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Analysis of Transient Conduction Heat Loss of Solid Sphere between Constant and Variable Free Convection
상수 또는 변수의 대류 경계조건을 가지는 구의 과도열전도 손실에 대한 해석

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주요용어 : 과도전도(Transient Conduction), 자연대류(Free Convection), 비오 수(Biot Number), 집중열용량법(Lumped Capacitance Model), 무차원 온도(Dimensionless Temperature)

Abstract : 본 연구는 구의 과도 열전도에 의한 열손실을 계산하는 데 있어, 외부의 경계조건인 대류의 조건에 해당하는 상황을 상수 및 변수로 가정하였을 경우의 열전달문제를 해석한 것이다. 이 문제를 해결하기 위해 집중열용량법을 사용하고 있으며, 대류열전달계수의 값이 온도의 함수로 변한다고 가정하여 계산하였다. 계산을 수행한 결과 대류경계조건의 값을 상수로 가정한 경우가 열손실이 높이 평가된다는 것을 알았고, 이러한 경향을 상관식으로 정리하였다.

Nomenclature

A : Heat transfer area [m^2]
Bi : Biot number [-]
C_t : Lumped thermal capacitance of sphere [J/K]
Cp : Specific heat [J/(kg·K)]
d : Diameter of sphere [m]
Fo : Fourier number [-]
Gr : Grashof number [-]
g : Gravitational acceleration [m/s^2]
h : Coefficient of convective heat transfer [watt/(m^2·K)]
K : Inverse of thermal time constant, $\left(\frac{\rho \cdot V \cdot Cp}{h \cdot A}\right)^{-1} = (R_t \cdot C_t)^{-1}[s^{-1}]$
k : Thermal conductivity [watt/(m·K)]
Nu : Nusselt number [-]
Pr : Prandtl number [-]
Ra : Rayleigh number [-]
R_t : Resistance to convection heat transfer [K/watt]
T : Temperature [K]
t : Time [s]
V : Volume of sphere [m^3]
α : Thermal diffusivity [m/s^2]
β : Thermal expansion rate [K^{-1}]
θ : Temperature difference [K]
υ : Kinetic viscosity [m^2/s]
ξ : Dimensionless temperature, $\frac{T-T_w}{T_i-T_w}$
ρ : Density [kg/m^3]

Subscript

c : Constant heat transfer coefficient
I : Initial
l : Lumped thermal capacitance method
s : Sphere
υ : Variable heat transfer coefficient
w : Cooling water

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1. Introduction

In the industrial part, especially in ironworks, heat loss of solid metal is very important factor to make a good steel product. It is not too much to say that the forging process is the most important process[1]. And also to estimate the heat loss from solid surface to circumference is a very difficult work. However, the estimation of heat loss is easily calculated by considering that the thermal properties are constants[2~7].

Unfortunately, this assumption has a significant error that the heat transfer rate is over estimated at the transient heat transfer case.

During forging process in ironworks, the heat transfer mechanism is complicated by heat conduction and convection. Also according to the lapse of time the heat transfer coefficient controlled by temperature difference is changed. Therefore, to make the accurate estimation of heat loss, it is desirable for the heat transfer coefficient to be a variable parameter.

The goal of this study is to compare the difference between constant value of heat transfer coefficient and variable value of it in the case of transient heat transfer.

2.1 The lumped thermal capacitance method and analytic solution

In this study, the transient heat transfer problem is one in which a solid sphere experiences a sudden change in its thermal environment. Consider a hot metal forging that is initially at a uniform temperature $T(t)$ and is quenched by immersing it in a water of lower temperature T_W (Fig. 1).

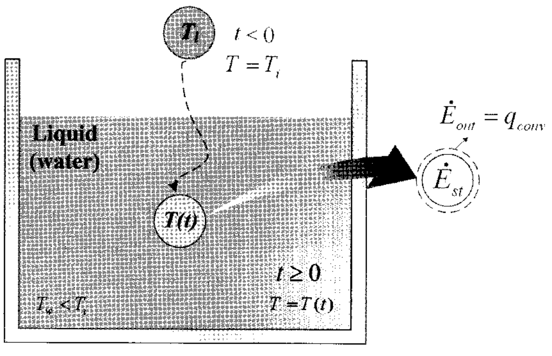

Fig. 1 Cooling of a hot metal sphere forging

If the quenching is said to begin at time $t = 0$, the temperature of the solid sphere will decrease for $t > 0$, until it eventually reaches T_W. This reduction is due to convection heat transfer at the solid-liquid interface. The lumped thermal capacitance method has the assumption that the temperature of the solid is spatially uniform at any instant time during the transient process[8]. And this assumption is reasonable that the cooling bath is very large so the cooling water has sufficient heat capacity compare to solid sphere. Consequently, this assumption implies that temperature gradients within the solid sphere are negligible.

