Journal of the Korean Data & Information Science Society
2010, **21**(4), 819–829

한국데이터정보과학회지

Properties of variable sampling interval control charts †

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Received 6 June 2010, revised 9 July 2010, accepted 13 July 2010

Abstract

Properties of multivariate variable sampling interval (VSI) Shewhart and CUSUM charts for monitoring mean vector of related quality variables are investigated. To evaluate average time to signal (ATS) and average number of switches (ANSW) of the proposed charts, Markov chain approaches and simulations are applied. Performances of the proposed charts are also investigated both when the process is in-control and when it is out-of-control.

Keywords: ANSS, ANSW, ASWR, ATS, variable sampling intervals.

1. Introduction

The purpose of a control chart is to detect assignable causes of variation so that these causes can be found and eliminated. During the control process, one wishes to detect any departure from a satisfactory state as quickly as possible and identify which attributes are responsible for the deviation.

A control chart is maintained by taking samples from the process and plotting in time order on the chart some control statistic which is a function of the samples. The operation of a control chart in detecting process changes can be described simply in terms of a control statistic and two disjoint regions, the signal region and the in-control region.

Traditional practice in using a control chart is to take samples from the process at fixed sampling interval (FSI). In recent years, application of VSI control charts has become quite frequent and several papers have been published about them in which the sampling interval is varied as a function of what is observed from the process.

One disadvantage of VSI procedure is that frequent switching between different sampling intervals requires more cost and effort to administer the process than corresponding FSI procedure. Amin and Letsinger (1991) studied the switching behavior of CUSUM charts with and without runs rules. They showed that VSI CUSUM chart has a much lower frequency of switching between the two sampling intervals then the VSI Shewhart chart in univariate $\overline{X}$-chart.

† This research is financially supported by Changwon National University in 2009-2010.

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In this paper, we investigate the properties VSI scheme for monitoring mean vector in terms of ATS, ANSS and ANSW of the proposed multivariate Shewhart and CUSUM charts. By Markov chain method or simulation, we found that performances of VSI CUSUM chart is more efficient then corresponding Shewhart chart in terms of ANSS, ATS and ANSW for small or moderate changes.

2. Description of some control procedures

Assume that the process of interest has p ($p \geq 2$) quality variables represented by the random vector $\underline{X}' = (X_1, X_2, \cdots, X_p)$ and we take a sequence of random vectors $\underline{X}_1, \underline{X}_2, \underline{X}_3, \cdots$ where $\underline{X}_i = (\underline{X}'_{i1}, \cdots, \underline{X}'_{in})'$ is a sample of observations at the sampling time i ($i = 1, 2, \cdots$) and $\underline{X}_{ij} = (X_{ij1}, \cdots, X_{ijp})'$. It will be also assumed that the successive observation vectors are distributed independent multivariate normal distribution with $N_p(\underline{\mu}, \Sigma)$. Hence, the distribution of $\underline{X}$ is indexed by a set of parameters $\underline{\theta} = (\underline{\mu}, \Sigma)$ where $\underline{\mu}$ is the mean vector and Σ is the dispersion matrix of $\underline{X}$. Let $\underline{\theta}_0 = (\underline{\mu}_0, \Sigma_0)$ be the known target values for $\underline{\theta}$.

2.1. Evaluating sample statistic

We can consider control procedures as a sequence of independent tests where each test is actually equivalent to a sequential probability ratio test (SPRT) for testing whether the process is in-control or out-of-control state.

