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Reliability-Based Topology Optimization Based on Bidirectional Evolutionary Structural Optimization

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양방향 진화적 구조최적화를 이용한 신뢰성기반 위상최적화

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Abstract

This paper presents a reliability-based topology optimization (RBTO) based on bidirectional evolutionary structural optimization (BESO). In design of a structure, uncertain conditions such as material property, operational load and dimensional variation should be considered. Deterministic topology optimization (DTO) is performed without considering the uncertainties related to the design variables. However, the RBTO can consider the uncertainty variables because it can deal with the probabilistic constraints. The reliability index approach (RIA) and the performance measure approach (PMA) are adopted to evaluate the probabilistic constraints in this study. In order to apply the BESO to the RBTO, sensitivity number for each element is defined as the change in the reliability index of the structure due to removal of each element. Smoothing scheme is also used to eliminate checkerboard patterns in topology optimization. The limit state indicates the margin of safety between the resistance (constraints) and the load of structures. The limit State function expresses to evaluate reliability index from finite element analysis. Numerical examples are presented to compare each optimal topology obtained from RBTO and DTO each other. It is verified that the RBTO based on BESO can be effectively performed from the results.

Key Words : Reliability-Based Topology Optimization(신뢰성기반 위상최적화), Reliability Index Approach(신뢰성 지수 접근법), Performance Measure Approach(목표성능치 접근법), Bidirectional Evolutionary Structural Optimization(양방향 진화적 구조최적화), Sensitivity number(민감도수), Reliability Index(신뢰성지수)

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1. Introduction

Design variables such as material property, external load and dimensional variation may have uncertainties in actual designs due to errors in manufacturing process and actual working state. Since deterministic design optimization (DO) can not consider the effect of the uncertainties on objective function or constraint, the reliability of an optimum design to satisfy the given constraint becomes degenerated. In RBDO, probabilistic constraints are formulated so as to construct approximated linear (or quadratic) functions to closely represent the nonlinear limit state functions for the reliability index (or safety index) calculation and optimization by using the appropriate transformations[(1~5)].

There are two methods to perform reliability analysis, which are sampling and fast probability integration methods. Monte Carlo simulation[(6,7)] as a typical sampling method uses response function of a system as it is, so that this method is relatively simple, easy and accurate comparing to other reliability analysis techniques. But it requires so many numbers of experiments or simulations to secure accuracy of reliability analysis. Fast probability integration method is to obtain reliability index as a relative index of failure probability by approximating response function of a system as linear or parabolic functions. There are two typical approaches[(8)] in the fast probability integration method. One is reliability index approach (RIA) with reliability constraint expressed by reliability index. The other is performance measure approach (PMA), which calculates probabilistic constraint using the inverse function of reliability index[(9,10)].

Structural problems are considered for topology optimization in this paper. Homogenization[(11,12)] and density distribution[(13)] methods are very well known topology optimization methods. Another topology optimization technique, ESO[(14,15)] (evolutionary structural optimization) method has been developed by Xie and Steven. As ESO considers only removing elements and those removed ones cannot be brought back in the later evolution, the final topology must be affected by the history of the previous evolution. It is known that the optimal topology might be differently obtained, according to removal ratio of elements and the element number of the initial design. BESO allows for the elements to be added as well as removed, so that the final optimal topology can be reached to almost the same topology. As the result, BESO a clear and robust indication of structural efficiency of the resulting topologies. Also, BESO is more flexible in choosing the initial design, and any design complying with the loading and boundary conditions can be used as an initial design[(16)].