From the Fourier's law, heat conduction in the absence of a temperature gradient implies the existence of infinite thermal conductivity. Such a condition is clearly impossible. However, although the conditions is never satisfied exactly, it is closely approximated if the resistance to conduction within the solid is small compared with the resistance to heat transfer between the solid and its surroundings.

In neglecting temperature gradients within the solid, it can be no longer consider the problem from within the framework of the heat balance equation. Instead, the transient temperature response is determined by formulating an overall energy balance on the solid. This balance must relate the rate of heat loss at the surface to the rate of change of the internal energy as expressed in Eq.(1).

$$-h \cdot A_s \cdot (T - T_w) = \rho_s \cdot V_s \cdot Cp_s \cdot \left(\frac{d}{dt}T\right) \qquad (1)$$

Introducing the temperature difference, $\theta = T - T_W$ and recognizing that $\frac{d}{dt}\theta = \frac{d}{dt}T$, it follows like Eq.(2).

$$\frac{\rho_s \cdot V_s \cdot Cp_s}{h \cdot A_s} \cdot \left(\frac{d}{dt}\theta\right) = -\theta \qquad (2)$$

Separating variables and integrating from the initial condition, for which t=0 and $T(0)=T_i$, the Eq.(2) can be rewritten as Eq.(3).

$$\frac{\rho_s \cdot V_s \cdot Cp_s}{h \cdot A_s} \cdot \int_{\theta_i}^{\theta} \frac{1}{\theta} d\theta = -\int_0^t 1 dt \tag{3}$$

where, $\theta_i = T_i - T_w$

Evaluating the integrals it follows like Eq.(4) or (5).

$$\frac{\rho_s \cdot V_s \cdot Cp_s}{h \cdot A_s} \cdot \ln\left(\frac{\theta_i}{\theta}\right) = t \tag{4}$$

$$\frac{\theta}{\theta_i} = \frac{T - T_w}{T_i - T_w} = \exp\left(\frac{h \cdot A_s}{\rho_s \cdot V_s \cdot Cp_s} \cdot t\right) \tag{5}$$

The Eq.(4) and (5) may be used to determine the time required for the solid to reach a instantaneous temperature(T) or, conversely, may be used to compute the temperature reached by the solid at some instantaneous time(t). And the total heat transfer(Q_{loss}) occurring up to some time is derived as Eq. (6)

$$Q_{loss} = \rho_s \cdot V_s \cdot Cp_s \cdot (T_i - T_w) \cdot \left(1 - \exp\left(\frac{-t}{K_s^{-1}}\right)\right) \tag{6}$$

2.2. Solution for constant heat transfer coefficient

From the sphere, considered in this study, the energy loss by convection to the cooling water at T_W is equal to the time rate of decrease in internal energy in the solid sphere at $T(t)$, and the energy gained by the cooling water is equal to the increase in internal energy in the water. So the energy balance of each side is expressed in Eq.(7).

$$\begin{aligned} h \cdot A_s \cdot (T_s - T_w) &= \rho_s \cdot V_s \cdot Cp_s \cdot \left(\frac{d}{dt} T_s\right) \\ h \cdot A_s \cdot (T_s - T_w) &= \rho_w \cdot V_w \cdot Cp_w \cdot \left(\frac{d}{dt} T_w\right) \end{aligned} \tag{7}$$

Let constants $K_s = \frac{h \cdot A_s}{\rho_s \cdot V_s \cdot Cp_s}$ and $K_w = \frac{h \cdot A_s}{\rho_w \cdot V_w \cdot Cp_w}$. The two coupled, linear differential equations can then be written as Eq.(8), where T_s and T_W are both function of time.

$$\begin{aligned} \frac{d}{dt} T_s + K_s \cdot T_s &= K_s T_w \\ \frac{d}{dt} T_w + K_w \cdot T_w &= K_w T_s \end{aligned} \tag{8}$$

As illustrated in Fig.2, the Biot number(Bi) provides a measure of the temperature drop in the solid sphere relative to the temperature difference between the surface and the cooling water. The result in Fig. 2 suggests that it is reasonable to assume a uniform temperature distribution across the solid sphere at any time during a transient heat conduction process.

