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Noninformative priors for the common location parameter in half-normal distributions

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Abstract

In this paper, we develop the reference priors for the common location parameter in the half-normal distributions with unequal scale paramters. We derive the reference priors as noninformative prior and prove the propriety of joint posterior distribution under the general prior including the reference priors. Through the simulation study, we show that the proposed reference priors match the target coverage probabilities in a frequentist sense.

Keywords: Half-normal distribution, location parameter, nonregular case, reference prior.

1. Introduction

Consider X and Y are independently distributed random variables according to the half-normal distribution $\mathcal{HN}(\xi, \eta_1)$ with the location parameter ξ and the scale parameter η_1, and the half-normal distribution $\mathcal{HN}(\xi, \eta_2)$ with the location parameter ξ and the scale parameter η_2. Then the half-normal distributions of X and Y are given by

$$f(x|\xi, \eta_1) = \sqrt{\frac{2}{\pi}} \frac{1}{\eta_1} \exp\left\{ -\frac{1}{2\eta_1^2}(x-\xi)^2 \right\}, x \geq \xi, -\infty < \xi < \infty, \eta_1 > 0, \tag{1.1}$$

and

$$f(y|\xi, \eta_2) = \sqrt{\frac{2}{\pi}} \frac{1}{\eta_2} \exp\left\{ -\frac{1}{2\eta_2^2}(x-\xi)^2 \right\}, y \geq \xi, -\infty < \xi < \infty, \eta_2 > 0, \tag{1.2}$$

respectively. Here the parameter ξ is the common location parameter. The present paper focuses on the reference priors for the common location parameter.

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This common parameter plays an important role in this model, because this parameter is a threshold value and also represents a location of the distribution. We want to develop noninformative priors of this parameter for Bayesian inferences. Once reference priors for common location parameter are developed, one can use this prior for analyzing data based on objective Bayesian inferences.

This half-normal distribution has been used as a model for truncated data from application areas as diverse as fibre buckling (Haberle, 1991, Johnson *et al.*, 1994), blowfly dispersion (Dobzhansky and Wright, 1943), sports science physiology (Pewsey, 2002, 2004) and stochastic frontier modeling (Aigner *et al.*, 1977; Meeusen and van den Broeck, 1977). Likelihood based inference for the half-normal distribution has been considered by Pewsey (2002, 2004). Wiper *et al.* (2008) gave Bayesian inference for the half-normal using conjugate prior. They show that a generalized version of the normal-gamma distribution is conjugate to the half-normal likelihood. Most of the Bayesian inferences related with half-normal distribution used subjective priors. Though there is an inferential convenience to use conjugate or subjective priors in Bayesian analysis, but it is too subjective. So there is a strong necessity for developing an objective priors.

In recent years, the notion of a noninformative prior has attracted much attention. There are different notions of noninformative prior, the reference prior approach of Bernardo (1979), which extended by Berger and Beranrdo (1989, 1992), and the approach of matching the posterior and frequentist probabilities of confidence intervals. Ghosh and Mukerjee (1992), and Berger and Bernardo (1992) gave a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. The matching idea goes back to Welch and Peers (1963). Interest in such priors revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may any cite work of DiCiccio and Stern (1994), Datta and Ghosh (1995), Datta (1996), Kang *et al.* (2008), Kim *et al.* (2009a), Kim *et al.* (2009 b) and Mukerjee and Ghosh (1997). Although this matching can be justified only asymptotically, simulation results in many works indicate that this is indeed achieved for small or moderate sizes as well. Quite often reference prior is the probability matching prior.

Most of the works about objective priors mentioned above are developed under the regular families of distributions. However, nonregular families, such as the uniform, shifted exponential or half-normal, are also important in many practical problems. The method developed for regular families of distribution can not be applied to nonregular distributions. In developing reference priors, an asymptotic expansion of the posterior density is required. The expansion of the posterior in nonregular case is quite different from regular case. Through this expansion, reference priors are obtained by maximizing the expected information from the data.

Ghosal and Smanta (1997) developed the reference priors for the case of one parameter families of discontinuous densities. Ghosal (1997) derived the reference priors for the multiparameter nonregular cases that the family of densities have discontinuities at some points which depend on one component of the parameter, while the family is regular with respect to the other parameters. Also Ghosal (1999) developed the probability matching prior for one parameter and two parameter cases under nonregular families.

