Journal of the Korean Data & Information Science Society
2010, **21**(4), 777–782

한국데이터정보과학회지

A note for hybrid Bollinger bands

Jungsoo Rhee[1]

[1]Department of Mathematics, Pusan University of Foreign Studies

Received 10 May 2010, revised 5 July 2010, accepted 16 July 2010

Abstract

We introduce some techniques to decompose the impulse (the unit sample) into several dilated pieces in the discrete time domain. From the decomposition of the impulse, we obtain localized moving averages. Thus we construct hybrid Bollinger bands that may give various strategies for stock traders. By simulations, we report that more than 94% of stock prices of companies in KOSPI 200 are inside this hybrid Bollinger band.

Keywords: Bollinger bands, impulse, localized moving averages.

1. Introduction

Bollinger bands introduced by John Bollinger (Bollinger, 2002) in the early 1980s are well known in stock markets as a popular technical analysis tool.

Bollinger bands are plotted two standard deviations above and below a 20-day simple moving average. The data used to calculate the standard deviation are the same data as those used for the simple moving average. In essence, Bollinger is using moving standard deviations to plot bands around a moving average (http://www.bollingerbands.com).

The default choice for the average is the simple moving average, but other types of averages can be employed as needed. Exponential moving average or weighted moving average are common second choice (Kim *et al.*, 2008; Park *et al.*, 2009). Also Bollinger bands provide a relative definition of high and low. By definition, prices are high at upper band and low at the lower band. Since the standard deviation is a measure of volatility, the interval between the upper and lower bands and middle band is determined by volatility of stock prices. When the market becomes more volatile, the bands widen and the bands become narrow during less volatile period.

The use of Bollinger bands varies widely among traders. Some traders buy when price touches the lower Bollinger band and exit when price touches the moving average in the center of the bands. Other traders buy when price breaks above the upper Bollinger band or sell when price falls below the lower Bollinger band.

But security prices have no known statistical distribution, normal or otherwise; they are known to have fat tails (Rachev *et al.*, 2005). In the Bollinger bands, the sample size

[1] Associate professor, Department of Mathematics, Pusan University of Foreign Studies, Pusan 608-738, Korea. E-mail: rhee@pufs.ac.kr

typically used, 20, is too small for conclusions derived from statistical techniques like the Central Limit Theorem to be reliable. Such techniques usually require the sample to be independent and identically distributed which is not the case for a time series like security prices. It is incorrect to assume that the percentage of the data outside the Bollinger Bands will always be limited to a certain amount.

Instead of finding about 95% of data inside the bands that would be the expectation with the default parameters if the data were normally distributed, one will typically find less; how much less is a function of the security's volatility. To show theoretically the improvement of the probability of data inside Bollinger bands, a weighted moving average of Black-Scholes stock price model was used in the paper (Liu *et al.*, 2006).

In our paper, replacing the simple moving average by a localized moving average filter, we suggest to make hybrid Bollinger bands. And we find that this method improves the probability of data inside Bollinger bands empirically.

2. Bollinger bands and localized moving averages

Bollinger bands consist of a set of three curves :
Middle Bollinger band being 20-day simple moving average,
Upper Bollinger band being Middle Bollinger band + 2 (20-day standard deviation),
Lower Bollenger band being Middle Bollinger band - 2 (20-day standard deviation).

The general moving average system (Lim, 1990; Oppenheim *et al.*, 1997) is defined by the equation

$$y(n) = \frac{1}{m+M+1}\sum_{j=-m}^{M} x(n-j).$$

This system computes the nth sample of the output sequence as the average of (m+M+1) samples of input sequence around the nth sample. This system is a linear time invariant system, i.e., the moving average is the convolution between $x(n)$ the given data and $h(n)$ the moving average of the impulse. Hence we have

$$y(n) = x(n) * h(n),$$

where $h(n) = \sum_{j=-m}^{M} \delta(n-j)/(m+M+1)$. Here $\delta(n)$ is the impulse that is defined by $\delta(0) = 1,\ \delta(n) = 0,\ n = \pm1, \pm2, ...$

Thus 20-day moving average is defined usually by $\bar{s}(l) = \sum_{j=l-19}^{l} f(j)/20,\ l = 1, 2, ..., p$ for sampled data $f(j)$. we put the standard deviation derived from $\bar{s}(l)$ as

$$\sigma(l) = (\frac{1}{20}\sum_{j=l-19}^{l} (f(j) - \bar{s}(l))^2)^{\frac{1}{2}}.$$

Then we get the upper Bollinger band as $ub(l) = \bar{s}(l) + 2\sigma(l)$ and the lower Bollinger band as $lb(l) = \bar{s}(l) - 2\sigma(l)$, $l = 1, 2, ...$,respectively.

