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A Seat Inventory Management Model in the Presence of Dependent Demands

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■ Abstract ■

When airlines sell the same seats on an air flight at different fares, demand for a fare class depends on demand for other fare classes due to demand dependency. Demand dependencies occur when customers will buy other fare class tickets if the originally requested fare were unavailable, or when customers postpone their purchase decisions in anticipation of reopening of the lower fare in the next period. Demand dependency as a result customer buying behavior has a considerable profit implication, which was ignored in many earlier studies. We investigate the impact of demand dependency on the optimal booking limits and the expected revenues under a single-period and a two-period setting. We show how to find optimal booking limits of the problem and provide numerical examples to illustrate the impact

Keywords : Airline Revenue Management, Airline Application, Demand Dependency

1. Introduction

Revenue management has made significant contributions to the profitability of service enterprises such as airlines. Airline revenue management is the application of information systems and pricing strategies to allocate the right capacity to the right customer at the right place

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at the right time (Kimes, 1989). There has been substantial literature in the form of journal articles, conference proceedings and literature surveys in airline revenue management. McGill and Van Ryzin (1999) classified the major areas of airline revenue management as forecasting, overbooking, seat inventory allocation and pricing. Seats on an air flight are products that can be offered to different fare class customers for different prices. Seat inventory allocation is concerned with the allocation of the finite seat inventory to the different fare classes over time. Through historical records in the various fare classes, an airline would adjust seat inventory levels between classes in order to maximize expected revenue. The first published research on airline seat inventory allocation (Littlewood, 1972) provided a useful analysis of a two-fare-class model of seat allocation on a single flight leg. Littlewood (1972) suggested that the point of closing down the low fare class was when the expected marginal revenue from selling an additional low fare seat was exceeded by the expected marginal revenue of selling the same seat at the high fare. Belobaba (1987, 1989) extended Littlewood (1972) work to the cases of multiple fare classes and developed the EMSR (Expected Marginal Seat Revenue) heuristic approach for nested booking classes in which Littlewood's (1972) rule was applied sequentially, known generally as "EMSR-a." The booking limit is defined to be the maximum number of seats that might be sold at the low price, and the protection level is the number of seats that would be sold to high fare customers because high fare customers might book later in the booking period. Belobaba also developed a variant of the EMSR method, known as "EMSR-b" (Belobaba, 1992). The EMSR-b heuristic model provided seat protection levels closer to optimal values than those from the EMSR-a. The solutions by the EMSR-b are calculated jointly for all high fare classes relative to a given low fare class based on a weighted combination of all classes above the one for which a booking limit is calculated. The EMSR-b heuristic model can be repeatedly applied to allow for multiple periods.

Methods for obtaining optimal booking limits with more than two fare classes are provided in Curry (1990), Brumelle and McGill (1993), Wollmer (1992), and Robinson (1995). These authors pointed out that EMSR solutions were not optimal when considered more than two fare classes, and developed optimal rules for multi-fare-class problems independently : The basic logic of all these methods is that the discount seats are offered until the contribution of the sale is less than the expected contribution from the sale of seats to all the remaining fare classes while EMSR approach considers the next highest fare only. All of these studies were performed independently and the demands between fare classes are assumed to be independent. However, many flexible travelers would take a high fare ticket if low fare tickets were unavailable. This type of customer buying behavior, which results in demand dependencies, is referred to as customer diversion (Pfeifer, 1989). Specifically, buy- up refers to customers' buying a high fare when low fares are closed whereas buy-down refers to the substitution of a low fare ticket for a high fare customer when low fares are still open. Waiting is denoted for situations when customers postpone their purchase decisions in anticipation of opening of the low price ticket in the next period, and it is referred to as a kind of strategic customer behavior. Actually, customer might decide to wait for the reopening of a low fare in the future if the low fare tickets were sold out in the current period.

Pfeifer (1989) examined a two-fare, single-period airline seat allocation problem and developed heuristic decision rules to incorporate customer diversion, where a customer might buy a more expensive fare ticket if a less expensive fare ticket was not available. Pfeifer's approach was to find the booking limit for the low-fare seats (q) for which the total expected profit at $q+1$ was less than that at q using marginal analysis. Bodily and Weatherford (1995) developed heuristic decision rules for a single-period model with more than three fare classes including customer diversion (which was based on Pfeifer's (1989) model) and presented numerical results using simulation. Belobaba and Weatherford (1996) developed new heuristic decision rules for multi-period models. They extended the decision rule derived by Bodily and Weatherford (1995), and they also developed a combined heuristic decision rule for more than three fare classes; the decision algorithm combined the seat allocation rule of the EMSR-b model with joint protection logic and the extended decision rule derived by Bodily and Weatherford (1995).

An individual customer is interested in buying the same ticket as cheap as possible. Customers' knowledge of seat inventory allocation process and prices as a function of time has made new opportunities for buying cheap tickets possible. Research on strategic customer behavior can be an important in airline seat allocation area because customers might engage in strategic purchasing behavior. Lazear (1986) studied strategic customer behavior and pricing strategy under the retailing settings and concluded customers might wait for low price items unless they perceive that there is a large demand for the item. Anderson and Wilson (2003) studied multi-period seat inventory allocation with waiting (as strategic customer behavior) and buy-down, and the booking limits were set by the EMSR-b. They investigated the impact of customer diversion on the total expected profit in their model. They argued that this was the case for the low demand flights or flights with drastically discounted fare seats.