The outline of the remaining sections is as follows. In Section 2, we develop reference priors for the common location parameter. In Section 3, we provide that the propriety of the posterior distribution for the general prior including the reference priors. In Section 4, simulated frequentist coverage probabilities under the derived priors are given.

2. The reference priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. Ghosal (1997) derived the reference prior in sense of Bernardo (1979) for multiparameter nonregular cases. In this section, we derive the reference priors for different groups of orderings of (ξ, η_1, η_2) by following Ghosal (1997).

Let $X_i, i = 1, \cdots, n_1$ denote observations from the half-normal distribution $\mathcal{HN}(\xi, \eta_1)$, and $Y_i, i = 1, \cdots, n_2$ denote observations from the half-normal distribution $\mathcal{HN}(\xi, \eta_2)$. Then likelihood function is given by

$$f(\mathbf{x}, \mathbf{y}|\xi, \eta_1, \eta_2) = \left(\frac{2}{\pi}\right)^{\frac{n_1+n_2}{2}} \eta_1^{-n_1}\eta_2^{-n_2} \exp\left\{-\frac{\sum_{i=1}^{n_1}(x_i-\xi)^2}{2\eta_1^2} - \frac{\sum_{i=1}^{n_2}(y_i-\xi)^2}{2\eta_2^2}\right\}, \quad (2.1)$$

where $-\infty < \xi < \infty$, $\eta_1 > 0$ and $\eta_2 > 0$.

We firstly derived the reference prior when ξ is parameter of interest. The reference prior is developed by considering a sequence of compact subsets of the parameter space, and taking the limit of a sequence of priors as these compact subsets fill out of the parameter space. The compact subsets were taken to be Cartesian products of sets of the form

$$\eta_1 \in [a_1, b_1], \eta_2 \in [a_2, b_2].$$

In the limit a_1, a_2 will tend to 0 and b_1, b_2 will tend to ∞. Here, and below, a subscripted Q denotes a function that is constant and does not depend on any parameter but any Q may depend on the ranges of the parameters.

From the likelihood function (2.1), the matrix $F(\xi, \eta_1, \eta_2)$ is given by

$$F(\xi, \eta_1, \eta_2) = Diag\left\{2n_1\eta_1^{-2}, 2n_2\eta_2^{-2}\right\},$$

where $F(\xi, \eta_1, \eta_2) = \{4J_{jk}(\xi, \eta_1, \eta_2)\}, j, k = 1, 2,$

$$J_{jk}(\xi, \eta_1, \eta_2) = \int\int g_{\eta_j}(\mathbf{x}, \mathbf{y}; \xi, \eta_1, \eta_2) g_{\eta_k}(\mathbf{x}, \mathbf{y}; \xi, \eta_1, \eta_2) d\mathbf{x}d\mathbf{y},$$

$g_{\eta_j} = \partial g/\partial \eta_j$ and $g = f^{\frac{1}{2}}$. Thus the reference prior for (η_1, η_2) given ξ is

$$\begin{aligned}\pi(\eta_1, \eta_2|\xi) &= [\det F(\xi, \eta_1, \eta_2)]^{\frac{1}{2}} \\ &= 2n_1^{\frac{1}{2}} n_2^{\frac{1}{2}} \eta_1^{-1}\eta_2^{-1}. \end{aligned} \quad (2.2)$$

The normalizing constant $K_l(\xi)$ of the reference prior $\pi(\eta_1, \eta_2|\xi)$ is given by

$$\begin{aligned} K_l(\xi) &= \left(\int_{a_2}^{b_2}\int_{a_1}^{b_1} [\det F(\xi, \eta_1, \eta_2)]^{\frac{1}{2}} d\eta_1 d\eta_2\right)^{-1} \\ &= \left(\int_{a_2}^{b_2}\int_{a_1}^{b_1} 2n_1^{\frac{1}{2}} n_2^{\frac{1}{2}} \eta_1^{-1}\eta_2^{-1} d\eta_1 d\eta_2\right)^{-1} \\ &= 2^{-1} n_1^{-\frac{1}{2}} n_2^{-\frac{1}{2}} [\log(b_1/a_1)\log(b_2/a_2)]^{-1}, \end{aligned} \quad (2.3)$$

and so we obatain

$$p_l(\eta_1,\eta_2|\xi) = K_l(\xi)\pi(\eta_1,\eta_2|\xi) = [\log(b_1/a_1)\log(b_2/a_2)]^{-1}\eta_1^{-1}\eta_2^{-1}. \tag{2.4}$$