Many researches on RBTO (reliability-based topology optimization) has been published in recent years[(17~19)]. Since RBTO is strongly dependent on topology optimization techniques, it can be expected that the better result can be obtained when BESO is used rather than ESO. But since RBTO based on BESO has not been reported yet, RBTO based on BESO is studied in this paper. Sensitivity number, which is a measure on element removal in BESO, is defined by reliability index. And AFORM (advanced first order reliability method)[(8)] is used for calculating sensitivity number in this paper. In order to verify whether RBTO based on BESO is successfully applied for obtaining an optimal topology or not, RBTO for a short cantilever and MBB beams is performed under the stiffness constraint in the case of having uncertainties of material property, applied load and thickness. Then, each optimal topology obtained from RBTO and DTO(deterministic topology optimization) are compared each other. In order to calculate the sensitivity number defined by reliability index, finite element analysis is performed by using MATLAB[(20)]. Limit state function is formulated to evaluate reliability index from finite element analysis.

2. RBDO

Whereas, DO problem is formulated by using deterministic constraint, reliability-based design optimization problem is formulated by using probabilistic constraint as the following Eq. (1) to consider uncertainties of design variables.

$$\begin{aligned} &Min. \quad f(d) \\ &S.t. \quad P[G_i(d,x) \le 0] \le P_f \end{aligned} \tag{1}$$

where, f is the objective function, d is the design variable, and x is the random variables vector. Probabilistic constraints are described by the ith limit state function, $G_i(d,x) \cdot P$ is the probability of failure, and P_f is the target probability of failure. Probabilistic constraint means that the probability of failure of a system must be smaller than the target probability of failure. The probability of failure of a system indicates the probability not to satisfy the constraint, that is, to violate the constraint. As one of the possible topology optimization problems, the objective function, f, is chosen as the volume.

2.1 *Reliability Index Approach(RIA)*[8,21,22]

When probabilistic constraints are estimated in terms of the reliability index, the probability structural design optimization of Eq. (1) may be expressed as

$$\begin{aligned} &Min. \quad Volume \\ &S.t. \quad \beta \le \beta_{target} \end{aligned} \tag{2}$$

where, β is the reliability index. β_{target} is the target reliability index. In order to evaluate the probabilistic constraint for RIA, nested optimization loop is necessary. The definition of reliability index is the minimum distance form origin to approximate limit state function. Therefore the reliability index β is formulated as an optimization problem with an equality constraint in U-space as follows

$$\begin{aligned} &Min \quad \beta = \|u\| = \sqrt{u^T u} \\ &S.t. \quad G(u) = 0 \end{aligned} \tag{3}$$

where, u is an uncertainty variable transformed into a standard normal distribution coordinate system. β is the reliability index, G(u) is the limit state function in U-space. The optimum point on the failure surface is called the most probable point (MPP), $u^*_{G(u)=0}$ Either an MPP search algorithm that is specifically developed for first-order reliability analysis or general optimization algorithms, SLP or SQP etc., can be used to solve this equation. In this paper, the advanced first order reliability method[8,22] is employed to perform reliability analyses in RIA.

2.2 *Performance Measure Approach (PMA)*[8,21,22]

Reliability analysis in PMA is formulated as the inverse of reliability analysis in RIA. The probabilistic performance measure G(u) is obtained from a nonlinear optimization problem in U-space as

$$\begin{aligned} &Min \quad G(u) \\ &S.t. \quad \|u\| = \sqrt{u^T u} = \beta_{target} \end{aligned} \tag{4}$$

where, u is an uncertainty variable transformed into a standard normal distribution coordinate system. Rather than a general optimization algorithm, the advanced mean value (AMV), conjugate mean value (CMV), and hybrid mean value (HMV) methods[23,24] are commonly used to solve the problem in Eq. (4), since they do not require a line search. In this paper, the above three methods are also used to solve the inverse problem in PMA.

3. Sensitivity Analysis

3.1 *Sensitivity Number for DTO*

In the finite element method, the static behavior of a structure[15,16] is represented by

$$[K]\{\delta\} = \{P\} \tag{5}$$

where $[K]$ is the global stiffness matrix, $\{\delta\}$ is the global nodal displacement vector, and $\{P\}$ is the nodal load vector.