Therefore, control chart can be considered as a repetitive test of significance where each quality characteristic is defined by p quality variables $X_1, X_2, \cdots, X_p$, we can obtain a sample statistic for monitoring $\underline{\mu}$ by using the likelihood ratio test (LRT) statistic for testing $H_0 : \underline{\mu} = \underline{\mu}_0$ vs $H_1 : \underline{\mu} \neq \underline{\mu}_0$ where Σ_0 is known. The regions above the upper control limit (UCL) corresponds to the LRT rejection region. For the i th sample, likelihood ratio λ_i can be expressed as

$$\lambda_i = \exp\left[-\frac{n}{2}(\overline{\underline{X}}_i - \underline{\mu}_0)'\Sigma_0^{-1}(\overline{\underline{X}}_i - \underline{\mu}_0)\right].$$

Let us define $Z_i^2 = -2\ln\lambda_i$. Then,

$$Z_i^2 = n(\overline{\underline{X}}_i - \underline{\mu}_0)'\Sigma_0^{-1}(\overline{\underline{X}}_i - \underline{\mu}_0). \tag{2.1}$$

Thus, LRT statistic Z_i^2 can be used as the control statistic for monitoring $\underline{\mu}$ of p related quality variables. Alt (1982) described various types of multivariate Shewhart type T^2 charts based on Hotelling's $T^2 = n(\overline{\underline{X}}_i - \underline{\mu}_0)'S^{-}1(\overline{\underline{X}}_i - \underline{\mu}_0)$ statistic and provided recommendations for implementation where S is the covariance matrix of the sample.

2.2. ANSS of FSI control chart

Ability of a control chart to detect any changes in the process is determined by the length of time required to signal. Thus, a good control chart detects changes quickly in the process while producing few false alarms.

In traditional FSI chart, the length of the sampling interval between sampling times t_i and t_{i-1} is constant for all i ($i = 1, 2, \cdots$) and the expected time to signal is simply the product of the average number of samples to signal (ANSS) and the length of the fixed sampling interval. The ANSS has the same definition as the average run length (ARL) but it seems preferable to use ANSS because it is more descriptive.

2.3. ATS of VSI control chart

The basic idea of VSI control chart is that the time interval should be short if there is some indication of a process change and should be long if there is no indication of a process change. If the indication of a process change is strong enough, then the VSI chart signals in the same way as the FSI control chart. Hence, the idea of using VSI control chart is intuitively reasonable.

For VSI chart, the sampling times are random variables and the sampling interval $t_{i+1} - t_i$ depends on the past sample informations $\underline{X}_1, \underline{X}_2, \cdots, \underline{X}_i$. Hence, the time required to signal is not the product of the number of samples and a fixed sampling interval. Thus, for the performances of a VSI chart, it is necessary to keep track of both ANSS and ATS.

2.4. ANSW of VSI control chart

VSI procedures have been shown to be more efficient when compared to the corresponding FSI procedures with respect to the ATS. But, because of frequent switching between different sampling intervals, VSI scheme requires more cost and effort to administer the process than corresponding FSI scheme. Hence, frequent switching between the different sampling intervals can be a complicating factor in the application of control charts with VSI procedures.

Therefore, it is necessary to define the number of switches (NSW) as the number of switches made from the start of the process until the chart signals, and let the average number of switches (ANSW) be the expected value of the NSW. The ANSW of VSI chart with two sampling intervals can be obtained as follows

$$ANSW = (ANSS - d_0) \cdot P(\text{switch}) \tag{2.2}$$

And, the probability of switch is given by

$$P(\text{switch}) = P(d_1) \cdot P(d_2|d_1) + P(d_2) \cdot P(d_1|d_2) \tag{2.3}$$

where $P(d_i)$ is the probability of using sampling interval d_i, and $P(d_i|d_j)$ is the conditional probability of using sampling interval d_i in the current sample given that the sampling interval d_j $(d_i \neq d_j)$ was used in the previous sample. To quantify the amount of switching, average switching rate (ASWR) can be defined as

$$ASWR = ANSW/ANSS. \tag{2.4}$$

A low value of the ASWR will usually be desirable from the administrative point of view, but a value of the ASWR very close to zero may not be achievable in a chart that is responsive to changes in the process.