Thus the marginal reference prior for ξ is given by

$$\pi_l(\xi) = \exp\left\{\int_{a_2}^{b_2}\int_{a_1}^{b_1} p_l(\eta_1,\eta_2|\xi)\log c(\xi,\eta_1,\eta_2)d\eta_1 d\eta_2\right\} = Q(a_1,b_1,a_2,b_2), \tag{2.5}$$

where $c(\xi,\eta_1,\eta_2) = E_{\xi,\eta_1,\eta_2}[\partial\log f/\partial\xi] = \sqrt{\frac{2}{\pi}}(n_1\eta_1^{-1}+n_2\eta_2^{-1})$. Therefore the reference prior for (ξ,η_1,η_2), when ξ is parameter of interest, is given by

$$\begin{aligned}\pi_1(\xi,\eta_1,\eta_2) &= \lim_{l\to\infty}\left[\frac{K_l(\xi)\pi_l(\xi)}{K_l(\xi_0)\pi_l(\xi_0)}\right]\pi(\eta_1,\eta_2|\xi)\\ &\propto \eta_1^{-1}\eta_2^{-1},\end{aligned} \tag{2.6}$$

where ξ_0 is a fixed point. Also when both ξ and (η_1,η_2) are parameters of interest, the reference prior for (ξ,η_1,η_2) is given by

$$\begin{aligned}\pi_2(\xi,\eta_1,\eta_2) &= c(\xi,\eta_1,\eta_2)[\det F(\xi,\eta_1,\eta_2)]^{\frac{1}{2}}\\ &\propto \eta_1^{-1}\eta_2^{-1}(n_1\eta_1^{-1}+n_2\eta_2^{-1}).\end{aligned} \tag{2.7}$$

When ξ is parameter of interest, the reference prior for (ξ,η_1,η_2) based on an appropriate penalty term of Ghosh and Mukerjee (1992) (and also see Ghosal, 1997) is given by

$$\pi_3(\xi,\eta_1,\eta_2) = c(\xi,\eta_1,\eta_2) = (n_1\eta_1^{-1}+n_2\eta_2^{-1}). \tag{2.8}$$

3. Implementation of the Bayesian procedure

We investigate the propriety of posteriors for a general class of priors which include the reference priors (2.6), (2.7) and (2.8). We consider the class of priors

$$\pi_g(\xi,\eta_1,\eta_2) \propto \eta_1^{-a}\eta_2^{-b}(n_1\eta_1^{-1}+n_2\eta_2^{-1})^c. \tag{3.1}$$

where $a \geq 0, b \geq 0$ and $c \geq 0$. The following general theorem can be proved.

Theorem 3.1 The posterior distribution of (ξ,η_1,η_2) under the general prior (3.1) is proper if $n_1+a-2>0, n_2+b-1>0$ or $n_1+a-1>0, n_2+b-2>0$.

Proof: Under the general prior (3.1), the joint posterior for ξ,η_1,η_2 given $\mathbf{x}$ and $\mathbf{y}$ is

$$\begin{aligned}\pi(\xi,\eta_1,\eta_2|\mathbf{x},\mathbf{y}) &\propto \eta_1^{-n_1-a}\eta_2^{-n_2-b}(n_1\eta_1^{-1}+n_2\eta_2^{-1})^c\\ &\quad\times\exp\left\{-\frac{s_1^2+n_1(\bar{x}-\xi)^2}{2\eta_1^2}-\frac{s_2^2+n_2(\bar{y}-\xi)^2}{2\eta_2^2}\right\},\end{aligned} \tag{3.2}$$

where $\bar{x} = \sum_{i=1}^{n_1} x_i/n_1$, $\bar{y} = \sum_{i=1}^{n_2} y_i/n_2$, $s_1^2 = \sum_{i=1}^{n_1}(x_i - \bar{x})^2$ and $s_2^2 = \sum_{i=1}^{n_2}(y_i - \bar{y})^2$. Then integrating with respect to ξ in (3.2), we have the posterior