The strain energy of the structure, which is defined as

$$C=\frac{1}{2}\{P\}^T\{\delta\} \tag{6}$$

is commonly used as the inverse measure of the overall stiffness of the structure. It is obvious that maximizing the overall stiffness is equivalent to minimizing the strain energy.

Consider the removal of the i-th element from a structure comprising n finite elements. The stiffness matrix will change by $[\Delta K]=[K^*]-[K]=-[K^i]$, where $[K^*]$ is the stiffness matrix of the resulting structure after removal of the i-th element and $[K^i]$ is the stiffness matrix of the i-th element. It is assumed that the removal of the element has no effect on the load vector $\{P\}$. By ignoring a higher order term, we can find the change of the displacement vector from Eq. (5) as

$$\{\Delta\delta\}=-[K]^{-1}[\Delta K]\{\delta\} \tag{7}$$

From Eq. (6) and (7), the change of strain energy can be expressed by the following.

$$\begin{aligned}\Delta C&=\frac{1}{2}\{P\}^T[\Delta\delta]\\&=-\left(\frac{1}{2}\right)\{P\}^T[K]^{-1}[\Delta K]\{\delta\}\\&=\frac{1}{2}\{\delta^i\}^T[K^i]\{\delta^i\}\end{aligned} \tag{8}$$

where $\{\delta^i\}$ is the displacement vector of the i-th element. Therefore, the sensitivity number for a structure with static stiffness constraint can be defined as Eq. (9).

$$\alpha_i=\left(\frac{1}{2}\right)\{\delta^i\}^T[K^i]\{\delta^i\}\ \ (i=1,2,\ldots,n) \tag{9}$$

This indicates the change in the strain energy due to the removal of the i-th element. It should be noted that α_i is the element strain energy. Both C and α_i are always positive values. In general, when an element is removed, the stiffness of a structure reduces and correspondingly the strain energy increases. To achieve this objective through element removal, it is obviously most effective to remove the element which has the lowest value of α_i, so that the increase in C is minimum.

Checkerboard patterns are quite common in various finite element based structural optimization techniques. It causes severe numerical errors in topology optimization. To overcome this problem, an intuitive smoothing scheme[25] and checkerboard patterns occurrence decision algorithm[26] are used in this study. It is clear that the second order scheme in the smoothing scheme may provide a better correction to these numerical instabilities than the first order scheme. Therefore, the second order scheme was implemented.

3.2 *Sensitivity Number for RBTO*

Limit state function for calculating the reliability index for a structure with displacement constraint is defined from Eq. (10) as follows.

$$\begin{aligned}g&=\delta_{\max}-\delta_{\text{allow}}\\&=\{r\}\{P\}-\delta_{\text{allow}}\end{aligned} \tag{10}$$

where, g?is the limit state function. $\delta_{\max}$ is the maximum displacement. δ_{allow} is the allowable displacement. $\{r\}$ is the column vector consisted of the elements of column in $[K]^{-1}$ corresponding to the degree of freedom of the node where the maximum displacement occurs. The reliability index can be calculated by solving optimization problem defined as Eq. (5) by using Eq. (10) at that time.

The displacement vector when the i-th element is removed is expressed as Eq. (11) from Eq. (7), which indicates the change of displacement vector due to the removal of the i-th element.

$$\{\delta'\} = \{\delta\} + \{\Delta\delta\} = [K]^{-1}\{P\} - [K]^{-1}[\Delta K]\{\delta\} \tag{11}$$

The limit state function due to the i-th element removal is defined by the following.

$$g^i = \delta' - \delta_{\text{allow}} = \{r\}\{P\} - \{r\}[\Delta K]\{\delta\} - \delta_{\text{allow}} \tag{12}$$

The reliability index can be calculated by solving optimization problem defined as Eq. (5) by using Eq. (12) when the i-th element is removed. Since the stiffness of a structure is reduced as an element is removed, probability satisfying the stiffness constraint due to an element removal is also reduced. In order to obtain the probability satisfying the stiffness constraint due to an element removal, it is known that the element with the smallest value of the change of the reliability index should be removed.