3. Properties of Shewhart chart

Shewhart charts, one of the most widely used control chart, are simple to use and fast in detecting large shifts from the target value. FSI Shewhart chart for $\underline{\mu}$ based on the sample statistics Z_i^2 in (2.1) signals whenever

$$Z_i^2 \ \geq \ h_S. \tag{3.1}$$

And for VSI Shewhart chart based on Z_i^2, suppose that the sampling interval ;

$$d_1 \text{ is used when } Z_i^2 \ \in \ (g_S, h_S],$$
$$d_2 \text{ is used when } Z_i^2 \ \in \ (0, g_S],$$

where $g_S \leq h_S$ and $d_1 < d_2$. The percentage point of LRT statistic Z_i^2 can be obtained from chi-square distribution with p degrees of freedom, and the parameters g_S and h_S can be obtained to satisfy a desired ATS and ANSS. When the process is in-control, the statistic Z_i^2 has a chi-squared distribution. And when the process has shifted to $\underline{\mu}$ from the target $\underline{\mu}_0$, Z_i^2 has a non-central chi-square distribution with noncentrality parameter $\tau^2 = n(\underline{\mu} - \underline{\mu}_0)'\Sigma_0^{-}1(\underline{\mu} - \underline{\mu}_0)$.

3.1. ANSS of Shewhart chart

Let q be the probability that a control statistic falls out-of-control limits and d be a sampling interval for FSI scheme. Then, the time required to signal T is dN where N is the number of samples to signal. Since N is geometrically distributed with parameter q, ANSS is

$$E(N) = \frac{1}{q},$$

and the variance of N is given by

$$V(N) = \frac{(1-q)}{q^2}.$$

3.2. ATS of Shewhart chart

Reynolds and Arnold (1989) showed that if the consecutive observations are independent, then the VSI control chart with lower number of different sampling intervals is more efficient in terms of ATS and using two sampling intervals spaced as far apart as possible is optimal for one- and two-sided Shewhart chart.

Following Reynolds *et al.* (1990), we use two sampling intervals d_1, d_2 $(d_1 < d_2)$, and let ψ_i represent the expected number of samples before the signal that $d_i (i = 1, 2)$ is used. For simplicity, we assume that the interval used before the first sample, is a fixed constant, say d_0. Given the definitions of ATS, ANSS, ψ_1 and ψ_2, it is easy to see that

$$ANSS = 1 + \psi_1 + \psi_2 \text{ and } ATS = d_0 + d_1\psi_1 + d_2\psi_2.$$

3.3. ANSW of Shewhart chart

To evaluate $P(\text{switch})$ in (2.2), it is convenient when the distribution of sample statistic is known. When the control statistic is continuous, the continuous state space of the control statistic is partitioned into a finite number of discrete intervals and the probability distribution of the control statistic is discretized as the Markov chain approach. Let the interval of control statistic Y_k be divided into in-control region C and out-of-control region $C' = (h, \infty)$. Suppose that the region C is partitioned into r states $E_1, E_2, \cdots, E_r$ where each interval corresponds to a state of Markov chain and absorbing state $C' = \{\underline{x}|Y_k > h\}$ is a signal region. Since Y_k is continuous, let a discretized version $\widetilde{Y}_k$ of $Y_k \in E_i$ be the midpoint of E_i. The probability of moving from any state i to any other state j can be denoted as $p_{ij}(k) = P(Y_k + 1 \in E_j | Y_k \in E_i)$ for $i, j = 1, 2, \cdots, r+1$ and $k = 0, 1, 2, \cdots$. In this paper, $p_{ij}(k)$ will be written briefly as p_{ij}. The transition probability matrix $P = [p_{ij}]$ can be partitioned as

$$P = \begin{bmatrix} Q & (I-Q)\underline{1} \\ \underline{0}' & 1 \end{bmatrix} \tag{3.2}$$

where Q is the $r \times r$ transition matrix corresponding to the transient state, I is the identity matrix, $\underline{0}$ is an $r \times 1$ vector of $0'$ s and $\underline{1}$ is the $r \times 1$ vector of 1's. Here, we present $p'_{ij}s$ of P in VSI Shewhart chart with two sampling intervals based on Z_i^2 in (2.1), and we denote $F(\cdot)$ as the distribution function of sample statistic. Suppose that this chart signals when $Z_i^2 \in C'$, d_1 is used when $Z_i^2 \in (g, h]$ and d_2 is used when $Z_i^2 \in (0, g]$.