Research on finding optimal results (optimal booking limits) for the single-period, two-fare-class problem with customer diversion have been studied, but more robust analytical results with managerial implications were done by Sen and Zhang (1999). Sen and Zhang (1999) considered a single-item, single-period stochastic inventory problem or the newsvendor problem, where the item could be sold to different demand classes at different prices. Sen and Zhang (1999) decide both optimal values for the initial capacity and the booking limit simultaneously. Although Sen and Zhang's (1999) model provides a nice analytical work for airline seat allocation including diversion, they do not consider strategic customer behavior.

In this e-commerce age, customers are becoming more familiar with the existence of pricing and booking structures (or seat allocation) employed by the airline companies. Therefore, customer buying behavior is becoming more complex and has a meaningful profit implication. We explore how much additional revenue potential exists, and we study how an airline can change its strategy for setting up optimal booking limits that make adjustments to the EMSR type seat allocation method on the assumption that customer diversion occurs. The purpose of this study is to investigate the impacts of demand dependency on the optimal booking limit and the total expected.

2. The Mathematical Model

We extend Sen and Zhang's (1999) one-period model to a two-period model where the basic assumptions are the same. Sen and Zhang (1999) considered a single-period stochastic inventory problem with two demand classes segmented by time and price and suggested a booking limit type optimal policy for this problem. The originally independent demands are stochastic with known probability distributions and are assumed to be realized sequentially in a single period; low-fare customers arrive before high-fare customers. Once the low-fare class is closed, it will never reopen. The high-fare class customers buy only the high-fare tickets. In their model, overbooking is not allowed, and a customer does not buy more than two tickets, and customer diversion is modeled by assuming that a fixed portion of the unsatisfied demand for low fare would join the demand for high fare, which is usually defined as the willingness of potential customers to purchase tickets in a different fare class from one they originally requested. We develop a two-period model additionally assuming that a fraction of customers who cannot purchase a low-fare ticket in period 1 will wait until period 2 in the hope that a low-fare will become available, as well as we consider buying-up customers in each period. Under these assumptions we develop a two-period model where airline companies can have opportunities for the reallocation of seats to fare classes in each period. We use the following notation for the two-period problem :

C : capacity at the beginning of period 1

c : capacity at the beginning of period 2

r_1 : unit revenue from saver (low) fare

r_2 : unit revenue from full (high) fare

d_i : fraction of buying-up in period i

w : fraction of waiting from period 1 to period 2

D_{ij} : demand for fare class j in period i

$f_{ij}(D_{ij})$: p.d.f. of D_{ij}

$F_{ij}(D_{ij})$: c.d.f. of D_{ij}

l_i : booking limit in period i

$E(D_{ij})$: expected demand for fare class j in period i

$E[\pi_i(l_i)]$: total expected revenue from saver and full fare classes in period i

$\pi_i(l_i)$: total revenue from saver and full fare classes in period i

$ER_1(D_{11}^*)$: expected revenue from saver fare class in period 1

$ER_1(D_{12}^*)$: expected revenue from full fare class in period 1

$ER_2(c, l_1, l_2^*(c), D_{21}^*, D_{22}^*)$: the expected revenue from both fare classes in period 2 without censored demand from period 1-uncensored demand case

$ER_2(c, l_1, l_2^*(c, l_1), \hat{D}_{21}^*, \hat{D}_{22}^*)$: the expected revenue from both fare classes in period 2 without censored demand from period 1-censored demand case

$ER_{Total}(l_1)$: the total expected revenue from both period

2.1 Two-Period Model

For analytical interpretation of the two-period model, we divide the total expected revenue of the two-period model into :

1. The total expected revenue from period 1;
2. The total expected revenue from period 2 where there are waiting customers from period 1 (censored demand case), and;
3. The total expected revenue from period 2 where there are no waiting customers from period 1 (uncensored demand case).

For given demand parameters in period 1 and 2, the optimal booking limit in period 2 is different due to the remaining capacity at the end of period 1 and the number of waiting customers. In our two-period model, we find the first period booking limit to maximize the total expected revenue from both periods, which include both censored demand cases and uncensored demand cases in period 2. For the censored demand case, the actual demand in period 2 becomes the original demand plus the waiting customers from period 1. In this case, where all l_1 low-fare seats were sold out, the only thing we know is that the low-fare demand is bigger than the booking limit in period 1. For the uncensored demand case, we do not need to consider waiting customers from period 1 to calculate the expected revenue from period 2. Now we introduce the mathematical model for the two-period problem.

