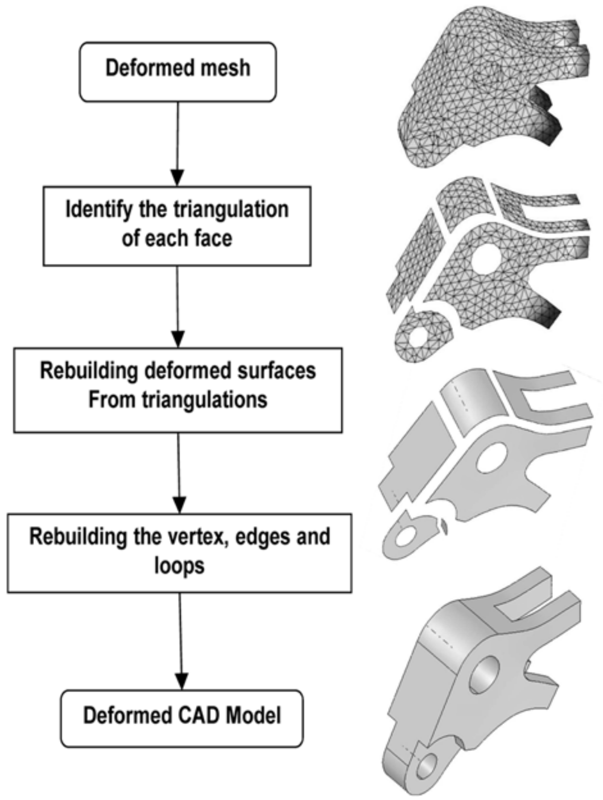

Fig. 2. General algorithm of reconstruction.

algorithm to reconstruct a CAD model from a deformed mesh [12]. The main steps of this algorithm are the following:

- Identification of the triangulation associated with each face of the model.
- Reconstruction of the bearing surfaces of the faces.
- Reconstruction of the contours, edges and vertices for each intersection of the reconstructed faces.

As a second step, we have to proceed according to whether the deformations are small or large. Based on the Finite Element results (Nodes displacement: N_d), the algorithm calculates the ratio $\frac{N_d}{\sqrt[3]{V_{part}}}$, where V_{part}: part volume. If $\frac{N_d}{\sqrt[3]{V_{part}}}$ ($\varepsilon = 0.001$) then we are in the case of of small deformation; else, we are in the large deformation.

If they are small, an algorithm has been developed in previous works and details are presented in [13]. In the case of large deformations, where the faces change their geometry and topology, will be considered in this paper.

In this algorithm (Fig. 2), the rebuilding deformed surfaces from triangulations step is the most difficult step. It is detailed in the section 4. The energetic method is firstly used to rebuild surfaces and explained in the section 4.1. The section 4.2 develop the faces reconstruction algorithm (addition of loops to the surfaces rebuilt in section 4.1).

3. Background: From mesh surfaces to CAD surfaces

Many years ago, research was conducted to reconstruct CAD surfaces from a scatter diagram and from many curves which construct the boundaries. Kruth [14] and Zhongwei [15] have developed a new reconstruction method of a surface from a scatter diagram (for the most part regular cases) generated by a numerical control machine. In the same context, Piegl [16] has developed a method of parameterisation and smoothing surfaces starting from four boundary curves and an ordered set of vertices [17].

In 1998, Volpin [18] developed an algorithm to reconstruct a single-surface NURBS from a mesh (Fig. 4.). The method was based on three phases:

- The simplification of the initial mesh model by constructing restricted zones in the curves conforming to the model.
- The creation of a quadrilateral mesh of the model (the restricted zones).
- The creation of smooth surfaces from quadrilateral meshes using an energetic method.

In this subject, various works used the approximation of a surface by the subdivision of the corresponding mesh [19, 20]. A remeshing algorithm was developed to iteratively refine a given mesh to estimate the true form of the meshed object.

It inserts a new node between two consecutive ones and each triangle is divided into four smaller ones.