Assume that the interval $(0, g]$ is divided into m states and $(g, h]$ is divided into $(r - m)$ states then ω and v are g/m and $(h - g)/(r - m)$, respectively.

Then the probability of switch $P(\text{switch})$ can be expressed as

$$\begin{aligned} P(\text{switch}) = &\sum_{i=m+1}^{r} P(Z_k^2 \in E_i) \cdot \left\{ \sum_{j=1}^{m} P(Z_{k+1}^2 \in E_j | Z_k^2 \in E_i) \right\} \\ &+ \sum_{i=1}^{m} P(Z_k^2 \in E_i) \cdot \left\{ \sum_{j=m+1}^{r} P(Z_{k+1}^2 \in E_j | Z_k^2 \in E_i) \right\}. \end{aligned} \tag{3.3}$$

Because the Shewhart chart uses only the information from the last sample and the successive observation vectors are independent, the conditional probabilities in (3.3) can be expressed as $\sum_{j=1}^{m} P(Z_{k+1}^2 \in E_j | Z_k^2 \in E_i) = \sum_{j=1}^{m} P(Z_{k+1}^2 \in E_j)$.

Then the transition probability p_{ij} is as follows: For i=1, 2, $\cdots$, m,

$$P[Z_i^2 \in E_i] = F(jw) - F[(j-1)w],$$

and

$$p_{ij} = F[(g + (j-m)v] - F[g - (j-m-1)v], (j = m+1, m+2, \cdots, r).$$

And, for $i = m+1, m+2, \cdots, r$,

$$P[Z_i^2 \in E_i] = F[(g + (i-m)v] - F[g - (i-m-1)v]$$

and

$$p_{ij} = F(jw) - F[(j-1)w], (j = 1, 2, \cdots, m).$$

The percentage point of LRT statistic Z_i^2 can be obtained from the chi-square distribution with p degrees of freedom.

4. Properties of CUSUM chart

The CUSUM control chart is a good alternative to the Shewhart control chart when small shifts are important. Vargas *et al.* (2004) presented a comparative study of the performance of CUSUM and EWMA charts in order to detect small changes of process average.

4.1. FSI and VSI CUSUM procedures

For monitoring mean vector of quality variables, the CUSUM statistic can be considered as a function of the sample means. The most direct and obvious FSI CUSUM chart for $\underline{\mu}$ based on Z_i^2 $(i = 1, 2, \cdots)$ can be constructed as

$$Y_i = \max\{Y_{i-1}, 0\} + (Z_i^2 - k), \tag{4.1}$$

where $Y_0 = \omega I_{(\omega \geq 0)}$ and reference value k ($k \geq 0$). This chart signals whenever $Y_i \geq h_C$. And for VSI CUSUM chart, suppose that the two sampling intervals;

$$\begin{aligned} d_1 \text{ is used when} Y_i &\in (g_C, h_C], \\ d_2 \text{ is used when} Y_i &\in (-k, g_C], \end{aligned}$$

where $g_C \leq h_C$ and $d_1 < d_2$.

When the process is in-control or mean vector $\underline{\mu}$ has changed, the performances of the CUSUM chart in (4.1) and the design parameters g_C and h_C can be obtained to satisfy a desired ATS and ANSS by Markov chain approach. And the ANSW values of the VSI chart were obtained by simulation.

4.2. Markov chain method for evaluating ANSS and ATS

To design and evaluate performances of the proposed multivariate CUSUM charts, we use the conditions in section 3.3. From the transition matrix P in (3.2), we can obtain the fundamental matrix M as

$$M = (I - Q)^{-1} = [m_{ij}], \tag{4.2}$$

where m_{ij} is the expected number of visits to the transient state j before absorption, given that the Markov chain starts in transient state i.