Let $l_2^*(c)$ be the optimal booking limit in period 2 where the remaining capacity from period 1 is c, and the first period booking limit l_1 is not reached in period 1, and let $l_2^*(c, l_1)$ be the optimal booking limit in period 2 where the remaining capacity at the end of period 1 is c and the booking limit in period 1 is reached. $ER_1(D_{11}^*)$ is the expected revenue from the saver-fare class in period 1, and $ER_1(D_{12}^*)$ is the expected revenue from the full-fare demand in period 1 where D_{11}^* and D_{12}^* are the actual sales for each fare class in period 1 :

$$D_{11}^* = min\ \{D_{11}, l_1\}$$

$$D_{12}^* = \begin{cases} min\ \{C - D_{11}, D_{12}\} & \text{if } D_{11} \le l_1 \\ min\ \{C - l_1, D_{12} + d_1(D_{11} - l_1)\} & \text{if } D_{11} > l_1 \end{cases}$$

$ER_2(c, l_1, l_2^*(c), D_{21}^*, D_{22}^*)$ is the total expected revenue from both demand classes in period 2 where the booking limit in period 1, l_1, is not reached and the remaining capacity at the end of period 1 is c. Therefore, there are no waiting customers. In this case, the optimal booking limit in period 2 is determined by the remaining capacity because there are no waiting customers. $ER_2(c, l_1, l_2^*(c, l_1), \hat{D}_{21}^*, \hat{D}_{22}^*)$ is the total expected revenue in period 2 where the booking limit in period 1, l_1, is reached and the remaining capacity at the end of period 1 is c. In this case, the optimal booking limit in period 2 for a specific problem is determined by the remaining capacity because and the number of waiting customers. $\hat{D}_{21}^*$ and $\hat{D}_{22}^*$ are the actual sales for each fare class in period 2 :

$$\hat{D}_{21}^* = min\ \{\hat{D}_{21}^*, l_2\}$$

$$\hat{D}_{22}^* = \begin{cases} min\ \{c - \hat{D}_{21}, D_{22}\} & \text{if } \hat{D}_{21} \le l_2 \\ min\ \{c - l_2, D_{22} + d_2(\hat{D}_{21} - l_2)\} & \text{if } \hat{D}_{21} > l_2 \end{cases}$$

where $\hat{D}_{21} = D_{21} + w(D_{11} - l_1)^+$

$c = (C - l_1 - D_{12} - d_1(D_{11} - l_1)^+)^+$

Then, the total expected revenue from both periods $ER_{Total}(l_1)$ (as a function of l_1) is given by

$$ER_{Total}(l_1) = ER_1(D_{11}^*) + ER_1(D_{12}^*) \quad (2.1)$$
$$+\int_0^{\infty}\int_0^{l_1} ER_2(c,\ l_1,\ l_2^*(c),\ D_{21}^*,\ D_{22}^*)$$
$$f_{11}(D_{11})dD_{11}f_{12}(D_{12})dD_{12}$$
$$+\int_0^{\infty}\int_{l_1}^{\infty} ER_2(c, l_1,\ l_2^*(c,\ l_1),\ \hat{D}_{21}^*,\ \hat{D}_{22}^*)$$
$$f_{11}(D_{11})dD_{11}f_{12}(D_{12})dD_{12}$$

In (2.1), the total expected revenue from period 1 is expressed :

$$ER_1(D_{11}^*) + ER_1(D_{12}^*) = r_1E(D_{11}^*) + r_2E(D_{12}^*)$$

where

$$E(D_{11}^*) = \int_0^{l_1} D_{11}f_{11}(D_{11})dD_{11} + \int_{l_1}^{\infty} l_1f_{11}(D_{11})dD_{11}$$

$$E(D_{12}^*) = \int_0^{l_1}\int_0^{C-D_{11}} D_{12}f_{12}(D_{12})dD_{12}f_{11}(D_{11})dD_{11}$$
$$+\int_0^{l_1}\int_{C-D_{11}}^{\infty}(C-D_{11})f_{12}(D_{12})dD_{12}f_{11}(D_{11})dD_{11}$$
$$+\int_0^{C-l_1}\int_{\infty}^{\frac{C-l_1+d_1l_1-D_{12}}{d_1}}(D_{12}+d_1(D_{11}-l_1))$$
$$f_{11}(D_{11})dD_{11}f_{12}(D_{12})dD_{12}$$
$$+\int_0^{C-l_1}\int_{\frac{C-l_1+d_1l_1-D_{12}}{d_1}}^{\infty}(C-l_1)f_{11}(D_{11})dD_{11}$$
$$f_{12}(D_{12})dD_{12}$$
$$+\int_{C-l_1}^{\infty}\int_{l_1}^{\infty}(C-l_1)f_{11}(D_{11})dD_{11}f_{12}(D_{12})dD_{12}$$

For the uncensored demand case in period 2, we have :

$$ER_2(c,\ l_1,\ l_2^*(c),\ D_{21}^*,\ D_{22}^*) = r_1E(D_{21}^*) + r_2E(D_{22}^*)$$

where D_{21}^* and D_{22}^* are given by

$$E(D_{21}^*) = \int_0^{l_2^*(c)} D_{21}\cdot f_{21}(D_{21})dD_{21} + \int_{l_2^*(c)}^{\infty} l_2^*(c)\cdot f_{21}(D_{21})dD_{21}$$