Other works have been based on using Bezier triangles to evaluate a surface from a mesh. These works calculate the triangular surface (Bezier triangle) [21, 22] which corresponds to each triangle of the mesh, in such a way as to be continuous with the adjoining surfaces (that correspond with the neighbouring triangles of the mesh) [23, 24]. The continuity is of the type C1 or C2 [25, 26, 27].

Many research projects have focused on the evaluation of the surface especially around one node or a triangle of the mesh. In the present work, we are interested in the evaluation and the reconstruction of the surface, as a part of the BREP, using its corresponding triangulation. We have adapted previous methods to obtain a global surface formulation using the NURBS model.

4. Reconstruction of CAD surface from triangulated surface

This approach is based on the energetic method, which permits to hold a surface to a set of curves and points [21].

4.1 Energetic method

After calculating the inertia plane, constraint points are projected on the plan (Fig. 3). Then, the surface is

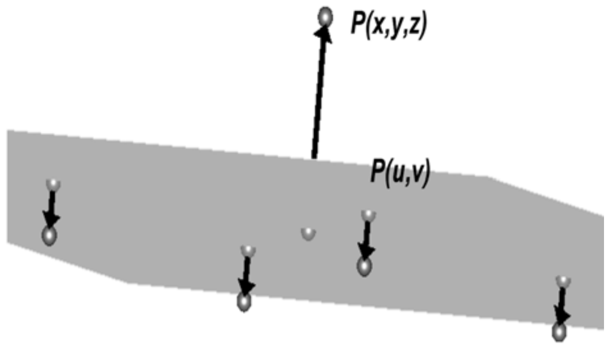

Fig. 3. Projection of points on the inertia plane.

calculated. Each constraint must verify a set of two conditions:

- Space constraint: the resulting surface must pass through the 3D constraint point;
- Parametric constraint: the crossing point at the surface must be defined by specific parameters (u, v);

Then, the inertia plan (mean square plane) is approximated by a B-Spline surface. To meet the constraints, displacement is applied on the control points of the surface (Fig. 4). In the next section, we give details of the process of the surface deformation.

Various works have investigated the problem of surface deformation. These include the Bloor work [28], which introduces the deformable B-Splines, and the D-NURBS (Dynamic NURBS) developed by Qin [29]. In 2004, LaGreca [30] have dealt with the deformed B-Spline problem in its research on geometric modelling.

The calculation process of the deformed surface using the energetic method is iterative (Fig. 5). The global deformation process of the surface is that the entire surface is moved to its constraints for each iteration. The shift applied to each iteration aims to decrease the distance between the surface and all constraints.

Considering N constraint points G_k $(k \in [1..N])$.

And $n \times m$ control points P_{ij} of the initial surface (which approximates the plan).

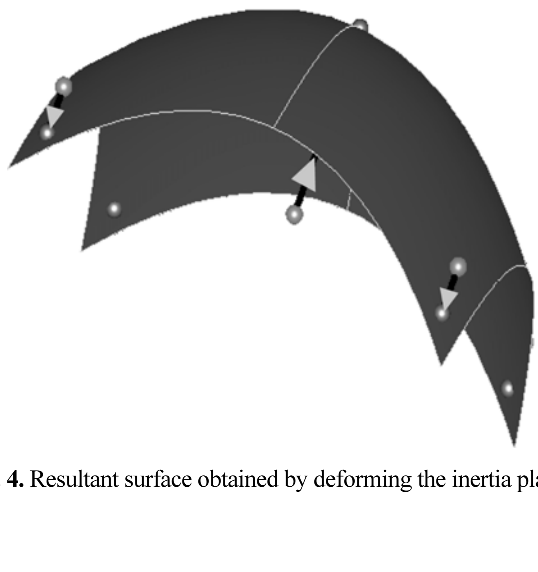

Fig. 4. Resultant surface obtained by deforming the inertia plane.

