

# Characteristic Analysis for IPMSM Considering Flux-Linkage Ripple

Dong-Kyun Woo<sup>†</sup>, Sang-Yeop Kwak\*, Jang-Ho Seo\* and Hyun-Kyo Jung\*

**Abstract** – In a multi-layer interior permanent magnet synchronous motor, the  $d$ - and  $q$ -axis parameters vary nonlinearly according to different load conditions, consequently changing the level of saturation. The flux-linkage of  $d$ - and  $q$ -axis conveys ripple characteristics resulting from mechanical structure and degree of magnetic saturation. If the calculated flux-linkage is correct, the torque using the Maxwell stress tensor method is the same torque calculated by the flux-linkage. However, discrepancy between results exists. In this paper, the  $d$ - and  $q$ -axis flux-linkage, in consideration of the ripple characteristic, is calculated. Simulation results are then compared with experimental results.

**Keywords:** Flux-linkage, IPMSM, Motor, Parameter, Ripple

## 1. Introduction

The interior permanent magnet synchronous motor (IPMSM) has many advantages, such as high power density and wide speed range, due to its salient rotor structure [1]. The multi-layer IPMSM adopting a multi-layer structure in rotor offers additional reluctance torque given its large saliency. The multi-layer IPMSM can also provide additional improved characteristics, such as high starting torque and constant power operation, in the wide speed range by using field weakening control [2], [3]. However, in order to control this device, predicting the accurate machine parameters in the design stage is necessary [4].

In the multi-layer IPMSM,  $d$ - and  $q$ -axis parameters vary nonlinearly according to different load conditions, consequently changing the level of saturation.

The flux-linkage of  $d$ - and  $q$ -axis presents a ripple characteristic resulting from mechanical structure and degree of magnetic saturation. For superior performance, a multi-layer IPMSM should be operated in a magnetic saturation region.

The aforementioned problem was investigated using a previously proposed analysis and an experimental method that calculates the  $d$ - and  $q$ -axis flux-linkage in multi-layer IPMSM. In this paper, the analysis method used to calculate the flux-linkage, with specific focus on the ripple characteristic, is presented.

## 2. Analysis Model

The analysis model presented in this paper is a multi-layer IPMSM characterized by a high power density and wide speed range. Fig. 1 shows the cross-sectional view of

the analysis model.

In the rotor structure, permanent magnets are partially embedded into each layer in order to obtain the sinusoidal back-EMF waveform. The design specification of the analysis model is summarized in Table 1.

Fig. 1 shows two different flux paths and a magnetic reluctance difference generated in  $d$ - and  $q$ -axis. The reluctance difference of the two axes is used to attain the high power density and the extended speed range during motor operation.

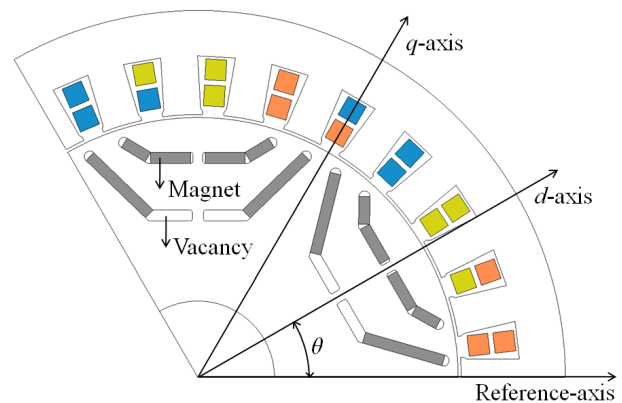


Fig. 1. Analysis model.

Table 1. Design specifications

Number of poles		6
Number of slots		27
Stacking length [mm]		230
Stator inner diameter [mm]		172
Stator outer diameter [mm]		240
Rotor inner diameter [mm]		50
Rotor outer diameter [mm]		170
Power	Rated power [kW]	50
	Maximum power [kW]	100
Maximum torque [Nm]		300
Maximum speed [rpm]		12000

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### 3. Previous Method of Analysis

The flux-linkage can be calculated by

$$\lambda_d = \frac{2}{3} [\cos(\theta_e)\lambda_a + \cos(\theta_e - \frac{2}{3}\pi)\lambda_b + \cos(\theta_e + \frac{2}{3}\pi)\lambda_c] \quad (1)$$

$$\lambda_q = \frac{2}{3} [-\sin(\theta_e)\lambda_a - \sin(\theta_e - \frac{2}{3}\pi)\lambda_b - \sin(\theta_e + \frac{2}{3}\pi)\lambda_c] \quad (2)$$