$$E(D_{22}^*) = \int_0^{l_2^*(c)}\int_0^{C-D_{21}} D_{22}f_{22}(D_{22})dD_{22}f_{21}(D_{21})dD_{21}$$
$$+\int_0^{l_2^*(c)}\int_{C-D_{21}}^{\infty}(C-D_{21})f_{22}(D_{22})dD_{22}$$
$$f_{21}(D_{21})dD_{21}$$
$$+\int_0^{C-l_2^*(c)}\int_{l_2^*(c)}^{\frac{(C-l_2^*(c)+d_2l_2^*(c)-D_{22})}{d_1}}$$
$$(D_{22}+d_2(D_{21}-l_2^*(c)))f_{21}(D_{21})$$
$$dD_{21}f_{22}(D_{22})dD_{22}$$
$$+\int_0^{C-l_2^*(c)}\int_{\frac{C-l_2^*(c)+d_2l_2^*(c)-D_{22}}{d_1}}^{\infty}$$
$$(C-l_2^*(c))f_{21}(D_{21})dD_{21}$$
$$f_{22}(D_{22})dD_{22}$$
$$+\int_{C-l_2^*(c)}^{\infty}\int_{l_2^*(c)}^{\infty}(C-l_2^*(c))f_{21}(D_{21})dD_{21}$$
$$f_{22}(D_{22})dD_{22}.$$

For the censored demand case, $ER_2(c,\ l_1,\ l_2^*(c,\ l_1),\ \hat{D}_{21}^*,\ \hat{D}_{22}^*)$ is expressed as

$$ER_2(c, l_1, l_2^*(c, l_1), \hat{D}_{21}^*, \hat{D}_{22}^*) = r_1E(\hat{D}_{21}^*) + r_2E(\hat{D}_{22}^*)$$

where $\hat{D}_{21}^*$ and $\hat{D}_{22}^*$ are given by

$$\hat{D}_{21}^* = min\ \{\hat{D}_{21}^*,\ l_2(c,\ l_1)\}$$
$$\hat{D}_{22}^* = \begin{cases} min\ \{c-\hat{D}_{21}, D_{22}\} & \text{if } \hat{D}_{21} \le l_2^*(c,\ l_1) \\ min\ \{c-l_2^*(c,\ l_1), D_{22}+d_2(\hat{D}_{21}-l_2^*(c,\ l_1))\} & \\ & \text{if } \hat{D}_{21} > l_2^*(c,\ l_1) \end{cases}$$
$$\hat{D}_{21} = D_{21} + w(D_{11}-l_1)^+,$$
$$c = (C-l_1-D_{12}-d_1(D_{11}-l_1)^+)^+$$

Therefore, the expected sales from the low-fare class in period 2 is given by

$$E(\hat{D}_{21}^*) = \int_0^{l_2^*(c,l_1)} \hat{D}_{21}\cdot f_{21}(\hat{D}_{21})d\hat{D}_{21}$$
$$+\int_{l_2^*(c,l_1)}^{\infty} l_2\cdot f_{21}(\hat{D}_{21})d\hat{D}_{21},$$

and the expected sales from the high-fare class in period 2 is

$$E(\hat{D}_{22}^{*}) = \int_{0}^{l_2^*(c,l_1)} \int_{0}^{c-\hat{D}_{21}} \hat{D}_{22} f_{22}(\hat{D}_{22}) d\hat{D}_{22} f_{21}(\hat{D}_{21}) d\hat{D}_{21}$$
$$+ \int_{0}^{l_2^*(c,l_1)} \int_{c-\hat{D}_{21}}^{\infty} (c-\hat{D}_{21}) f_{22}(\hat{D}_{22}) d\hat{D}_{22} f_{21}(\hat{D}_{21}) d\hat{D}_{21}$$
$$+ \int_{0}^{c-l_2^*(c,l_1)} \int_{l_2}^{\frac{(c-l_2^*(c,l_1)+d_2 l_2^*(c,l_1)-\hat{D}_{22})}{d_2}} (\hat{D}_{22} + d_2(\hat{D}_{21} - l_2^*(c,l_1))) f_{21}(\hat{D}_{21}) d\hat{D}_{21} f_{22}(\hat{D}_{22}) d\hat{D}_{22}$$
$$+ \int_{0}^{c-l_2^*(c,l_1)} \int_{\frac{(c-l_2^*(c,l_1)+d_2 l_2^*(c,l_1)-\hat{D}_{22})}{d_2}}^{\infty} (c-l_2^*(c,l_1)) f_{21}(\hat{D}_{21}) d\hat{D}_{21} f_{22}(\hat{D}_{22}) d\hat{D}_{22}$$
$$+ \int_{c-l_2^*(c,l_1)}^{\infty} \int_{l_2^*(c,l_1)}^{\infty} (c-l_2^*(c,l_1)) f_{21}(\hat{D}_{21}) d\hat{D}_{21} f_{22}(\hat{D}_{22}) d\hat{D}_{22}.$$