$$\begin{aligned}\pi(\eta_1, \eta_2|\mathbf{x}, \mathbf{y}) \propto & \left(1 - \text{Erf}\left[\left(\frac{\frac{n_1}{2\eta_1^2}\bar{x} + \frac{n_2}{2\eta_2^2}\bar{y}}{\frac{n_1}{2\eta_1^2} + \frac{n_2}{2\eta_2^2}} - z\right)\left(\frac{n_1}{2\eta_1^2} + \frac{n_2}{2\eta_2^2}\right)^{\frac{1}{2}}\right]\right) \\ & \times \eta_1^{-n_1-a-c+1}\eta_2^{-n_2-b-c+1}(n_1\eta_2 + n_2\eta_1)^c \left(n_1\eta_2^2 + n_2\eta_1^2\right)^{-\frac{1}{2}} \\ & \times \exp\left\{-\frac{s_1^2}{2\eta_1^2} - \frac{s_2^2}{2\eta_2^2} - \frac{n_1 n_2}{2(n_1\eta_2^2 + n_2\eta_1^2)}(\bar{x} - \bar{y})^2\right\},\end{aligned} \tag{3.3}$$

where $z = min\{x_1, \cdots, x_{n_1}, y_1, \cdots, y_{n_2}\}$ and $\text{Erf}(w) = \frac{2}{\sqrt{\pi}}\int_0^w e^{-t^2}dt$. From the posterior density (3.3), we obtain the following inequality.

$$\begin{aligned}\pi(\eta_1, \eta_2|\mathbf{x}, \mathbf{y}) & \leq k_1\eta_1^{-n_1-a-c+1}\eta_2^{-n_2-b-c}(n_1\eta_1 + n_2\eta_2)^c \exp\left\{-\frac{s_1^2}{2\eta_1^2} - \frac{s_2^2}{2\eta_2^2}\right\} \\ & \equiv \pi'(\eta_1, \eta_2|\mathbf{x}, \mathbf{y}),\end{aligned} \tag{3.4}$$

where k_1 is a constant. We will prove the propriety of the function (3.4). For $0 < \eta_1 < 1$ and $0 < \eta_2 < 1$,

$$\pi'(\eta_1, \eta_2|\mathbf{x}, \mathbf{y}) \leq k_1(n_1 + n_2)^c\eta_1^{-n_1-a-c+1}\eta_2^{-n_2-b-c} \exp\left\{-\frac{s_1^2}{2\eta_1^2} - \frac{s_2^2}{2\eta_2^2}\right\}. \tag{3.5}$$

Thus the function (3.4) is proper if $n_1 + a + c - 2 > 0, n_2 + b + c - 1 > 0$. For $0 < \eta_1 < 1$ and $\eta_2 \geq 1$,

$$\pi'(\eta_1, \eta_2|\mathbf{x}, \mathbf{y}) \leq k_1(n_1 + n_2)^c\eta_1^{-n_1-a-c+1}\eta_2^{-n_2-b} \exp\left\{-\frac{s_1^2}{2\eta_1^2} - \frac{s_2^2}{2\eta_2^2}\right\}. \tag{3.6}$$

Thus the function (3.4) is proper if $n_1 + a + c - 2 > 0, n_2 + b - 1 > 0$. For $\eta_1 \geq 1$ and $0 < \eta_2 < 1$,

$$\pi'(\eta_1, \eta_2|\mathbf{x}, \mathbf{y}) \leq k_1(n_1 + n_2)^c\eta_1^{-n_1-a+1}\eta_2^{-n_2-b-c} \exp\left\{-\frac{s_1^2}{2\eta_1^2} - \frac{s_2^2}{2\eta_2^2}\right\}. \tag{3.7}$$

Thus the function (3.4) is proper if $n_1 + a - 2 > 0, n_2 + b + c - 1 > 0$. For $\eta_1 \geq 1$ and $\eta_2 \geq 1$,

$$\pi'(\eta_1, \eta_2|\mathbf{x}, \mathbf{y}) \leq k_1(n_1 + n_2)^c\eta_1^{-n_1-a+1}\eta_2^{-n_2-b} \exp\left\{-\frac{s_1^2}{2\eta_1^2} - \frac{s_2^2}{2\eta_2^2}\right\}. \tag{3.8}$$