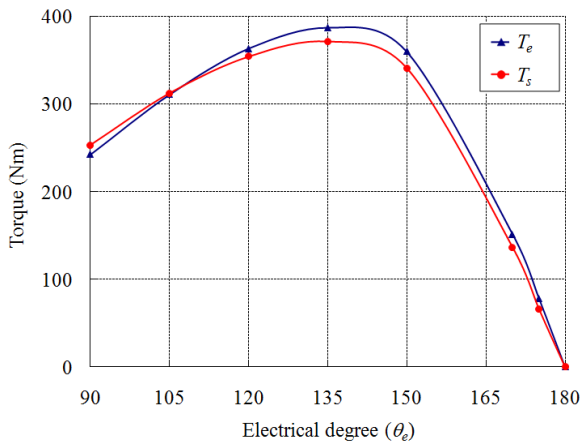
where  $\lambda_d$  and  $\lambda_q$  are  $d$ - and  $q$ -axis flux-linkage;  $\lambda_a, \lambda_b,$  and  $\lambda_c$  are the flux-linkages per phase; and the rotor positional angle  $\theta_e$  is the electrical angle between the  $d$ -axis and reference axis, as shown in Fig. 1 [5]. Previous analysis involved a fixed angle between two references, and hence, calculation was conducted on the flux-linkage based only on the change in current angle. If the calculated flux-linkage were correct, the torque  $T_s$  using the Maxwell stress tensor method would be the same as torque  $T_e$ . This can be calculated by

$$T_e(i_d, i_q) = \frac{3}{2} \frac{P}{2} [\lambda_d(i_d, i_q)i_q - \lambda_q(i_d, i_q)i_d] \quad (3)$$

where  $i_d$  and  $i_q$  are  $d$ - and  $q$ -axis currents, and  $P$  is the number of poles. However, dissimilarity was observed in the two analysis results, as well as discrepancy in the different rotor positions. Fig. 2 shows the comparative result between  $T_s$  and  $T_e$  according to the current angle. The result implies that the previous analysis method has not reflected the electromagnetic characteristics accurately. Table 2 shows an error of about 30 Nm between the calculated

**Table 2.** Torque results according to current angle

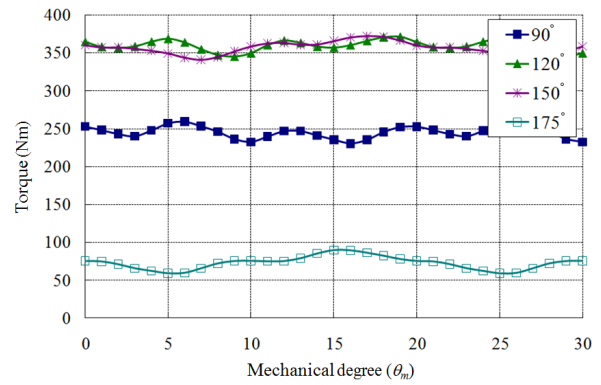
Current angle [deg]	120	150	175
$T_{s, avg}$ [Nm]	360.3	358.4	74.9
$T_{s, diff}$ [Nm]	26.1	31.7	30.6
Torque ripple [%]	7.2	8.8	40.8



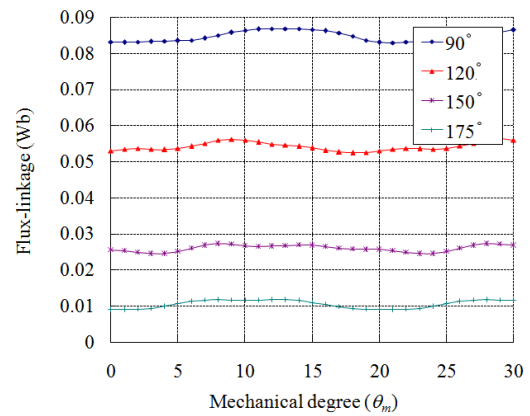
**Fig. 2.** Comparison between  $T_s$  and  $T_e$  according to current angle ( $\theta_m = 0^\circ, i_{peak-to-peak} = 640$  A).

torque results when the current angle was considered.