Now we consider the first derivative of the total expected revenue (2.1) with respect to l_1. After considerable algebraic simplification, we obtain the first derivative of (2.1) with respect to l_1. The first derivative of the expected revenue from period 1 with respect to l_1 is :

$$\frac{\partial(ER_1(D_{11}^*) + ER_1(D_{12}^*))}{\partial l_1} = (r_1 - d_1 r_2)(1-F_{11}(l_1)) \quad (2.2)$$
$$-r_2(1-d_1)(1-F_{11}(l_1))(1-F_{12}(C-l_1))$$
$$-r_2(1-d_1)\int_0^{C-l_1}\left\{1-F_{11}\left\{\frac{(C-l_1+d_1 l_1-D_{12})}{d_1}\right\}\right\}, f_{12}(D_{12})dD_{12}$$

and first derivative of the total expected revenue from period 2 is expressed as

$$\frac{\partial}{\partial l_1}\left\{\begin{aligned}&\int_0^{\infty}\int_0^{l_1} ER_2[c, l_1, l_2^*(c), D_{21}^*, D_{22}^*] f_{11}(D_{11})dD_{11} f_{12}(D_{12})dD_{12}\\ &\int_0^{\infty}\int_{l_1}^{\infty} ER_2[c, l_1, l_2^*(c, l_1)\, \hat{D}_{21}^*, \hat{D}_{22}^*] f_{11}(D_{11})dD_{11} f_{12}(D_{12})dD_{12}\end{aligned}\right\} \quad (2.3)$$

$$= -r_1 w \int_0^{\infty}\int_{l_1}^{\infty} f_{11}(D_{11})dD_{11} f_{12}(D_{12})dD_{12}$$
$$-r_2 w \int_0^{\infty}\int_{l_1}^{\infty}\left\{\begin{aligned}&\int_0^{l_2^*(c,l_1)}[1-F_{22}(c-w(D_{11}-l_1)^+-D_{21})^+] f_{21}(D_{21})dD_{21}\\ &+\int_0^{\hat{c}}(1-F_{21}(\overline{C})) f_{22}(D_{22})dD_{22}\\ &+\int_{\hat{c}}^{\infty}(1-F_{21}(l_2^*(c, l_1))) f_{22}(D_{22})dD_{22}\end{aligned}\right\} f_{11}(D_{11})dD_{11} f_{12}(D_{12})dD_{12}$$

where $c \equiv (C-l_1-D_{12}-d_1(D_{11}-l_1)^+)^+$,

$\delta \equiv 1-d_1-w$,

$\hat{c} \equiv (C-l_1-D_{12}-d_1(D_{11}-l_1)^+ - w(d_{11}-l_1)^+ - l_2^*(c, l_2))^+$,

$$\overline{C} \equiv \frac{(C-l_1-D_{12}-d_1(D_{11}-l_1)^+ - w(D_{11}-l_1)^+ - l_2^*(c, l_1) + d_2 l_2^*(c, l_1) - D_{22})^+}{d_2}$$

The first derivative of the total expected profit from both periods is (2.2) plus (2.3). From (2.3), we know that the first derivative on l_1 is always negative for any parameters and demands, which means the slope of the expected revenue function on l_1 is negative. Considering the two-period model with the same parameters of the one-period model, the optimal booking limit is always less than that of the one-period model because the first derivative of the second period expected revenue with respect to l_1 is always negative. As l_1 increases, fewer customers will wait for reopening low fare in period 2. Therefore, the expected revenue function of the second period is monotonically decreasing as l_1 increases. Next propositions present the conditions that segmentation of seat inventory for different fare classes is not guaranteed.

Proposition 2.1

If $r_1 - d_1 r_2 - w r_1 < 0$ or $\frac{r_1}{r_2} < \frac{d_1}{1-w}$, then the optimal booking limit in period 1 for the two-period model is 0.

Proof : From (2.2) and (2.3), we have the first derivative as

$$
\begin{aligned}
&\frac{\partial ER_{Total}(l_1)}{\partial l_1} = \\
&(r_1 - d_1 r_2 - r_1 w)(1 - F_{11}(l_1)) \\
&- r_2(1-d_1)(1-F_{11}(l_1))(1-F_{12}(C-l_1)) \\
&- r_2(1-d_1)\int_0^{C-l_1}\left\{1 - F_{11}\left\{\frac{(C-l_1+d_1 l_1 - D_{12})}{d_1}\right\}\right\} \\
&\qquad f_{12}(D_{12})dD_{12} \\
&- r_2\delta\int_0^{\infty}\int_{l_1}^{\infty}\left\{\begin{array}{l}\int_0^{l_2^*}(1-F_{22}(c-w(D_{11-l_1})^+ - D_{21})) \\ \qquad f_{21}(D_{21})dD_{21} \\ + \int_0^{\hat{c}}(1-F_{21}(\bar{C}))f_{22}(D_{22})dD_{22} \\ + \int_{\theta}^{\infty}(1-F_{21}(l_2^*(c, l_1)))f_{22}(D_{22})dD_{22}\end{array}\right\} \\
&\qquad f_{11}(D_{11})dD_{11}f_{12}(D_{12})dD_{12}
\end{aligned}
$$

If $r_1 - d_1 r_2 - w r_1 < 0$, then $\frac{\partial ER_{Total}(l_1)}{\partial l_1} < 0$ for all C and l_1. Hence, $l_1^* = 0$. ■

From (2.2), for the single-period model, if $r_1 - d_1 r_2 < 0$, then $l_1^* = 0$: Product segmentation of seat inventory for different fare classes is not warranted unless the fraction willing to upgrade from low fare to high fare is greater than the ratio of unit revenues (saver versus full) in a single-period model. From proposition 3.1, if we consider both waiting and buying-up, the condition that makes the optimal booking limit in period 1 for the two-period model zero will be tighter compared to the case of the single-period with the same parameters.