Thus the function (3.4) is proper if $n_1 + a - 2 > 0, n_2 + b - 1 > 0$. This completes the proof. □

Theorem 3.2 Under the reference prior π_1, the marginal posterior density of ξ is given by

$$\pi(\xi|\mathbf{x}, \mathbf{y}) \propto \left[s_1^2 + n_1(\bar{x} - \xi)^2\right]^{-\frac{n_1}{2}} \left[s_2^2 + n_2(\bar{y} - \xi)^2\right]^{-\frac{n_2}{2}}. \tag{3.9}$$

Under the reference prior π_2, the marginal posterior density of ξ is given by

$$\pi(\xi|\mathbf{x},\mathbf{y}) \propto n_1 \left[s_1^2 + n_1(\bar{x}-\xi)^2\right]^{-\frac{n_1+1}{2}} \left[s_2^2 + n_2(\bar{y}-\xi)^2\right]^{-\frac{n_2}{2}} + n_2 \left[s_1^2 + n_1(\bar{x}-\xi)^2\right]^{-\frac{n_1}{2}} \left[s_2^2 + n_2(\bar{y}-\xi)^2\right]^{-\frac{n_2+1}{2}}. \quad (3.10)$$

Under the reference prior π_3, the marginal posterior density of ξ is given by

$$\pi(\xi|\mathbf{x},\mathbf{y}) \propto n_1 \left[s_1^2 + n_1(\bar{x}-\xi)^2\right]^{-\frac{n_1}{2}} \left[s_2^2 + n_2(\bar{y}-\xi)^2\right]^{-\frac{n_2-1}{2}} + n_2 \left[s_1^2 + n_1(\bar{x}-\xi)^2\right]^{-\frac{n_1-1}{2}} \left[s_2^2 + n_2(\bar{y}-\xi)^2\right]^{-\frac{n_2}{2}}. \quad (3.11)$$

Note that normalizing constant for the marginal density of β requires an one dimensional integration. Therefore we can have the marginal posterior density of β and so we compute the marginal moment of β. In Section 4, we investigate the frequentist coverage probabilities for the reference priors π_1, π_2 and π_3, respectively.

4. Numerical study

We investigate the frequentist coverage probability by investigating the credible interval of the marginal posteriors density of ξ under the noninformative prior π given in Section 3 for several configurations (ξ, η_1, η_2) and (n_1, n_2). That is to say, the frequentist coverage of a $100(1-\alpha)\%th$ posterior quantile should be close to $1-\alpha$. This is done numerically. Table 4.1 gives numerical values of the frequentist coverage probabilities of 0.05 (0.95) posterior quantiles for the proposed priors. The computation of these numerical values is based on the following algorithm for any fixed true (ξ, η_1, η_2) and any prespecified value α. Here α is 0.05 (0.95). Let $\xi^\pi(\alpha|\mathbf{x},\mathbf{y})$ be the posterior α-quantile of ξ given $\mathbf{x}$ and $\mathbf{y}$. That is to say, $F(\xi^\pi(\alpha|\mathbf{x},\mathbf{y})|\mathbf{x},\mathbf{y}) = \alpha$, where $F(\cdot|\mathbf{x},\mathbf{y})$ is the marginal posterior distribution of ξ. Then the frequentist coverage probability of this one sided credible interval of ξ is

$$P_{(\xi,\eta_1,\eta_2)}(\alpha;\xi) = P_{(\xi,\eta_1,\eta_2)}(0 < \xi < \xi^\pi(\alpha|\mathbf{x},\mathbf{y})). \quad (4.1)$$

The estimated $P_{(\xi,\eta_1,\eta_2)}(\alpha;\xi)$ when $\alpha = 0.05(0.95)$ is shown in Tables 4.1. In particular, for fixed (ξ, η_1, η_2), we take 10,000 independent random samples of $\mathbf{X}$ and $\mathbf{Y}$ from the model (2.1).

For the cases presented in Table 4.1, we see that the reference prior π_1 and π_3 matches the target coverage probability much more accurately than the reference prior π_2 for values of (ξ, η_1, η_2) and values of (n_1, n_2). In particular, the reference prior π_3 meet very well the target coverage probabilities in small samples. Note that the reference prior π_1 is the prior when β is parameter of interest, and the results of tables are not much sensitive to change of the values of (η_1, η_2). Thus we recommend to use the reference priors π_1 and π_3 in the sense of asymptotic frequentist coverage property.