For VSI chart, if we use a finite number of interval lengths $d_1, d_2, \cdots, d_\eta$ where $d_1 < d_2 < \cdots < d_\eta$. The region C be partitioned into η regions $C_1, C_2, \cdots, C_\eta$ where C_i is the region in which the interval d_i is used when $Y_j \in C_i$.

Let $\underline{b} = (b_1, b_2, \cdots, b_r)$, $\underline{N} = (N_1, N_2, \cdots, N_r)$ and $\underline{T} = (T_1, T_2, \cdots, T_r)$ are the vectors of sampling interval, NSS and TS, respectively. The ANSS vector is

$$E(\underline{N}) = M\underline{1} \tag{4.3}$$

and

$$V(\underline{N}) = (2M - I) \cdot E(\underline{N}) - [E(\underline{N})]^{(2)}, \tag{4.4}$$

where $[E(\underline{N})]^(2)$ is a vector whose i th component is the square of the i th component of $E(\underline{N})$. Hence when the process starts in state i, the ANSS N_i and the variance $V(N_i)$ is given as

$$E(N_i) = \sum_{j=1}^{r} m_{ij}$$

and

$$V(N_i) = 2\sum_{k=1}^{r}\sum_{j=1}^{r} m_{ik}m_{kj} - \sum_{j=1}^{r} m_{ij} - (\sum_{j=1}^{r} m_{ij})^2$$

Following Reynolds (1988), the ATS vector is

$$E(\underline{T}) = M\underline{b} \tag{4.5}$$

and

$$V(\underline{T}) = MB(2M - I)\underline{b} - [M\underline{b}]^{(2)}, \tag{4.6}$$

where B is a diagonal matrix with elements of corresponding sampling interval and $[M\underline{b}]^{(2)}$ is a vector whose i th component is the square of the i th component of $M\underline{b}$. Hence when the process starts in state i, the ATS T_i and the variancce $V(T_i)$ is given as

$$E(T_i) = \sum_{j=1}^{r} m_{ij}b_j \tag{4.7}$$

and

$$V(T_i) = 2\sum_{k=1}^{r}\sum_{j=1}^{r} m_{ik}m_{kj}b_kb_j - \sum_{j=1}^{r} m_{ij}b_j^2 - (\sum_{j=1}^{r} m_{ij}b_j)^2 \tag{4.8}$$

4.2.1. ANSS for FSI CUSUM chart

Assume that the in-control region $C = (-\infty, h]$ is devided r states such that the interval $(-\infty, 0]$ is 1st state and the interval $(0, h]$ is divided into $(r-1)$ states then $\omega = h/(r-1)$ and we denote $F(\cdot)$ as the distribution function of control statistic.

Then the transition probability p_{ij} in (3.2) is as follows: For $i = 1$,

$$p_{1j} = \begin{cases} F[k] & j = 1 \\ F[(j-1)w + k] - F[(j-2)w + k] & j = 2, 3, \cdots, r. \end{cases}$$

For $i = 2, 3, \cdots, r$,

$$p_{ij} = \begin{cases} F[-(i-\frac{3}{2})w+k] & j=1 \\ F[(j-i+\frac{1}{2})w+k] - F[(j-i-\frac{1}{2})w+k] & j=2,3,\cdots,r. \end{cases}$$

4.2.2. ATS for VSI CUSUM chart

For the two sampling intervals VSI CUSUM chart based on Z_i^2 in (4.1), suppose that this chart signals when $Y_i \in C'$, the sampling interval d_2 is used when $Y_i \in (g, h]$ and d_1 is used when $Y_i \in (-\infty, g]$. Let the interval $(-\infty, \infty)$ be divided into in-control region $C_1 = (-\infty, g]$, $C_2 = (g, h]$ and out-of-control region $C' = (h, \infty)$.