Proposition 2.2

For a fixed C, the corresponding optimal $l_1^* = 0$ if and only if

$$r_2(1-d_1)P(d_1 D_{11} + D_{12} < C) \le r_2 - r_1 + r_1 w \quad (2.4)$$

Proof : see Appendix A. ■

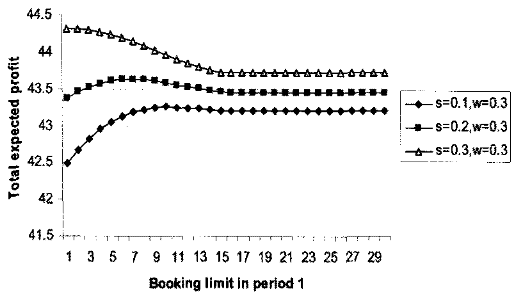
From proposition 2.2 we derive an important analytic insight. We rewrite (2.4) as :

$$
\begin{aligned}
r_2 P(d_1 D_{11} + D_{12} > C) + r_2 d_1 P(d_1 D_{11} + D_{12} \le C) \\
\ge r_1 - r_1 w \qquad (2.5)
\end{aligned}
$$

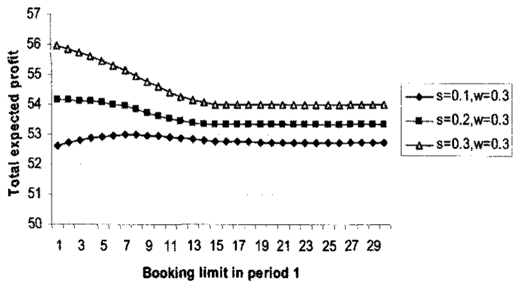
This formula (2.5) implies that there exists a C_0 such that $l_1^* = 0$ if $C \le C_0$ and $l_1^* > 0$ if $C > C_0$. We have an expected marginal interpretation from (2.5). The right hand side of (2.5) is the marginal revenue when l_1 increasing from 0 to 1 (we gain r_1 but also lose $w r_1$). The left hand side of (2.5) is the expected marginal revenue lost when the total high fare demand is bigger than C or when the total high fare demand is less than C. Note that (2.5) is more general than the EMSR heuristic algorithm (Belobaba, 1989), Pfeifer's (1989) rule, and the decision rule by Belobaba and Weatherford (1996).

3.3 Computational Study for the Two-Period Model

Let C = 35 and $r_1 = 1$. We assume that the fraction of buying-up in each period has the same value : $d_1 = d_2 = s$. Basically, assume that $s = 0.3$ and $w = 0.3$. The demand is assumed to follow a normal distribution with a mean of 10 and a standard deviation of 3 for each fare class in each period. <Figure 1> and <Figure 2> present the expected revenue curves as a function of the fraction of buying-up s at a given fraction of waiting w = 0.3 for the cases of $r_2 = 1.5$ and $r_2 = 2.0$. In both cases, increasing s will change the optimal booking in period 1, significantly and it will also change the optimal expected revenue significantly. As the fare ratio (full vs. saver) increases, the optimal booking limit decreases.

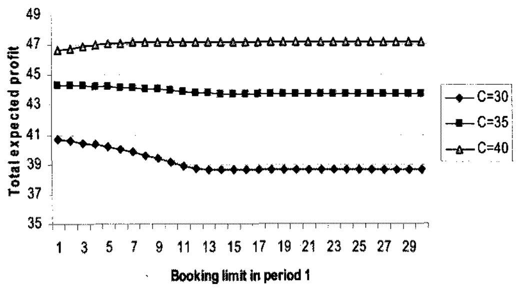


〈Figure 1〉 Total expected revenue as a function of the booking limit in period 1 with different combinations of s and w where the full fare is 1.5



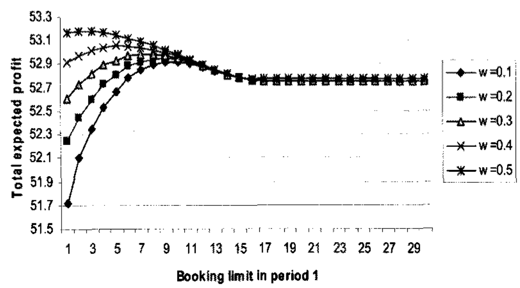
〈Figure 2〉 Total expected revenue as a function of the booking limit in period 1 with different combinations of s and w where the full fare is 2.0

Continuing the above numerical example under the same parameter settings for demand distributions, <Figure 3> display the optimal booking limits in period 1 and the expected revenues as a function of the initial capacity where the full fare is 1.5. As the capacity increases, the optimal booking limit and its expected revenue change significantly.