Therefore, the sensitivity number α^i for RBTO can be defined as the change of the reliability index when the i-th element is removed.

$$\alpha^i = \beta - \beta^i \tag{13}$$

where, β is the reliability index for the present topology, and β^i is the reliability for the topology after removing the i-th element. RBTO for a structure is performed through the following procedure[27]. And a flowchart for RBTO based on BESO is shown in Fig. 1.

Step 1. Discretise the structure using a fine mesh of finite elements.

Step 2. Perform the finite element analysis for the given load.

Step 3. Calculate the reliability index β for the present topology by using Eq. (3) and (10) for each element.

Step 4. Calculate the sensitivity numbers, i.e. the change of the reliability index due to the i-th element removal by use of Eq. (13) for each element.

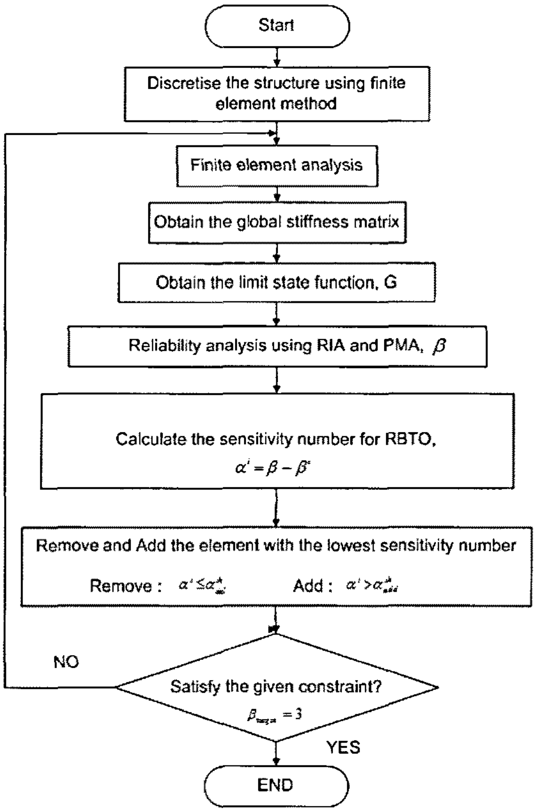


Fig. 1 Flowchart of reliability-based topology optimization based on BESO

Step 5. Remove and add some elements with the smallest sensitivity numbers due to the threshold sensitivity numbers for removing and adding elements. Removing constraint is $\alpha^i \le \alpha^{th}_{del}$ and adding constraint is $\alpha^i \le \alpha^{th}_{add}$.

Step 6. Repeat step 3 to step 5 until the probability constraint (Eq. (3) and Eq. (4)) is satisfied.

4. Numerical Examples

4.1 *RBTO for a Short Cantilever Beam*

RBTO and DTO for the cantilevered beam subjected to applied load at the right end as shown in Fig. 2 are performed, and each optimal topology obtained from RBTO and DTO are compared each other. The dimen-