Suppose that states $1, 2, \cdots, m$ used d_2 and states $m+1, \cdots, r$ used d_1. Consider first the case $g > 0$. Then the state 1 corresponds to $Y_i \le 0$ and $\widetilde{Y}_i = 0$. Let $w = g/(m-1)$. Then for $j = 2, 3, \cdots, m$ state j corresponds to $(j-2)w < Y_i \le (j-1)w$ and $\widetilde{Y}_i = (j-3/2)w$. Let $v = (h-g)/(r-m)$. Then for $j = m+1, \cdots, r$, the state j corresponds to $g+(j-m-1)v < Y_i \le g+(j-m)v$ and $\widetilde{Y}_i = g+(j-m-0.5)v$.

The transition probability p_{ij} for Q is as follows : For $i = 1$,

$$p_{1j} = \begin{cases} F(k) & j=1 \\ F[(j-1)w+k] - F[(j-2)w+k] & j=2,\cdots,m \\ F[g+(j-m)v+k] - F[g+(j-m-1)v+k] & j=m+1,\cdots,r. \end{cases}$$

For $i = 2, 3, \cdots, m$,

$$p_{ij} = \begin{cases} F[-(i-\frac{3}{2})w+k] & j=1 \\ F[(j-i+\frac{1}{2})w+k] - F[(j-i-\frac{1}{2})w+k] & j=2,3,\cdots,m \\ F[(m-i+\frac{1}{2})w+(j-m)v+k] & \\ \qquad -F[(m-i+\frac{1}{2})w+(j-m-1)v+k] & j=m+1,\cdots,r. \end{cases}$$

For $i = m+1, m+2, \cdots, r$,

$$p_{ij} = \begin{cases} F[-g-(i-m-\frac{1}{2})v+k] & j=1 \\ F[-(m-j)w-(i-m-\frac{1}{2})v+k] & \\ \qquad -F[-(m-j+1)w-(i-m-\frac{1}{2})v+k] & j=2,3,\cdots,m \\ F[(j-i+\frac{1}{2})v+k] - F[(j-i-\frac{1}{2})v+k] & j=m+1,\cdots,r. \end{cases}$$

For the case $g < 0$ two states are needed for nonpositive values Y_i. State 1 corresponds to $Y_i \leq g$ where d_2 is used, and state 2 corresponds to $g < Y_i \leq 0$ where d_1 is used. Thus $m = 2$. Let $w = h/(r-2)$. Then for state 1

$$p_{1j} = \begin{cases} F(g+k) & j = 1 \\ F(k) - F(g+k) & j = 2 \\ F[(j-2)w+k] - F[(j-3)w+k] & j = 3, \cdots, r. \end{cases}$$

For state 2, $p_{2j} = p_{1j}$ for $j = 1, 2, \cdots, r$.
For $i = 3, \cdots, r$,

$$p_{ij} = \begin{cases} F[g-(i-\frac{5}{2})w+k] & j = 1 \\ F[-(i-\frac{5}{2})w+k] - F[g-(i-\frac{5}{2})w+k] & j = 2 \\ F[(j-i+\frac{1}{2})w+k] - F[(j-i-\frac{1}{2})w+k] & j = 3, 4, \cdots, r. \end{cases}$$

For the case, $g = 0$. only one state corresponding to $Y_i \leq 0$ is needed to use d_2 and thus $m = 1$. Let w be $h/(r-1)$. Then for $i = 1$

$$p_{ij} = \begin{cases} F[-(i-\frac{3}{2})w+k] & j = 1 \\ F[(j-i+\frac{1}{2})w+k] - F[(j-i-\frac{1}{2})w+k] & j = 2, 3, \cdots, r. \end{cases}$$

For $i = 2, 3, \cdots, r$,

$$p_{ij} = \begin{cases} F[-(i-\frac{3}{2})w+k] & j = 1 \\ F[(j-i+\frac{1}{2})w+k] - F[(j-i-\frac{1}{2})w+k] & j = 2, 3, \cdots, r. \end{cases}$$

5. Concluding remarks

In order to evaluate the properties of the proposed charts when the process are in-control or changed, some kinds of standards for comparison are necessary. For simplicity in our computation, we assume that the target mean vector $\underline{\mu}_0 = \underline{0}'$, all diagonal and off-diagonal elements of Σ_0 are 1 and 0.3, respectively. The numerical results were obtained when the ANSS and ATS of the in-control state was approximately equal to 200.0, $d_0 = 1$ and the sample size for each variable was five for $p = 3$.