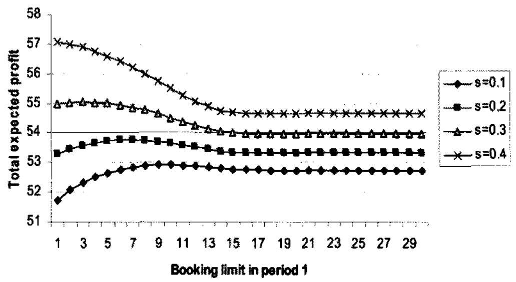
〈Figure 3〉 Total expected revenue as a function of period 1 booking limit with different capacities where the full fare is 1.5

Next we consider the case where the fraction of buying-up s is 0.1, the saver fare is 1.0 and the full fare is 2.0. <Figure 4> displays optimal booking limits and expected revenues as a function of the fraction of waiting. As the fraction of waiting increases from 0.1 to 0.5, the optimal booking limit in period 1 decreases from 9 to 2. For the case where the fraction willing to wait is 0.1, <Figure 5> displays the optimal booking limits and the expected revenue as a function of the fraction of buying-up. When the fraction of buy-up is 0.4, the optimal booking limit in period 1 is zero. <Figure 4> and <Figure 5> illustrate that optimal booking limits decrease significantly in the presence of customer diversion and strategic customer behavior. Note



〈Figure 4〉 Total expected revenue as a function of period 1 booking limit where the fraction of waiting changes from 0.1 to 0.5

that the booking limit calculated by the EMSR is 15 if no strategic customer behavior is assumed.



⟨Figure 5⟩ Total expected revenue as a function of period 1 booking limit where the fraction of buying-up changes from 0.1 to 0.4

Compared with results using EMSR type approach, all the figures illustrate that the optimal booking limits decrease significantly in the presence of customer diversion and strategic customer behavior. <Table 1> compares the total expected revenues when EMSR type decision rule including customer diversion effect is used (waiting could not be considered when EMSR type decision rule is used) with those when our algorithm is used. If strategic customer behavior and diversion are not considered, the booking limit in period 1 will be 15 and the booking limit in period 2 is 25 by the EMSR. As shown in the table, ignoring strategic customer behavior may result in 8% revenue loss.

⟨Table 1⟩ Total Expected Revenue Using EMSR Rule vs. the Optimal Booking Limit

Diversion (%)	Waiting (%)	EMSR with diversion	Optimal	Gain(%)
10	10	52.03	52.92	1.71
20	10	52.44	53.76	2.52
30	10	52.77	55.05	4.32
40	10	53.05	57.06	7.56
10	10	52.03*	52.92	1.71
10	20	52.03*	52.95	1.77
10	30	52.03*	52.98	1.83
10	40	52.03*	53.06	1.98

4. Conclusion

Seat inventory allocation has been a major research area in airline revenue management. Customer diversion for one-period models has been investigated by many researchers. Finding optimal booking limits for multi-period revenue management models has proven to be difficult and many of the multi-period models in the literature are heuristic in nature. In this research, we develop a two-period model for airline seat allocations considering waiting (as strategic customer behavior) and customer diversion effects. As the assumptions become more realistic, the solutions become less tractable. If we consider customer diversion or strategic customer behavior, then the optimal solutions are totally different from solutions using EMSR type methods, and revenue losses may exceed 8% under certain circumstances. The main contribution of this research is to extend previous research to allow customers to behave strategically by waiting to see whether a cheaper product will become available in the next period, and solve one-dimensional problems for finding optimal booking limits for a two-period model. The application of this method to the case of multi-period problems having three or more periods requires more complex mathematical derivations, although dynamic program may offer a feasible alternative when developing optimal algorithms for these multi-period models.

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⟨Appendix A⟩

The first derivative of the total expected revenue of the two-period problem with respect to l_1 is :

$$\frac{\partial ER_{Tatol}(l_1)}{\partial l_1} = (r_1 - d_1 r_2)(1 - F_{11}(l_1)) - r_2(1-d_1)(1-F_{11}(l_1))(1-F_{12}(C-l_1))$$
$$-r_2(1-d_1)\int_0^{C-l_1}\left\{1-F_{11}\left\{\frac{(C-l_1+d_1l_1-D_{12})}{d_1}\right\}\right\}f_{12}(D_{12})dD_{12}$$
$$-r_1 w\int_0^{\infty}\int_{l_1}^{\infty} f_{11}(D_{11})dD_{11}f_{12}(D_{12})dD_{12}$$
$$-r_2\delta\int_0^{\infty}\int_{l_1}^{0}\left\{\begin{array}{l}\int_0^{l_2^*(c,l_1)}(1-F_{22}(c-w(D_{11}-l_1)^+ - D_{21})f_{21}(D_{21})dD_{21}\\ +\int_0^{\hat{c}}(1-F_{21}(\overline{C}))f_{22}(D_{22})dD_{22}\\ +\int_{\hat{c}}^{\infty}(1-F_{21}(l_2^*(c,l_1)))f_{22}(D_{22})dD_{22}\end{array}\right\}f_{11}(D_{11})dD_{11}f_{12}(D_{12})dD_{12}$$

where $c \equiv (C - l_1 - D_{12} - d_1(D_{11} - l_1)^+)^+$, $\hat{c} \equiv (C - l_1 - D_{12} - d_1(D_{11}-l_1)^+ - w(D_{11}-l_1)^+ - l_2^*(c, l_1))^+$

$$\overline{C} \equiv \frac{(C - l_1 - D_{12} - d_1(D_{11}-l_1)^+ - w(D_{11}-l_1)^+ - l_2^*(c, l_1) + d_2 l_2^*(c, l_1) - D_{22})^+}{d_2}$$ and $\delta \equiv 1 - d_1 - w.$