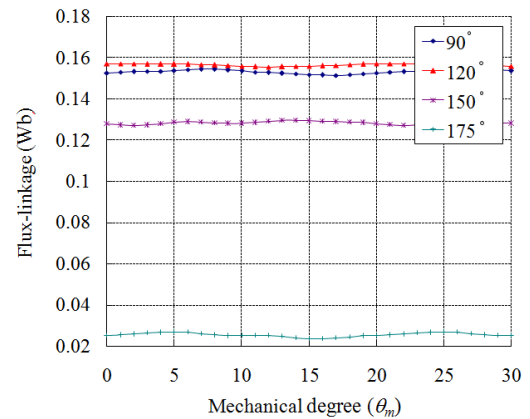
Fig. 3 shows the characteristics of the torque ripple based on the change in rotor position at different current angles. Fig. 4 and Fig. 5 present the flux-linkage variation for the torque ripple period in Fig. 3. As shown by results presented in from Fig. 3 to Fig. 5, torque ripple and flux-linkage variation were within the same period. The  $T_s$  generated with a specified current is presented as an average torque  $T_{s, avg}$  for a period of torque ripple. Therefore, in this paper, the same method was applied in the flux-linkage calculation.



**Fig. 3.** Torque ripple characteristics according to change in rotor position at different current angles.



**Fig. 4.** Ripple characteristics of  $d$ -axis flux-linkage.



**Fig. 5.** Ripple characteristics of  $q$ -axis flux-linkage.

#### 4. Flux-Linkage Calculation and Ripple Characteristics

Flux-linkage was calculated based on the change in rotor position. The angle between two reference frames would change with rotor rotation. The flux-linkages for each angle were calculated while the rotor was operating. Table 3 presents the ripple characteristics of the flux-linkage and its corresponding values for torque ripple. The table also shows similar torque ripples and flux-linkage ripple characteristics for different current angles.

In this paper, the  $d$ - and  $q$ -axis flux-linkage are introduced, taking into account the ripple characteristics:

$$\lambda_d^*(i_d, i_q) = \frac{\sum_{\theta=0}^{N-1} \lambda_d(i_d, i_q, \theta)}{N} \quad (4)$$

$$\lambda_q^*(i_d, i_q) = \frac{\sum_{\theta=0}^{N-1} \lambda_q(i_d, i_q, \theta)}{N} \quad (5)$$

where  $N$  is the number of calculation point during the period of flux-linkage ripple. Based on the average flux-linkage for a ripple period, the average torque  $T_{e\_avg}$  can then be analyzed by

$$T_{e\_avg}(i_d, i_q) = \frac{3P}{2} [\lambda_d^*(i_d, i_q)i_q - \lambda_q^*(i_d, i_q)i_d] \quad (6)$$

#### 5. Evaluation of the Proposed Method

The voltage equation for the IPMSM is presented by

$$V_d = i_d R_s + L_d \frac{di_d}{dt} - \omega \lambda_q \quad (7)$$

$$V_q = i_q R_s + L_q \frac{di_q}{dt} + \omega \lambda_d \quad (8)$$

where  $V_d$  and  $V_q$  are the  $d$ - and  $q$ -axis components of terminal voltage;  $L_d$  and  $L_q$  are the inductances along  $d$ - and  $q$ -axis; and  $R_s$  is the armature winding resistance.

Since the measurement was conducted in the steady state, derivative terms were neglected. Therefore,  $\lambda_d$  and  $\lambda_q$  can be defined by

$$\lambda_d = \frac{V_q - i_q R_s}{\omega}, \quad \lambda_q = -\frac{V_d - i_d R_s}{\omega} \quad (9)$$

However,  $R_s$  includes both phase and system resistance. To avoid measurement errors for flux-linkage from  $R_s$  in (9), voltage equations were employed. The voltage equations corresponding to both generating and motoring modes were calculated as

$$V_d^+ = i_d R_s - \omega L_q i_q = i_d R_s - \omega \lambda_q \quad (10)$$

$$V_q^+ = i_q R_s + \omega(L_d i_d + \lambda_f) = i_q R_s + \omega \lambda_d \quad (11)$$

$$V_d^- = i_d R_s + \omega L_q i_q = i_d R_s + \omega \lambda_q \quad (12)$$

$$V_q^- = -i_q R_s + \omega(L_d i_d + \lambda_f) = -i_q R_s + \omega \lambda_d \quad (13)$$

where  $V_d^+$  and  $V_q^+$  are the  $d$ - and  $q$ -axis voltages measured in the motoring mode;  $V_d^-$  and  $V_q^-$  are the  $d$ - and  $q$ -axis voltages measured in the generating mode; and  $\lambda_f$  is the maximum flux linkage of the permanent magnet [6]. In both motoring and generation modes, the same magnitude of  $d$ - and  $q$ -axis current excitations would occur; the same magnetic saturation would also be distributed in the motor. Based on (10)-(13), the  $d$ - and  $q$ -axis flux-linkage can be expressed by

$$\lambda_q = \frac{V_d^- - V_d^+}{2\omega}, \quad \lambda_d = \frac{V_q^- + V_q^+}{2\omega} \quad (14)$$

Fig. 6 shows the prototype motor used to verify the proposed analysis method. From Fig. 7 to Fig. 10, the graphs show the comparative results for the proposed method and the experiment of the prototype motor. Based on these results, the torque and parameters corresponding to the combined currents in the multi-layer IPMSM were then evaluated as an average value while considering the ripple characteristics. Furthermore, in this paper, the mechanical characteristics and magnetic saturation were both taken into account in the proposed method for the  $d$ - and  $q$ -axis flux-linkage calculation.