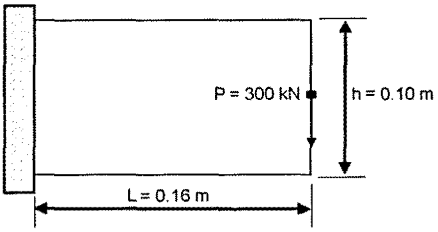


Fig. 2 Design domain for a cantilever beam

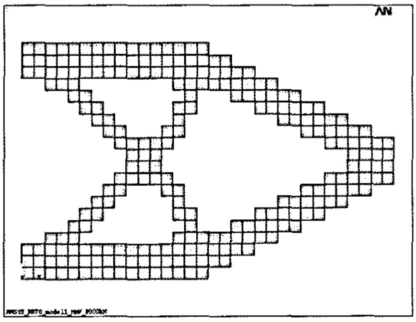
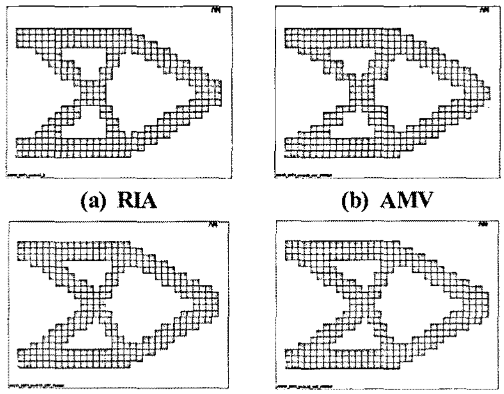


Fig. 3 Optimal topology of DTO for the cantilever beam

sions of the beam are L=0.16m, h=0.10m and t=0.005m. The properties of the material are E=200GPa, Poisson's ratio ν=0.3. The applied load at the free end is P=300 kN. The displacement constraint is that the maximum displacement at the free end should be less than 1.5mm. Design domain was divided into 32×20 rectangular quadrilateral elements. BESO is used for topology optimization. RIA and PMA are used for RBTO. $AR_{\max}$ and ER in performing BESO are defined as 5% and 1%, respectively. $AR_{\max}$ and ER indicate the maximum additional element and element removal ratios, respectively[27].

In order to perform RBTO, the probabilistic variables defined in this problem are assumed that they have normal distribution characteristics and 10% standard deviations with respect to each average value, and are independent in probability each other. The formulation of DTO for this example is as follows.

$$\begin{aligned} &Min. \quad Volume \\ &S.t. \quad G = \delta_{\max} - \delta_{\text{allow}} \le 0 \end{aligned} \tag{14}$$



(a) RIA (b) AMV

(c) CMV (d) HMV

Fig. 4 Optimal topology of RBTO for the cantilever beam for one uncertainty variable (E)

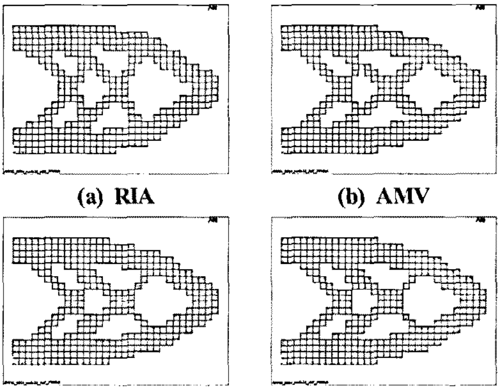
Table 1 One uncertainty variable (E)

	Reliability index, (β)	Displacement (δ)	Volume
RIA	3.015	1.411	41.25%
AMV	2.994	1.401	41.87%
CMV	3.002	1.365	41.87%
HMV	3.002	1.365	41.87%
DTO	1.289	1.611	37.81%

where, $\delta_{\max}$ is the maximum displacement of the present topology, and δ_{allow} is set to be 1.5.

Target reliability index, β_{target} is defined as 3, and the formulation of RBTO is same as Eq. (2). Probability of failure corresponding the target reliability index 3 in Eq. (2) is P_f=0.136%. Fig. 3 shows the result of DTO with the removal ratio 1% for the given problem.

In the case of one uncertainty variable of Young's modulus E, the results of RBTO are shown in Fig. 4 and summarized in Table 1 compared with that of DTO. Optimal topologies having reliability indices between 2.994 and 3.015 very close to the target reliability index 3, are obtained by RBTO. And optimal topology in the case of DTO has the reliability index 1.289 for the volume of 37.81%. Likewise, the displacements of RBTO



(a) RIA (b) AMV

(c) CMV (d) HMV

Fig. 5 Optimal topology of RBTO for the cantilever beam for three uncertainty variable (E, P, t)

Table 2 Three uncertainty variables (E, P, t)

	Reliability index, (β)	Displacement (δ)	Volume
RIA	3.002	1.157	50.00%
AMV	3.015	1.154	50.93%
CMV	2.999	1.167	50.62%
HMV	2.999	1.167	50.62%
DTO	1.289	1.611	37.81%

show less than that of DTO as expected. From the results of Table 1, Fig. 3 and Fig. 4, it is verified that optimal topologies of RBTO are better than those of DTO in the case of one uncertainty variable. Also, it is known that the volume of the optimal topology obtained by RBTO in the case of one uncertainty variable is larger than that of DTO as expected.