We rewrite $\frac{\partial ER_{Tatol}(l_1)}{\partial l_1}$ in a marginal expression as :

$$\begin{aligned}&r_2(1-d_1)P(d_1(D_{11}-l_1)+D_{12} < C - l_1,\ D_{11} > l_1\\ &\quad -(r_2 - r_1 + r_1 w)P(D_{11} > l_1)\\ &\quad -r_2\delta\left\{\begin{array}{l}P(D_{22} > c - w(D_{11}-l_1)^+ - D_{21},\ D_{21} < l_2^*(c, l_1),\ D_{11} > l_1)\\ +P(D_{21} > \overline{C},\ D_{22} < \hat{c},\ D_{11} > l_1\\ +P(D_{21} > l_2^*(c, l_1), D_{22} > \hat{c},\ D_{11} > l_1\end{array}\right\}\end{aligned} \quad \text{(A1)}$$

We show that if $\partial ER_T(l_1)/\partial l_1 \le 0$ at $l_1 = 0$, then $\partial ER_T(l_1)/\partial l_1 \le 0$ for all $l_1 > 0$ for a given C. The first derivative of $ER_T(l_1)$ at $l_1 = 0$ is given by

$$r_2(1-d_1)P(d_1D_{11}+D_{12} < C) - (r_2 - r_1 + r_1 w) - r_2\delta P(C, c, D_{11}, D_{12}, D_{21}, D_{22}, 0, l_2^*(c, 0))$$

where $P(C, c, D_{11}, D_{12}, D_{21}, D_{22}, 0, l_2^*(c, 0)) = \left\{\begin{array}{l}P(D_{22} > c - wD_{11} - D_{21},\ D_{21} < l_2^*(c, 0))\\ +P(D_{21} > \overline{C},\ D_{22} < \hat{c})\\ +P(D_{21} > l_2^*(c,\ 0), D_{22} > \hat{c})\end{array}\right\}$

$\hat{c} \equiv (C - D_{12} - d_1D_{11} - wD_{11} - l_2^*(c, 0))^+$, $c \equiv (C - D_{12} - d_1 D_{11})^+$

$\overline{C} \equiv \frac{(\hat{c} + d_2 l_2^*(c, 0) - D_{22})^+}{d_2}$, and $\delta \equiv 1 - d_1 - w$

If

$$r_2(1-d_1)P(d_1D_{11}+D_{12}<C)-(r_2-r_1+r_1w)\leq 0 \qquad \text{(A2)}$$

then it is also true that

$$r_2(1-d_1)P(d_1D_{11}+D_{12}<C)-(r_2-r_2+r_1w)-r_2\delta P(C,c,D_{11},D_{12},D_{21},D_{22},0,l_2^*(c,0))\leq 0$$

Therefore, substituting (A2) into (A1) leads to

$$\begin{aligned}\frac{\partial ER_{Tatol}(l_1)}{\partial l_1}&\leq r_2(1-d_1)P(D_{11}-l_1)+D_{12}<C-l_1,D_{11}>l_1)\\&-r_2(1-d_1)P(d_1D_{11}+D_{12}<C)P(D_{11}>l_1)\\&-r_2\delta P(C,c,D_{11},D_{12},D_{21},D_{22},l_1,l_2^*(c,l_1))\\&\leq r_2(1-d_1)P(d_1D_{11}+D_{12}<C-l_1+d_1l_{11})P(D_{11}>l_1)\\&-r_2(1-d_1)P(d_1D_{11}+D_{12}<C)P(D_{11}>l_1)\\&-r_2\delta P(C,c,D_{11},D_{12},D_{21},D_{22},l_1,l_2^*(c,l_1))\\&\leq 0.\end{aligned}$$

It can be easily shown that for two independent random variables A and B, if Pr{B > b}>0, then Pr {A+B ≤ a | B > b} ≤ Pr{A+B ≤ a} for all a. Hence,

$$\begin{aligned}&r_2(1-d_1)P(d_1(D_{11}-l_1)+D_{12}<C-l_1,D_{11}>l_1)\\&=r_2(1-d_1)P(d_1D_{11}+D_{12}<C-l_1+d_1l_1|D_{11}>l_1,D_{11}>l_1)\\&\leq r_2(1-d_1)P(d_1D_{11}+D_{12}<C-l_1+d_1l_1)P(D_{11}>l_1)\end{aligned}$$

From the above, for a given C, if (A2) holds, then $\partial ER_T(l_1)/\partial l_1\leq 0$ at $l_1=0$, and $\partial ER_T(l_1)/\partial l_1\leq 0$ for all $l_1>0$. Therefore, a sufficient condition is

$$r_2(1-d_1)P(d_1D_{11}+D_{12}<C)\leq r_2-r_1+r_1w,$$

for the optimal booking limit in period 1 being 0. This completes the proof. ■