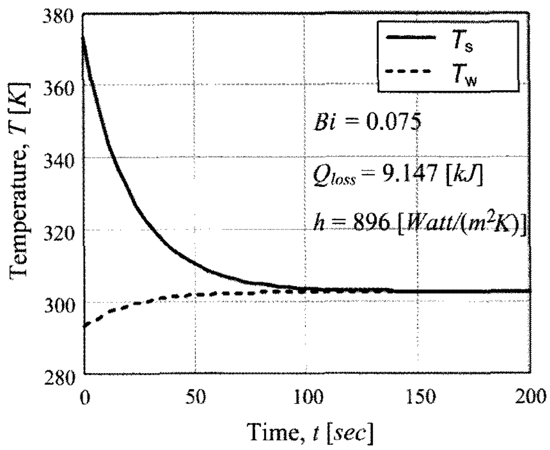

Fig. 2 Solution of constant heat transfer coefficient

This result may also be associated with interpretation of the Biot number as a ratio of thermal resistances. The Biot number in Fig. 2 is Bi = 0.075, it is much less than 1. This means that the resistance to conduction within the solid is much less than the resistance to convection across the fluid boundary layer. Hence the assumption of a uniform temperature distribution is reasonable.

2.3 Solution for variable heat transfer coefficient

Usually in forging process, there exists free convection due to buoyance force within the fluid. And the free convection flows typically originate from a thermal instability. That is, warmer, lighter fluid moves vertically upward relative to cooler heavier fluid.

The free convection boundary layer depends on the relative magnitude of the buoyance and viscous forces in the fluid. It is customary to correlate its occurrence in terms of the Rayleigh number(Ra), which is simply the product of the Grashof(Gr) and Prandtl numbers(Pr).

$$Ra = Gr \cdot \Pr = \frac{g \cdot \beta(T_s - T_w) \cdot d^3}{v \cdot \alpha} \tag{9}$$

$$\mathrm{Nu} = 2 + \frac{0.589 \cdot \mathrm{Ra}^{0.25}}{\left[1 + \left(\frac{0.469}{\mathrm{Pr}}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \tag{10}$$

The Eq.(9) indicates the Rayleigh number of sphere and Eq.(10) indicates the correlation of Nusselt number derived from Churchill[9]. Also the heat transfer coefficient(h) has relation with Nusselt number as Eq. (11).

$$h = \frac{k_S}{d} \cdot \mathrm{Nu} = \frac{k_S}{d} \cdot \left[2 + \frac{0.589 \cdot \mathrm{Ra}^{0.25}}{\left[1 + \left(\frac{0.469}{\mathrm{Pr}}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}\right] \tag{11}$$

Fig. 3 illustrates the calculated results between constant and variable heat transfer coefficients(h). The solid lines are cooling curves based upon a constant heat transfer coefficient using the initial temperature difference between the solid sphere and cooling water. And the dashed lines account for the variable heat transfer coefficient as a function of temperature difference between solid sphere and cooling water.

Comparing the results shown in Fig. 3, it is apparent that the case of variable heat transfer coefficient is under estimated at the middle region of time(10<t<160).

This means that the strength of free convection is typically originated from the temperature difference between solid sphere and cooling water. In other words, the case of variable heat transfer coefficient means that the temperature difference become smaller according to lapse of time. However, the case of constant heat transfer coefficient, it is only calculated with initial temperatures of solid sphere and cooling water. Therefore, the heat transfer of constant case is over estimated.

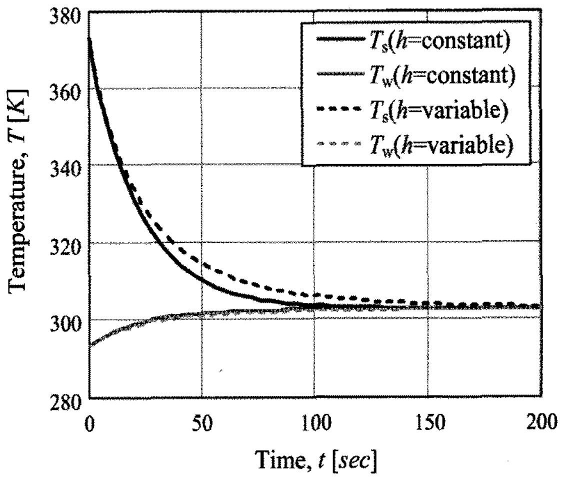

Fig. 3 Comparison of constant and variable heat transfer coefficients

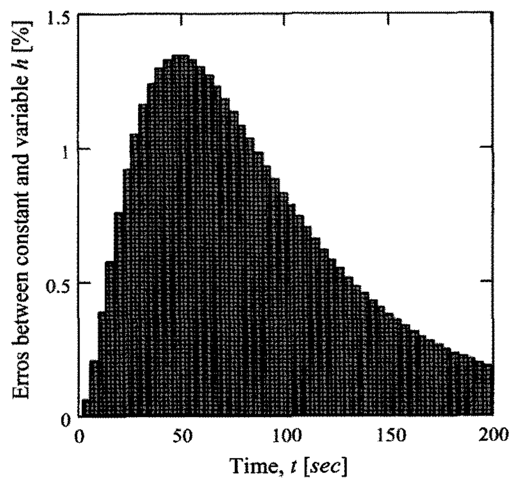

Fig. 4 Errors between constant and variable heat transfer coefficient

Fig. 4 indicates the errors between constant and variable heat transfer coefficient. As mentioned above, the heat transfer is over estimated in the case of constant h. Especially at t = 50[sec], the maximum error exists.