After the reference value of the proposed CUSUM charts have been determined, the design parameters $h's$ and $g's$ of the CUSUM charts were calculated by Markov chains with the number of transient states $r = 100$. And the ANSS, ATS and ANSW values when the process has changed were also obtained by Markov chains with the number of transient states $r = 100$ or simulation with 10,000 runs.

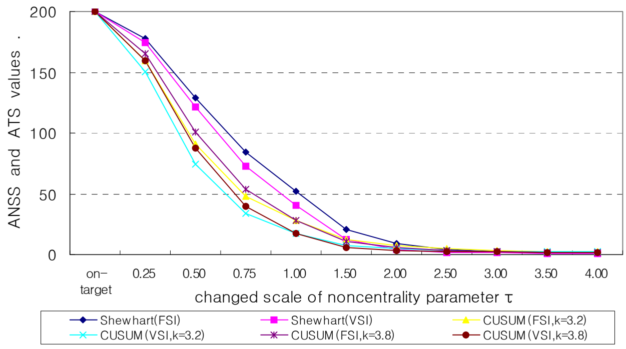

Figure 5.1 Performances of the proposed Shewhart and CUSUM charts (p=3)

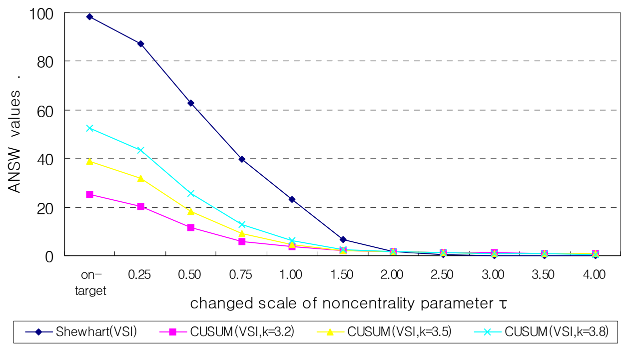

Figure 5.2 ANSW of the proposed Shewhart and CUSUM charts (p=3)

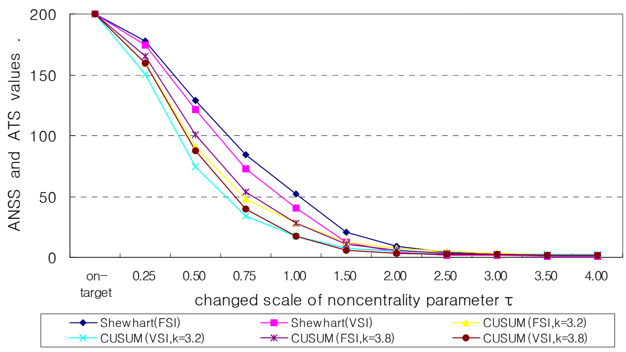

Figure 5.3 ASWR of the proposed Shewhart and CUSUM charts (p=3)

When r is greater than 100 for various p, we found that asymptotic ANSW and ATS using Markov chain method tends to be stabilized.

From the numerical results, we found the following properties. When small or moderate changes in the process have occurred, the ANSW and ASWR for CUSUM procedure is substantially less than those of Shewhart procedure.

The properties and comparison of the proposed procedures are given in Figure 4.1 through Figure 4.3. As illustrated in figures, small reference values are efficient for small shifts from the target value and vice versa in multivariate CUSUM charts in terms of ANSS, ATS and ANSW.

The optimal selection of reference value k in CUSUM procedure depends on the size of the shift in the mean vector to be detected quickly. And, it may be possible to improve the performance of the proposed chart at selected off-target conditions with alternate choices of k.

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