RBTO with three uncertainty variables of Young's modulus E, the applied load P and the thickness t is performed. In order to perform RBTO, the probabilistic variables defined in this problem are assumed that they have normal distribution characteristics and 10% standard deviations with respect to each average value, and are independent in probability each other. The results of RBTO are shown in Fig. 5 and summarized in Table 2 compared with that of DTO.

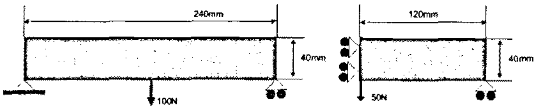


Fig. 6 Design domain and a half model of MBB beam

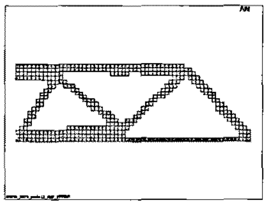
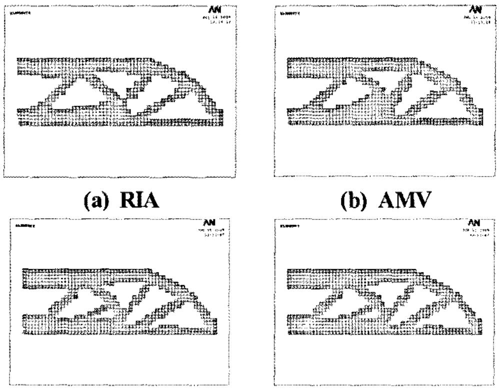


Fig. 7 Optimal topology of DTO for the MBB beam

Optimal topology having reliability index 2.999 very close to the target reliability index 3, is obtained by RBTO. And optimal topology in the case of DTO has the reliability index 1.289 for the volume of 37.81%. Likewise, the displacements of RBTO show less than that of DTO. From the results of Table 2, Fig. 3 and 5, it is verified that optimal topology of RBTO is the better than that of DTO in the case of one uncertainty variable. Also, it is known that the volume of the optimal topology obtained by RBTO in the case of three uncertainty variable is the larger than that in the case of one uncertainty variables as expected.

4.2 *RBTO for MBB Beam*

RBTO and DTO for MBB beam as shown in Fig. 6 are performed, and each optimal topology obtained from RBTO and DTO are compared each other. The dimensions of the beam are L=0.24m, h=0.04m and t=0.001m. The properties of the material are E=200GPa, Poisson's ratio ν=0.3. The applied load at the free end is P=200 N. The displacement constraint is that the maximum displacement at the center should be less than 1.3mm. Design domain was divided into 60×20 rectangular quadrilateral elements. BESO, RIA and PMA were used for topology optimization and RBTO, respectively. AR_{max} and ER in performing BESO are defined as 5% and 3%, respectively. Fig. 7 shows the result of DTO with the



(a) RIA (b) AMV (c) CMV (d) HMV

Fig. 8 Optimal topology of RBTO for the MBB beam for one uncertainty variable (E)

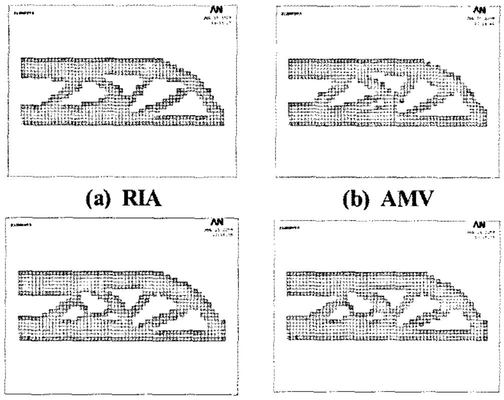
Table 3 One uncertainty variable (E)