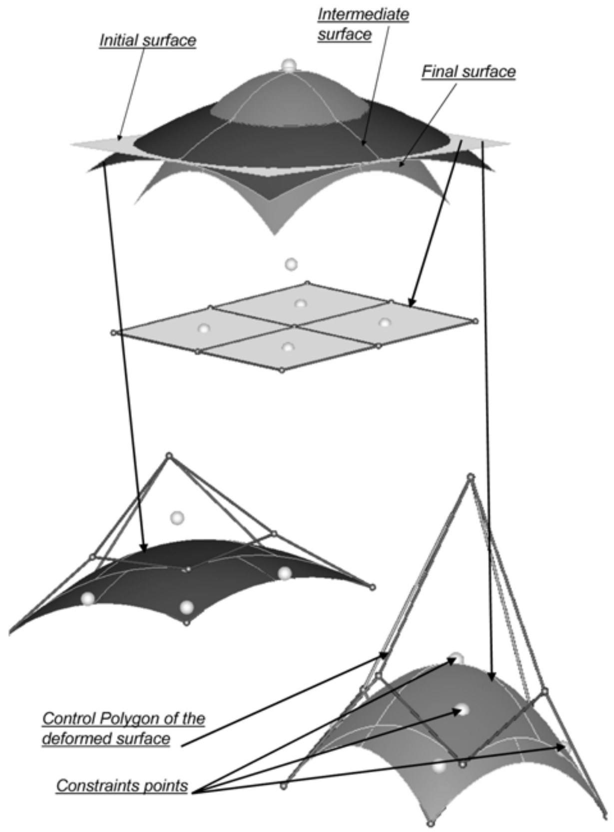

Fig. 5. Deformation of surface by energetic method

Note : At this level of analysis (for choosing the number of control points following u and v), a number of initial 9 x 9 control points is chosen for the surface.

In order to approach the surface to the constraints points, some control points (related to the surface) are inserted or removed based on the surface flexibility criteria. several works [31, 32, 33, 34] have studied the same problem (insertion and removal of nodes at vector knots).

We consider $e_{k,t}$ the error associated to a point G_k at iteration t. This is the Euclidean distance between the surface and the forced point.

The vector motion of the control point P_{ij} to meet a constraint G_k at iteration t is defined by:

$$\vec{d}_{i,j} = \frac{R_{i,j}(u,v)}{\sum_{a=0}^{n-1}\sum_{b=0}^{m-1}(R_{a,b}(u,v))^2} \tag{1}$$

$$R_{i,j}(u,v) = N_{j,p}(u,v) * N_{j,q}(u,v) \tag{2}$$

$N_{i,p}(u,v)$: are the interpolation functions of the B-Spline surface.

$(u,\ v)$: are the parameters of the projected force point on the surface at t iteration t.

Thus $P^{t}_{(i,j)(x,y,z)} = P^{t-1}_{(i,j)(x,y,z)} + d_{ij} * e^{t}(x,y,z)$ (3)

Which is developed to the following system :

$$\begin{pmatrix} P^{t}_{(1,1)(x,y,z)} \\ P^{t}_{(1,2)(x,y,z)} \\ P^{t}_{(1,3)(x,y,z)} \\ \cdots\cdots \\ P^{t}_{(2,1)(x,y,z)} \\ P^{t}_{(2,2)(x,y,z)} \\ P^{t}_{(2,3)(x,y,z)} \\ \cdots\cdots \\ P^{t}_{(n,m)(x,y,z)} \end{pmatrix} = \begin{pmatrix} P^{t-1}_{(1,1)(x,y,z)} \\ P^{t-1}_{(1,2)(x,y,z)} \\ P^{t-1}_{(1,3)(x,y,z)} \\ \cdots\cdots \\ P^{t-1}_{(2,1)(x,y,z)} \\ P^{t-1}_{(2,2)(x,y,z)} \\ P^{t-1}_{(2,3)(x,y,z)} \\ \cdots\cdots \\ P^{t-1}_{(n,m)(x,y,z)} \end{pmatrix} + \begin{pmatrix} d^{1}_{1,1}\ d^{2}_{1,1}\ d^{3}_{1,1}\ \ldots\ d^{N}_{1,1} \\ d^{1}_{1,2}\ d^{2}_{1,2}\ d^{3}_{1,2}\ \ldots\ d^{N}_{1,2} \\ d^{1}_{1,3}\ d^{2}_{1,3}\ d^{3}_{1,3}\ \ldots\ d^{N}_{1,3} \\ \cdots\cdots \\ d^{1}_{2,1}\ d^{2}_{2,1}\ d^{3}_{2,1}\ \ldots\ d^{N}_{2,1} \\ d^{1}_{2,2}\ d^{2}_{2,2}\ d^{3}_{2,2}\ \ldots\ d^{N}_{2,2} \\ d^{1}_{2,3}\ d^{2}_{2,3}\ d^{3}_{2,3}\ \ldots\ d^{N}_{2,3} \\ \cdots\cdots \\ d^{1}_{n,m}\ d^{2}_{n,m}\ d^{3}_{n,m}\ \ldots\ d^{N}_{n,m} \end{pmatrix} \times \begin{pmatrix} e^{1,t}(x,y,z) \\ e^{2,t}(x,y,z) \\ e^{3,t}(x,y,z) \\ \cdots\cdots \\ e^{N,t}(x,y,z) \end{pmatrix} \tag{4}$$