5. Concluding remarks

In the half-normal distributions, we have found reference priors for the location parameter. We derived the reference priors when ξ is parameter of interest, and both ξ and (η_1, η_2) are parameters of interest. We showed that the reference priors π_1 and π_3 perform better than the reference prior π_2 in matching the target coverage probabilities. Thus we recommend the use of the reference priors π_1 and π_3 for the Bayesian inference in two independent half-normal distributions with common locattion parameter.

Table 4.1 Frequentist Coverage Probabilities of 0.05 (0.95) Posterior Quantiles for ξ

			ξ=0.1			ξ=1.0	
η_1,η_2	(n_1,n_2)	π_1	π_2	π_3	π_1	π_2	π_3
1,1	5,5	0.063 (0.954)	0.085 (0.959)	0.046 (0.950)	0.072 (0.959)	0.094 (0.962)	0.052 (0.954)
	5,10	0.059 (0.950)	0.074 (0.954)	0.049 (0.947)	0.062 (0.954)	0.074 (0.956)	0.051 (0.951)
	10,10	0.060 (0.954)	0.068 (0.956)	0.052 (0.951)	0.062 (0.955)	0.072 (0.957)	0.055 (0.952)
	10,15	0.051 (0.953)	0.057 (0.954)	0.046 (0.951)	0.052 (0.950)	0.058 (0.953)	0.047 (0.949)
1,5	5,5	0.058 (0.954)	0.083 (0.961)	0.048 (0.951)	0.059 (0.950)	0.089 (0.956)	0.050 (0.947)
	5,10	0.063 (0.953)	0.087 (0.958)	0.052 (0.951)	0.058 (0.954)	0.084 (0.960)	0.048 (0.952)
	10,10	0.053 (0.958)	0.065 (0.961)	0.049 (0.957)	0.056 (0.955)	0.070 (0.958)	0.051 (0.953)
	10,15	0.056 (0.949)	0.067 (0.953)	0.052 (0.948)	0.056 (0.948)	0.070 (0.950)	0.051 (0.946)
1,10	5,5	0.056 (0.951)	0.090 (0.959)	0.050 (0.949)	0.056 (0.949)	0.089 (0.957)	0.051 (0.947)
	5,10	0.059 (0.955)	0.090 (0.960)	0.052 (0.952)	0.059 (0.952)	0.087 (0.958)	0.052 (0.951)
	10,10	0.057 (0.952)	0.069 (0.956)	0.055 (0.952)	0.054 (0.953)	0.071 (0.956)	0.053 (0.951)
	10,15	0.054 (0.954)	0.067 (0.958)	0.051 (0.953)	0.049 (0.949)	0.063 (0.953)	0.047 (0.949)
5,1	5,5	0.060 (0.951)	0.088 (0.958)	0.052 (0.947)	0.063 (0.954)	0.092 (0.960)	0.051 (0.949)
	5,10	0.052 (0.953)	0.066 (0.956)	0.047 (0.951)	0.054 (0.949)	0.068 (0.952)	0.049 (0.947)
	10,10	0.052 (0.949)	0.064 (0.952)	0.048 (0.948)	0.054 (0.948)	0.066 (0.951)	0.050 (0.946)
	10,15	0.054 (0.953)	0.063 (0.955)	0.052 (0.952)	0.053 (0.952)	0.061 (0.953)	0.049 (0.951)
5,5	5,5	0.067 (0.958)	0.087 (0.963)	0.048 (0.954)	0.070 (0.953)	0.091 (0.957)	0.051 (0.947)
	5,10	0.060 (0.952)	0.075 (0.956)	0.050 (0.949)	0.064 (0.955)	0.077 (0.958)	0.053 (0.951)
	10,10	0.062 (0.953)	0.069 (0.955)	0.055 (0.951)	0.059 (0.951)	0.068 (0.954)	0.049 (0.948)