To compare these supposed cases, the lumped thermal capacitance method usually used in analysis of transient heat transfer problem was used. And using the dimensionless temperature

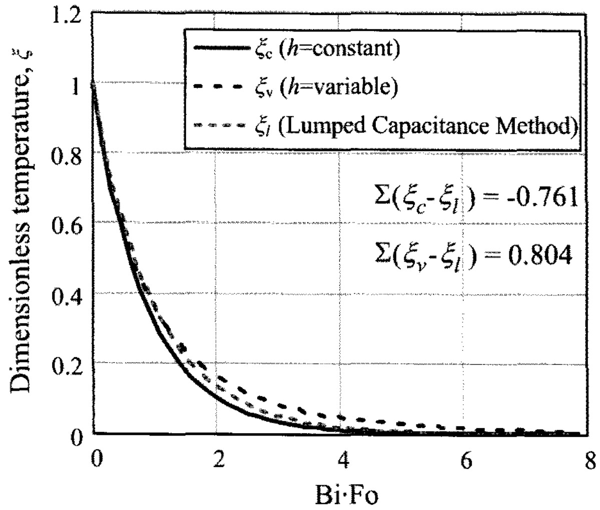

Fig. 5 Relationship between ξ and Bi·Fo

(ξ), the behavior of these data is arranged as shown in Fig. 5.

From this graph, the tendencies of ξ are well agreed with others although the sum of differences exist. And also it is cleared that the case of constant h is over estimated compare with lumped thermal capacitance method.

On the contrary, the case of variable h is under estimated. These results would be brought about the premised assumption of lumped thermal capacitance method that the cooling water temperature is not changed until forging process is done. So the most reliable calculated value may be the case of variable h.

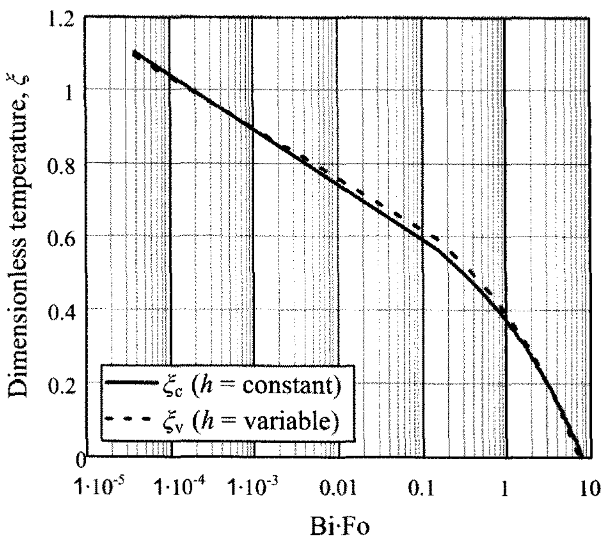

Fig. 6 Curve fitted data of ξ vs Bi·Fo

The tendencies of these temperatures show logarithmic change as shown in Fig.3 and 5. And to derive the correlations on each case(Eq.(12)), the logarithmic function is used and the acquired results are shown in Fig. 6.

$$\xi_c = -0.064 \cdot (Bi \cdot Fo) - 0.151 \cdot \sqrt{Bi \cdot Fo} + 0.458$$
$$\xi_v = -0.056 \cdot (Bi \cdot Fo) - 0.178 \cdot \sqrt{Bi \cdot Fo} + 0.532 \qquad (12)$$

3. Conclusions

In this study, the coupled ordinary differential equations were used to calculate the transient heat transfer problem. These equations compared with lumped thermal capacitance method. And also the dimensionless correlations were derived.

(1) The case of constant h, usually used in many cases, is over estimated to calculate the transient heat transfer. Because the constant h is based upon a initial temperature difference between the solid sphere and cooling water.

(2) It is apparent that the case of variable h is under estimated at the middle region of time($10<t<160$). This means that the strength of free convection is typically originated from the temperature difference between solid sphere and cooling water. In other words, the case of variable h means that the temperature difference become smaller according to lapse of time.

(3) The tendencies of ξ are well agreed with each other. And also it is cleared that the case of constant h is over estimated compare with lumped thermal capacitance method. On the contrary, the case of variable h is under estimated.

(4) The relationship between Bi·Fo and ξ was observed with logarithmic function. And the each correlation equation was derived.

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