4.2 Reconstruction of faces based on the energetic method

The Energetic Method corresponds well to our case. It is about rebuilding a surface that interpolates the mesh nodes and that adheres to the curves of the loop of the face, which are going to be rebuilt from the mesh.

The reconstruction algorithm of a deformed face from the mesh, which uses the energetic method, is based on six steps (Fig. 6).

The first step consists on the extraction of the triangulation corresponding to the deformed face. Then, the rebuilding process determines the set of nodes corresponding on the edges of the deformed face. Based on the nodes previously identified, the curves are building. The fourth step consists on determination of the nodes which are on the face but not on the edges in order to

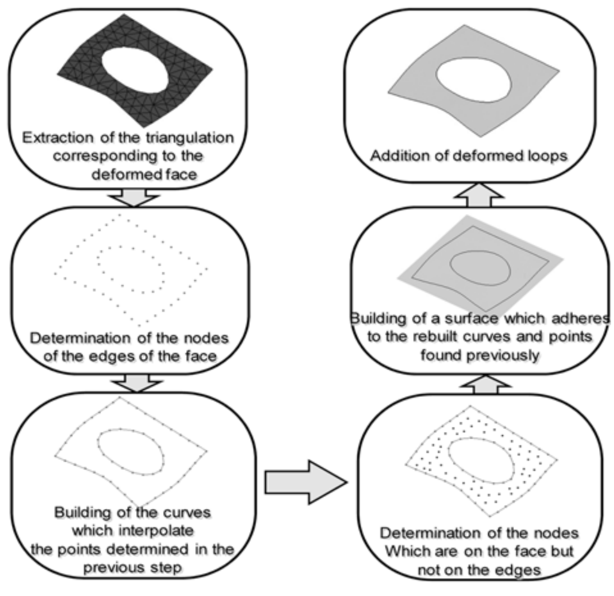

Fig. 6. Rebuilding faces algorithm.

build the surface which adheres to the rebuilt curves and nodes. Finally, the reconstruction process of deformed faces is achieved by the addition of deformed loops.

The process is stopped when the error between the calculated surface and each constraint point is less than a distance fixed by the user or when the maximum number of iterations is reached.

The result of the energetic method is a NURBS surface. This fits well with most cases of the deformed CAD model: the surface represents a deformed face, in most cases, as NURBS surface.

5. Implementation and validation

This approach is developed under the Open Cascade environment that contains some functions in its API (Application Programming Interface) which permit to approach a surface to a set of curves and points. These functions of Open Cascade are based on energetic methods [21].

Figure 7 represents the validation methodology of the proposed model. The latter is based on four steps detailed in the next sections.

In order to improve the proposed method, we choose a centrifugal pump as an example (Fig. 7, step 1). Constituents of the pump are presented in Figure 8.