	10,15	0.059 (0.952)	0.066 (0.954)	0.052 (0.950)	0.061 (0.951)	0.069 (0.953)	0.054 (0.950)
5,10	5,5	0.070 (0.954)	0.093 (0.959)	0.053 (0.950)	0.067 (0.954)	0.095 (0.959)	0.052 (0.949)
	5,10	0.063 (0.953)	0.079 (0.956)	0.051 (0.950)	0.062 (0.954)	0.080 (0.957)	0.050 (0.951)
	10,10	0.057 (0.951)	0.067 (0.954)	0.049 (0.950)	0.059 (0.953)	0.070 (0.956)	0.053 (0.951)
	10,15	0.053 (0.952)	0.062 (0.955)	0.047 (0.950)	0.057 (0.953)	0.065 (0.954)	0.051 (0.951)
10,1	5,5	0.054 (0.949)	0.090 (0.957)	0.048 (0.947)	0.055 (0.955)	0.093 (0.960)	0.051 (0.953)
	5,10	0.052 (0.951)	0.069 (0.954)	0.049 (0.950)	0.051 (0.956)	0.066 (0.958)	0.048 (0.954)
	10,10	0.050 (0.953)	0.065 (0.957)	0.048 (0.952)	0.052 (0.952)	0.066 (0.956)	0.048 (0.951)
	10,15	0.052 (0.948)	0.063 (0.952)	0.050 (0.948)	0.052 (0.953)	0.063 (0.955)	0.051 (0.952)
10,5	5,5	0.067 (0.957)	0.091 (0.961)	0.050 (0.952)	0.067 (0.953)	0.090 (0.958)	0.051 (0.949)
	5,10	0.059 (0.951)	0.070 (0.954)	0.051 (0.949)	0.058 (0.953)	0.070 (0.957)	0.048 (0.950)
	10,10	0.058 (0.953)	0.066 (0.955)	0.050 (0.951)	0.060 (0.952)	0.069 (0.954)	0.052 (0.950)
	10,15	0.053 (0.948)	0.059 (0.951)	0.049 (0.947)	0.056 (0.953)	0.064 (0.955)	0.050 (0.950)
10,10	5,5	0.063 (0.956)	0.085 (0.960)	0.047 (0.949)	0.062 (0.954)	0.082 (0.960)	0.045 (0.949)
	5,10	0.063 (0.955)	0.075 (0.958)	0.052 (0.952)	0.067 (0.953)	0.081 (0.957)	0.054 (0.950)
	10,10	0.058 (0.952)	0.067 (0.955)	0.049 (0.949)	0.057 (0.954)	0.067 (0.957)	0.049 (0.951)
	10,15	0.057 (0.955)	0.064 (0.956)	0.050 (0.952)	0.061 (0.947)	0.068 (0.949)	0.053 (0.945)
			ξ=5.0			ξ=10.0	
η_1,η_2	(n_1,n_2)	π_1	π_2	π_3	π_1	π_2	π_3
1,1	5,5	0.068 (0.951)	0.089 (0.955)	0.053 (0.946)	0.072 (0.955)	0.091 (0.960)	0.053 (0.950)
	5,10	0.060 (0.950)	0.074 (0.954)	0.049 (0.945)	0.061 (0.953)	0.074 (0.955)	0.051 (0.949)
	10,10	0.060 (0.954)	0.068 (0.956)	0.052 (0.952)	0.058 (0.953)	0.068 (0.954)	0.048 (0.950)
	10,15	0.062 (0.951)	0.068 (0.953)	0.054 (0.949)	0.059 (0.952)	0.065 (0.955)	0.054 (0.950)
1,5	5,5	0.061 (0.952)	0.092 (0.958)	0.051 (0.949)	0.056 (0.957)	0.086 (0.964)	0.047 (0.953)
	5,10	0.060 (0.954)	0.084 (0.957)	0.052 (0.950)	0.062 (0.953)	0.086 (0.958)	0.053 (0.950)
	10,10	0.053 (0.950)	0.067 (0.954)	0.049 (0.949)	0.051 (0.948)	0.065 (0.952)	0.048 (0.947)
	10,15	0.055 (0.952)	0.066 (0.954)	0.051 (0.950)	0.054 (0.948)	0.064 (0.952)	0.051 (0.947)
1,10	5,5	0.058 (0.953)	0.093 (0.960)	0.052 (0.952)	0.054 (0.953)	0.088 (0.960)	0.049 (0.952)
	5,10	0.055 (0.950)	0.090 (0.956)	0.049 (0.947)	0.057 (0.947)	0.088 (0.953)	0.051 (0.945)