Instead of using the sample mean $\bar{s}$ for Bollinger bands, we'll use a localized moving average as a middle Bollinger band. There are other ways to get a middle Bollinger band fitted to given sample data (Lee and Oh, 2007; Pak, 2003). In order to find a localized moving average, we introduce some interesting recovering theorems.

Theorem 2.1 Let $f \in S(R^n)$, where $S(R^n)$ is the set of Schwarz-class functions on R^n. Suppose that ϕ is a bounded radial function and suppose that ϕ is an integrable function with an admissibility condition satisfying $\int_0^\infty \frac{|\hat{\phi}(r)|}{r} dr < \infty$. Then, for $c_\phi = \int_0^\infty \frac{\hat{\phi}(r)}{r} dr$, we have

$$\int_0^\infty \int_{R^n} f(x-y) r^{n-1} \phi(ry) dy dr = c_\phi f(x),$$

for all $x \in R^n$.

Moreover, if we put $\Psi_0(x) = \int_0^1 r^{n-1}\phi(rx)dr, \Psi_1(x) = \int_1^2 r^{n-1}\phi(rx)dr$ and $\Psi_k(x) = 2^{n(k-1)}\Psi_1(2^{k-1}x)$, $k = 1, 2, ...$, we obtain

$$\sum_{k=0}^{\infty} f * \Psi_k(x) = c_\phi f(x).$$

For the proof, see Rhee's paper (Rhee, 2010). Now we construct a discrete type of the dilated convolution similar to Theorem 2.1 to decompose the impulse.

Theorem 2.2 Suppose that ϕ is a bounded even function that has a support in $[-1, 1]$ of R. Also we suppose that $\int_0^1 \phi(x)dx = 0$ and $\phi(0) = 1$. We set $\Phi_0(x) = \int_0^1 \phi(rx)dr$, $\Phi_1(x) = \int_1^2 \phi(rx)dr$ and $\Phi_k(x) = 2^{k-1}\Phi_1(2^{k-1}x),\ k = 1, 2, 3, ...$. Let T be a sampling period. Let $f(\frac{l}{T})$ be a given sampled data where $l = ... - m, -(m-1), ..., -1, 0, 1, 2, 3, ...p, (p+1), ..., (p+M), ...$ Then we recover $f(\frac{l}{T})$, $l = 1, 2, ..., p$, by the following procedure:

$$f(\frac{l}{T}) = \sum_{k=0}^{n} \sum_{j=l-m}^{l+M} f(\frac{j}{T}) \frac{1}{2^n} \Phi_k(\frac{l-j}{2^n}),\ n = 0, 1, 2, ...$$

For the proof, refer to Rhee's paper (Rhee, 2010). The recovering formula in Theorem 2.2 is a discrete typed version of $\sum_{k=0}^{\infty} f * \Psi_k(x) = c_\phi f(x)$. In this case, the dyadic dilation has an important role to decompose δ the impulse. The impulse is decomposed to be

$$\delta(l) = \sum_{k=0}^{n} \frac{1}{2^n} \Phi_k(\frac{l}{2^n}),\ l = 0, \pm 1, \pm 2, ...$$

by Theorem 2.2.

From the decomposition of the impulse δ, we can construct a localized moving average to be a finite sum of linear combinations of the kernels Φ_k and given data $f(j)$, i.e.,

$$\begin{aligned} m_a(l) &= \sum_{k=s}^{t} \sum_{j=l-m}^{l+M} f(j) \frac{1}{2^n} \Phi_k(\frac{l-j}{2^n}) \\ &= \sum_{j=l-m}^{l+M} \sum_{k=s}^{t} f(j) \frac{1}{2^n} \Phi_k(\frac{l-j}{2^n}),\ 0 \le s \le t \le n. \end{aligned}$$

Actually, m_a is a convolution combining two finite-length sequences. Instead of using 20-day moving average, we can put the middle Bollinger band as $m = m_a$.