According to the validation method illustrated in Fig. 7, the designer starts by specifying the mechanical part which is the object of the verification analysis of the Finite Elements Method (FEM) (Fig. 7, step 1). In the case of the validation example, the part is the transmission shaft (Fig. 8). This component will make the object of an eventual idealization in order to delete details (hole, etc.) considered as superfluous for the mechanical analysis (FEM). Figure 8 presents the idealised CAD model of the transmission shaft of the centrifugal pump.

The second step (Fig. 7, step 2) consists of a mechanical analysis of the transmission shaft by FEM. Figure 10 presents the board conditions defined on the idealised CAD model of the shaft and the FE method analysis results.

The purpose of the third step (Fig. 7, step3) is to rebuild a deformed CAD model, based on the cloud of points (Resulting from FE analysis) (Fig. 9). The

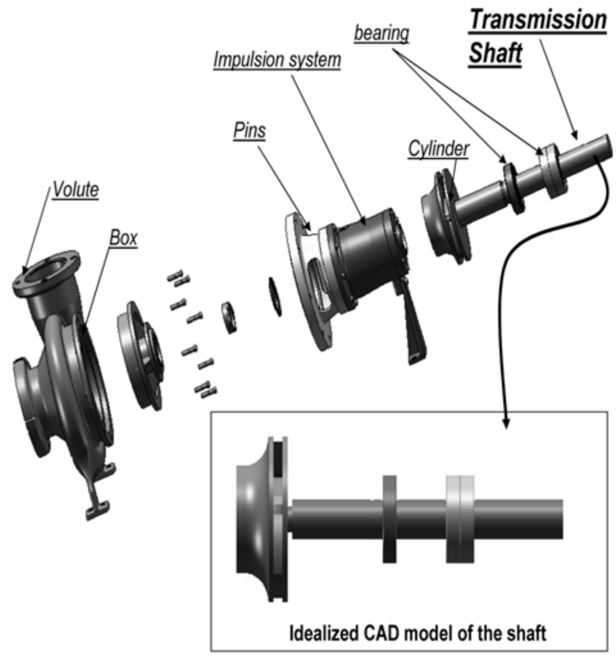

Fig. 8. CAD model of the centrifugal pump.

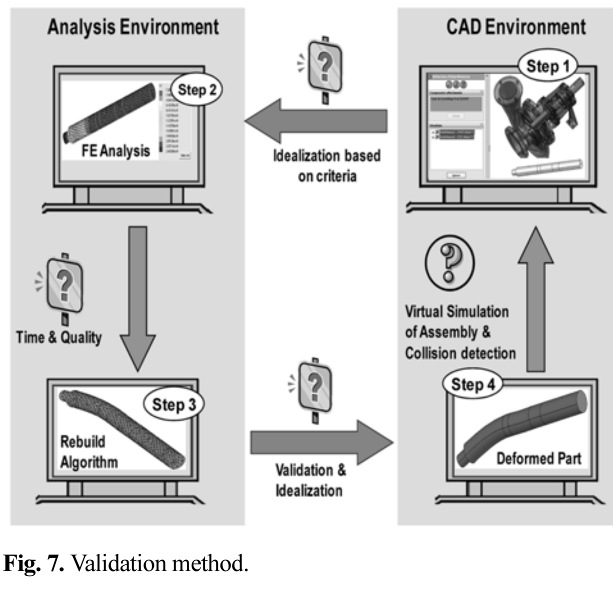

Fig. 7. Validation method.