	10,10	0.057 (0.952)	0.073 (0.955)	0.054 (0.951)	0.049 (0.948)	0.062 (0.952)	0.047 (0.947)
	10,15	0.051 (0.950)	0.064 (0.954)	0.048 (0.949)	0.052 (0.951)	0.064 (0.954)	0.050 (0.950)
5,1	5,5	0.059 (0.952)	0.089 (0.958)	0.049 (0.948)	0.059 (0.951)	0.090 (0.957)	0.049 (0.949)
	5,10	0.054 (0.954)	0.069 (0.957)	0.050 (0.952)	0.048 (0.954)	0.062 (0.956)	0.045 (0.953)
	10,10	0.056 (0.954)	0.070 (0.958)	0.053 (0.953)	0.055 (0.953)	0.067 (0.956)	0.051 (0.952)
	10,15	0.055 (0.952)	0.064 (0.954)	0.051 (0.951)	0.051 (0.953)	0.059 (0.955)	0.048 (0.952)
5,5	5,5	0.072 (0.955)	0.092 (0.960)	0.055 (0.951)	0.067 (0.953)	0.088 (0.958)	0.050 (0.949)
	5,10	0.059 (0.953)	0.070 (0.957)	0.049 (0.949)	0.064 (0.951)	0.077 (0.954)	0.053 (0.949)
	10,10	0.057 (0.955)	0.067 (0.957)	0.049 (0.952)	0.059 (0.952)	0.068 (0.955)	0.050 (0.950)
	10,15	0.056 (0.955)	0.063 (0.956)	0.048 (0.952)	0.055 (0.953)	0.062 (0.955)	0.047 (0.951)
5,10	5,5	0.064 (0.959)	0.086 (0.964)	0.049 (0.955)	0.062 (0.955)	0.086 (0.960)	0.050 (0.950)
	5,10	0.069 (0.956)	0.086 (0.961)	0.055 (0.953)	0.063 (0.953)	0.080 (0.956)	0.051 (0.950)
	10,10	0.051 (0.953)	0.062 (0.955)	0.043 (0.951)	0.058 (0.952)	0.069 (0.955)	0.051 (0.949)
	10,15	0.053 (0.952)	0.061 (0.954)	0.048 (0.950)	0.051 (0.951)	0.059 (0.953)	0.045 (0.948)
10,1	5,5	0.052 (0.950)	0.085 (0.958)	0.045 (0.948)	0.053 (0.951)	0.085 (0.958)	0.047 (0.949)
	5,10	0.054 (0.949)	0.068 (0.953)	0.051 (0.948)	0.054 (0.954)	0.071 (0.958)	0.051 (0.952)
	10,10	0.055 (0.950)	0.069 (0.954)	0.053 (0.950)	0.053 (0.952)	0.067 (0.957)	0.051 (0.951)
	10,15	0.053 (0.949)	0.064 (0.952)	0.052 (0.948)	0.050 (0.952)	0.059 (0.955)	0.049 (0.951)
10,5	5,5	0.068 (0.954)	0.094 (0.959)	0.053 (0.950)	0.065 (0.957)	0.090 (0.962)	0.049 (0.952)
	5,10	0.058 (0.950)	0.070 (0.953)	0.050 (0.948)	0.055 (0.951)	0.068 (0.954)	0.045 (0.948)
	10,10	0.054 (0.952)	0.065 (0.955)	0.048 (0.949)	0.057 (0.952)	0.067 (0.955)	0.050 (0.950)
	10,15	0.057 (0.954)	0.064 (0.956)	0.052 (0.953)	0.053 (0.945)	0.061 (0.947)	0.047 (0.944)
10,10	5,5	0.071 (0.956)	0.092 (0.959)	0.052 (0.951)	0.069 (0.956)	0.090 (0.961)	0.050 (0.951)
	5,10	0.064 (0.955)	0.076 (0.957)	0.051 (0.952)	0.064 (0.953)	0.076 (0.955)	0.054 (0.949)
	10,10	0.059 (0.954)	0.068 (0.956)	0.050 (0.951)	0.058 (0.954)	0.067 (0.956)	0.051 (0.952)
	10,15	0.056 (0.952)	0.062 (0.954)	0.050 (0.950)	0.056 (0.950)	0.064 (0.953)	0.049 (0.948)

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