	Reliability index, (β)	Displacement (δ)	Volume
RIA	3.024	0.91	55.00%
AMV	3.109	0.90	56.16%
CMV	2.999	0.92	56.16%
HMV	2.999	0.92	56.16%
DTO	0.768	1.35	34.50%

removal ratio 1% for the given problem[27].

In the case of one uncertainty variable of Young's modulus E, the results of RBTO are shown in Fig. 8 and summarized in Table 3 compared with that of DTO. Optimal topologies having reliability indices between 2.999 and 3.109 very close to the target reliability index 3, are obtained by RBTO. And optimal topology in the case of DTO has the reliability index 0.768 for the volume of 34.50%. Likewise, the displacements of RBTO show less than that of DTO as expected. From the results of Table 3, Fig. 7 and Fig. 8, it is verified that optimal topologies of RBTO are better than that of DTO in the case of one uncertainty variable. Also, it is known that the volume of the optimal topology obtained by RBTO in the case of one uncertainty variable is larger than that of DTO as expected.

RBTO with three uncertainty variables of Young's modulus E, the applied load P and the thickness t is



(a) RIA (b) AMV (c) CMV (d) HMV

Fig. 9 Optimal topology of RBTO for the MBB beam for three uncertainty variable (E, P, t)

Table 4 Three uncertainty variables (E, P, t)

	Reliability index, (β)	Displacement (δ)	Volume
RIA	2.998	0.77	65.33%
AMV	3.008	0.75	69.50%
CMV	3.009	0.76	69.50%
HMV	3.009	0.76	69.50%
DTO	0.768	1.35	34.50%

performed. In order to perform RBTO, the probabilistic variables defined in this problem are assumed that they have normal distribution characteristics and 10% standard deviations with respect to each average value, and are independent in probability each other. The results of RBTO are shown in Fig. 9 and summarized in Table 4 compared with that of DTO.

Optimal topology having reliability indices between 2.998 and 3.009 very close to the target reliability index 3, is obtained by RBTO. And optimal topology in the case of DTO has the reliability index 0.768 for the volume of 34.50%. Likewise, the displacements of RBTO show less than that of DTO. From the results of it is verified that optimal topology of RBTO is the better than that of DTO in the case of one uncertainty variable. Also, it is known that the volume of the optimal topology obtained by RBTO in the case of three uncertainty variable is the larger than that in the case of one uncertainty variables as expected.

The optimal topologies are very close to each other. It was found that the change of the reliability index is very small and very smooth as iteration proceeds. The reliability index is more gradually converged with very small change to the target reliability index as mesh size is smaller. It may result in high expensive computation cost.

5. Conclusions

Reliability-based topology optimization based on bidirectional evolutionary structural optimization, which provides a clear and robust indication of structural efficiency of the resulting topologies for a short cantilevered and MBB beams were performed. From the comparison of the results of RBTO and DTO, the following conclusions are obtained.

(1) It is verified that BESO using RIA and PMA can effectively be applied to RBTO.
(2) It is known that optimal topology of RBTO is the better than that of DTO in the case of one and three uncertainty variables. In other words, the reliability indices and the displacements of RBTO are larger and smaller than of DTO in the same volume, respectively.
(3) It is known that the volume of the optimal topology obtained by RBTO in the case of one uncertainty variable is larger than that of DTO as expected.
(4) It is known that the volume of the optimal topology obtained by RBTO in the case of three uncertainty variable for the same reliability index is the larger than that in the case of one uncertainty variables as expected.
(5) BESO provides a clear and robust indication of structural efficiency of the resulting topologies.

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