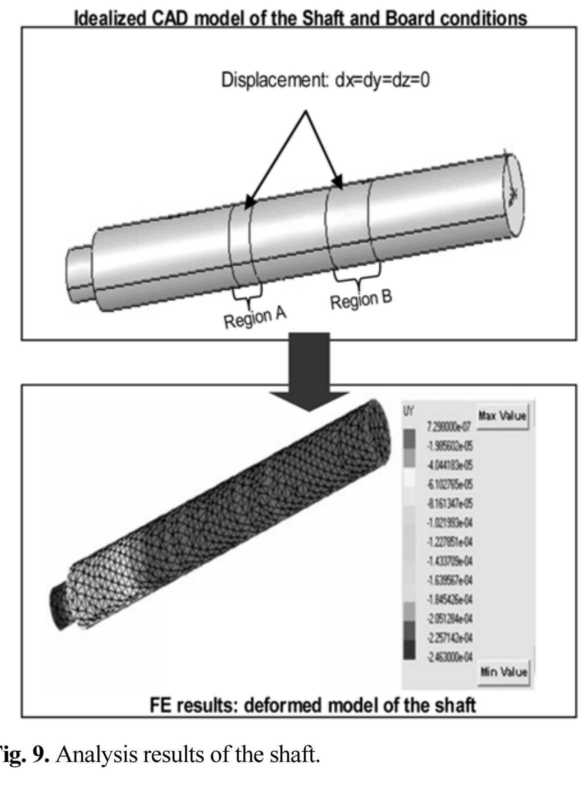

Fig. 9. Analysis results of the shaft.

reconstruction approach is based on the energetic method detailed in the previous sections. Four steps are necessary to reconstruct a deformed CAD model using the energetic method:

- Each curve constraint is discretized in a set of points. The inertia of all the points is built. An inertia plane is the plane that passes through the flow maximum points.
- The various constraints are projected onto the plan.
- An initial surface which is near to the calculated final surface is achieved by deforming the inertia plane. This is accomplished by minimising the distances between the constraint points and their projection on the plan of inertia. It can be considered that this step (the third step) represents a preparation phase of a surface to the fourth step.
- The construction of the final surface takes place over several iterations. However, in most cases one iteration is enough except for areas which are more complex peaks and sharp bends.

Based on these steps, the faces are identified and reconstructed one by one. The edges and loops are found by the intersection of all faces. The chosen CAD model contains cylindrical and planar faces, the identification of the triangulations related to each of these faces is achieved directly (without topology investigating), because the topology has not changed. All the reconstructed faces are of the NURBS type, except for the embedded faces, which are planar.

Figure 10 presents a rebuilding of the deformed CAD model based on the FE result (deformed mesh).

Based on the rebuilding of the deformed CAD model of the transmission shaft, the designer starts with the virtual simulation step of the functioning of the assembly pump using the deformed model of the shaft (Fig. 7, step 4). According to this simulation, the designer can discern possible collisions between parts, particularly between the shaft and volute, and between the shaft and cylinder.

The simulation results permit to discern collisions which are represented in Fig. 11. The designer uses this interoperability scenario between design and analysis processes in order to discern, from the early design steps and without manufacturing a real prototype of the centrifugal pump a serious problem of functioning of the assembly. These problems are caused by an extreme deformation of the shaft. The intervention of the designer can be double:

- Change the shaft material in order to choose a more resistant material. This choice can increase the pump cost.
- Change the section of the shaft in order to increase its resistance. According to the designer, this decision can be better.

In this situation, a new scenario allows the iteration of FE analysis and the rebuilding of the deformed CAD model using a new dimension of the shaft section. The new simulation of the assembly is approved because the virtual simulation of the assembly was performed without collisions between the shaft and the other parts. The new dimension of the shaft section is also approved.

Figure 12 shows a new pump simulation after the change of the section of the transmission shaft. The simulation is performed without collision and the assembly pump is valid.

6. Conclusions

In this article, an original method of interoperability between design and analysis (Finite Element Method FEM) has been introduced. This method supports the