3. Application and Bollinger band property

It is known that Black-Scholes stock price has Markov property: given the present, the future is independent of the past. However, the formulation, itself, of the Bollinger bands largely depends on the past. Also, some real stock prices and Black-Scholes model are inside the Bolliger band for more than 94% (Liu *et al.*, 2006). Hence we consider that security data possesses the Bolinger band property if data lie in the Bollinger bands with reasonable ratios. In this section, we show that stock data of KOSPI 200 have the Bollinger band property. One can refer to Table 3.1 and Table 3.2 for the ratios of the Bollinger band property.

In order to get a concrete decomposition of the impulse δ, we take ϕ as follows : let $\phi(x) = (-1)\chi_{[-1,-\frac{1}{2})}(x) + \chi_{[-\frac{1}{2},\frac{1}{2}]}(x) + (-1)\chi_{(\frac{1}{2},1]}(x)$. Then we get $\Phi_0(x) = 1$ on $[-\frac{1}{2}, \frac{1}{2}]$, and $\Phi_0(x) = -1 + \frac{1}{x}$ on $(\frac{1}{2}, 1]$. Also we have $\Phi_0(x) = -1 - \frac{1}{x}$ on $[-1, -\frac{1}{2})$ and $\Phi_0(x) = 0$ for all $|x| > 1$.

Let $n = 5$, $m = 4$ and $M = 0$. Since we have $\Phi_0(\frac{i}{32}) = 1$ for each $i = 0, \pm 1, ..., \pm 15$, we get the first projection,

$$\sum_{j=l-4}^{l} f(\frac{j}{T})\frac{1}{2^n}\Phi_0(\frac{l-j}{2^n}) = \frac{1}{2^5}\sum_{j=0}^{4} f(l-j),\ T = 1.$$

The above expression is a modified version of the moving average of 5 data.

In our example, we let

$$m_i(l) = \sum_{k=0}^{i}\sum_{j=l-m}^{l+M} f(j)\frac{1}{2^5}\Phi_k(\frac{l-j}{2^5}).$$

From now on, we put $m = 19, M = 0$, and we let β_i be the mean of ratios, i.e.,

$$\beta_i = \frac{1}{L}\sum_{l=1}^{L}\frac{m_{n-1}(l)}{m_i(l)},$$

where L is the length of data.

For the realistic implementation of localized moving averages, we replace m_a as $\beta_3 m_3$. Thus we take the middle Bollinger bend as $m(l) = \beta_3 m_3(l)$. Then we get the upper hybrid Bollinger band as

$$hub(l) = \beta_3 m_3(l) + 2\sigma(l)$$

and the lower hybrid Bollinger bands as

$$hlb(l) = \beta_3 m_3(l) - 2\sigma(l),$$

$l = 1, 2, ...,$respectively. Of course, you may take $\beta_2 m_2$ as the middle Bollinger bands. Then we can construct several different types of hybrid Bollinger bands.

In Figure 3.1, we draw hybrid Bollinger bands with $\pm\sigma$ and SAMSUNG C&T price in case of $\beta_2 = 1.318436$. In this graph, we find that data inside hybrid Bollinger bands is over 79% for 120 data from Aug. 31, 2009 to Feb. 22, 2010, where $hub(l) = \beta_2 m_2(l) + \sigma(l)$ and $hlb(l) = \beta_2 m_2(l) - \sigma(l)$. We also investigate that SAMSUNG C&T price inside hybrid Bollinger band with $\pm 2\sigma$ is over 96% with the same β_2. This phenomenon is something