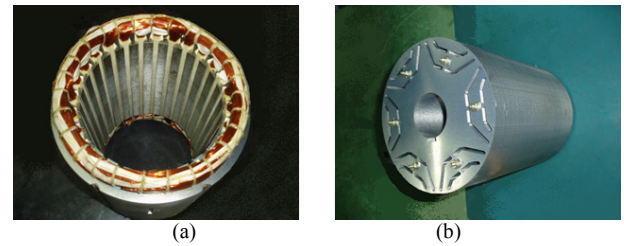


Fig. 6. Configuration of prototype motor: (a) stator and (b) rotor.

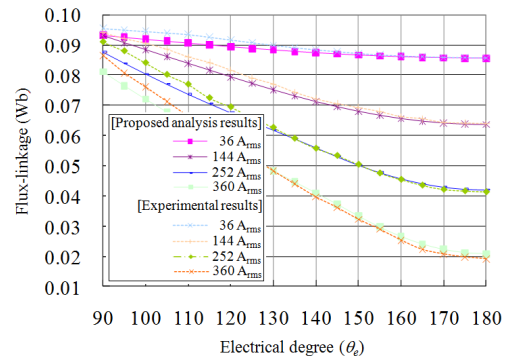


Fig. 7. Comparison between the proposed analysis and experimental results for the  $d$ -axis flux-linkage.

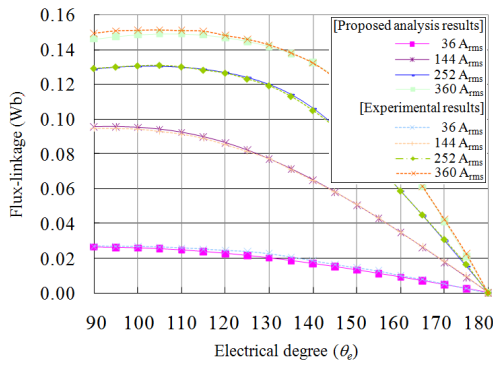


Fig. 8. Comparison between the proposed analysis and experimental results for the  $q$ -axis flux-linkage.

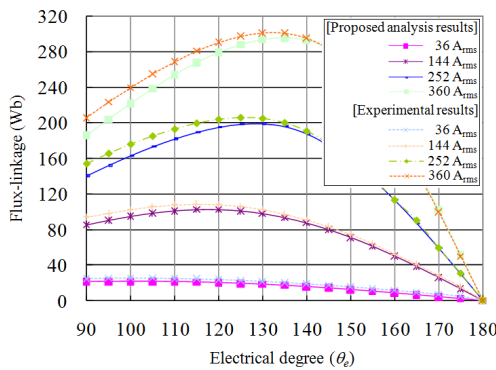


Fig. 9. Comparison between the proposed analysis and experimental results for the torque.

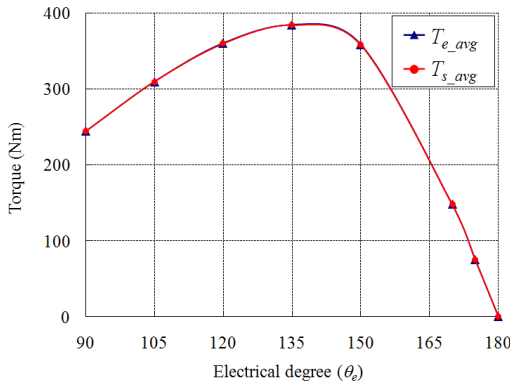


Fig. 10. Comparison between  $T_{s\_avg}$  and  $T_{e\_avg}$  according to the current angle ( $i_{peak-to-peak} = 640A$ ).

### 6. Conclusion

In this paper, we presented that the flux-linkage which is one of the most important parameters in an IPMSM has ripple characteristics as the rotor rotates. A method of calculating the flux-linkage variation using finite element analysis is suggested. Moreover, the behavior of the flux-linkage variation is analyzed with taking account of magnetic saturation and the mechanical structure.

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