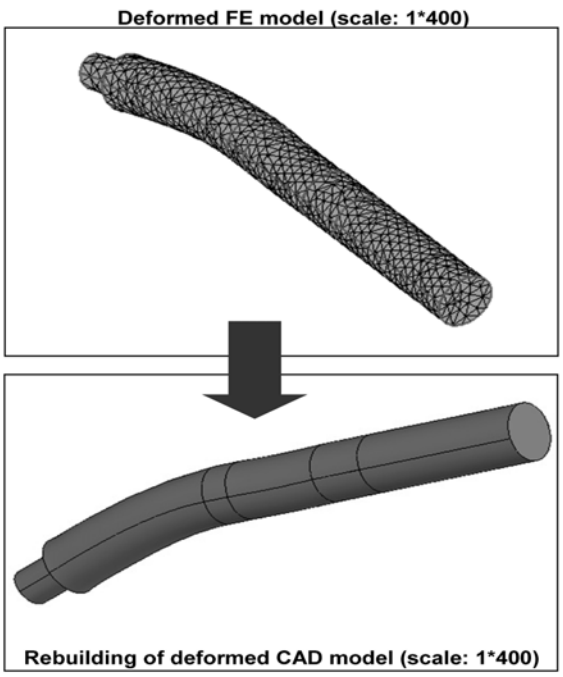

Fig. 10. Application of the Energetic method to reconstruct the deformed CAD model of the Shaft.

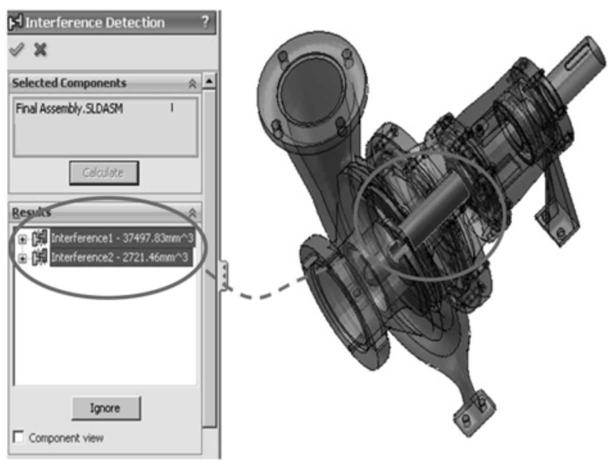

Fig. 11. Virtual collision detection in the pump model.

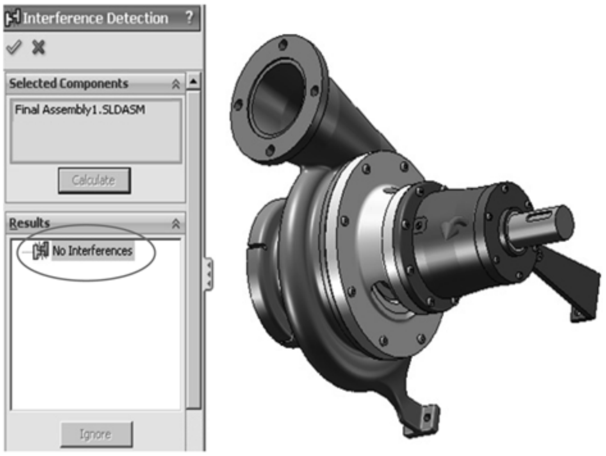

Fig. 12. Validation of the pump assembly.

designer in the loop of design – analysis – design. The transition process from the design environment to the analysis environment includes an important step of idealization of CAD model in order to avoid some geometric details (hole, chamfer, etc.) considered as superfluous for the FE analysis task. Once FE analysis is performed, the proposed method allows the rebuilding of deformed CAD model (BREP model). The rebuilding method is based on an energetic method which is described at the beginning of this paper.

From the deformed BREP model, the designer is able to simulate the functioning of the designed assembly in order to discern possible collisions between parts due to an extreme deformation of these ones (Fig. 12).

The proposed method was improved using an example of a centrifugal pump. The validation example allowed the implementation of the loop of design – analysis – design using the proposed method. It also made it possible to prove that the implementation of the integrated design-analysis method allows to the designer to discern from the early stages of the design process the risks of collisions between parts of the designed system.

In the case of the transmission shaft of the pump and using the actual dimensions of the shaft, the pump becomes non-functional because of the collisions between the shaft and the volute, and between the shaft and the pan. The increase of the diameter of the transmission shaft provides more resistance to the shaft and the pump becomes functional.

Our approach is a new proposal of the integrated design context. The simulation of the assembly system is performed in a virtual mode without using a real prototype of the centrifugal pump, which requires the manufacturing and the assembly of the different parts, thereby increasing the cost of the product.

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