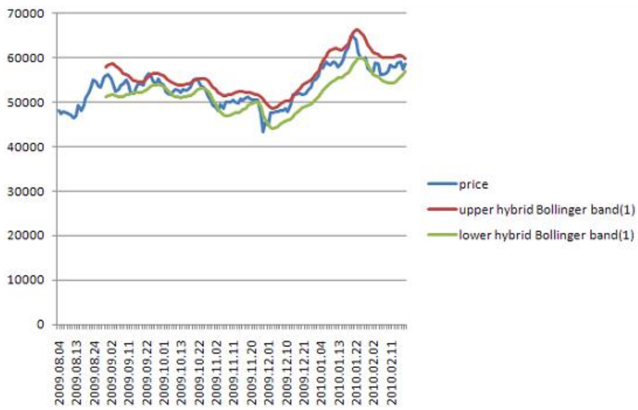

Figure 3.1 SAMSUNG C&T and hybrid bollinger bands

Table 3.1 Ratio of stock prices in KOSPI 200 with β_2

KOSPI 200	β_2	$\pm\sigma$	$\pm2\sigma$
MiraeAsset	1.312	79%	96%
LS Industrial System	1.316	85%	99%
HDS	1.317	78%	99%
Hundai Securities	1.311	77%	97%
Busan Bank	1.318	80%	99%
Woori Investment & Securities	1.313	78%	99%
Hanwha Chemical Corp.	1.320	80%	98%
Daegu Bank	1.316	79%	99%
CJ	1.324	84%	100%
Lotte Confectionary	1.320	79%	93%

different from the analysis of the original 20-day Bollinger bands which is widely used in real stock markets. We can see the remarkable ratios of the hybrid Bollinger bands with $\pm 2\sigma$ in Table 3.1 and Table 3.2 below.

Since the middle Bollinger band describes a trend for the given stock data, we guess that $\beta_2 m_2$ as middle Bollinger band is smoother than $\beta_3 m_3$ from the results of Table 3.1 and Table 3.2. Actually, this conclusion is based on the construction of m_i from Theorem 2.2.

We trace all the daily closing data of stock prices of 120 data of companies in KOSPI 200. Also we attache the specific results of ratios in various Bolinger bands for β_2 and β_3 respectively in Table 3.1 and Table 3.2. When we choose the middle Bollinger band for β_3, we observe that more than 94% of daily closing prices of all the KOSPI 200's companies are falling into hybrid Bollinger bands with $\pm 2\sigma$.

It is natural to ask which bands we should take for more benefits. As one guesses, this is not a simple problem. Actually we design many hybrid Bollinger bands to apply for real stock markets. We may recommend to use hybrid Bollinger bands of β_2 with $\pm\sigma$. In this case, stock traders have more opportunities to sell and to buy stocks than stock traders who choose hybrid Bollinger bands of β_3 with $\pm\sigma$, because many stock prices in hybrid Bollinger bands of β_2 with $\pm\sigma$ break through them in comparison with stock prices in hybrid Bollinger bands of β_3 with $\pm\sigma$ like in Figure 3.1. According to strategies and trading skills of stock traders, one may choose adaptive bands that reflect volatility of stock prices as in the above.

Table 3.2 Ratio of stock prices in KOSPI 200 with β_3

KOSPI 200	β_2	$\pm\sigma$	$\pm 2\sigma$
MiraeAsset	1.198	89%	100%
LS Industrial System	1.199	97%	100%
HDS	1.200	93%	99%
Hundai Securities	1.198	93%	100%
Busan Bank	1.200	89%	99%
Woori Investment & Securities	1.199	92%	100%
Hanwha Chemical Corp.	1.201	89%	99%
Daegu Bank	1.199	93%	100%
CJ	1.202	97%	100%
Lotte Confectionary	1.201	88%	98%

So when we use Bollinger bands to make decision, the default parameters β_i and standard deviations may be adjusted to suit one's purposes.

We inform the readers that the tables and the figure has been simulated and rendered by Microsoft Excel 2007. In these tables, we also find that every stock price of company in KOSPI 200 has good Bollinger band property when σ is in the practical range.

Through the decomposition of the unit sample, we can have lots of hybrid Bollinger bands. Thus these bands are of numerous different types by the selection of Φ_0 in Theorem 2.2.

Acknowlegment

The author thanks the editor and the referees for their valuable comments which have led to an improved presentation of the paper. We also thank for using the summarized data for Bollinger bands in Wikipedia.

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