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FUZZY WEAKLY r-M CONTINUOUS FUNCTIONS ON FUZZY r-MINIMAL STRUCTURES

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ABSTRACT. In this paper, we introduce the concept of fuzzy weakly r-M continuous function on r-minimal structures and investigate characterizations and properties for such functions.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [11]. Chang [2] defined fuzzy topological spaces using fuzzy sets. In [3, 8], Chattopadhyay, Hazra and Samanta introduced a smooth fuzzy topological space which is a generalization of fuzzy topological space.

In [9], Yoo et al. introduced the concept of fuzzy r-minimal space which is an extension of the smooth fuzzy topological space. The concepts of fuzzy r-open sets, fuzzy r-semiopen sets, fuzzy r-preopen sets, fuzzy r-β-open sets and fuzzy r-regular open sets were introduced in [1, 4, 5, 6], which are various kinds of fuzzy r-minimal structures. The concept of fuzzy r-M continuity was also introduced and investigated in [9]. Min and Kim [10] introduced and studied the concepts of fuzzy r-minimal compactness, almost fuzzy r-minimal compactness and nearly fuzzy r-minimal compactness. In this paper, we introduce and study the concept of fuzzy weak r-M continuity which is a generalization of fuzzy r-M continuity. Finally, we investigate the relationships between fuzzy weakly r-M continuous functions and several types of fuzzy r-minimal compactness.

2. Preliminaries

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Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a fuzzy set of X. By $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$ we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{\mathbf{1}} - A$. All other notations are standard notations of fuzzy set theory.

A *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha \text{ if } y = x, \\ 0 \text{ if } y \neq x. \end{cases}$$

A fuzzy point x_α is said to belong to a fuzzy set A in X, denoted by $x_\alpha \in A$, if $\alpha \leq A(x)$ for $x \in X$.

A fuzzy set A in X is the union of all fuzzy points which belong to A.

Let $f : X \to Y$ be a function and $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} \sup\limits_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

for $y \in Y$ and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x)), x \in X$.

A *smooth fuzzy topology* [8] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{\mathbf{0}}) = \mathcal{T}(\tilde{\mathbf{1}}) = 1$.
(2) $\mathcal{T}(A_1 \wedge A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$.
(3) $\mathcal{T}(\vee A_i) \geq \wedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth fuzzy topological space.*

Definition 1 ([9])**.** Let X be a nonempty set and $r \in (0,1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a *fuzzy r-minimal structure* if the family

$$\mathcal{M}_r = \left\{ A \in I^X \mid \mathcal{M}(A) \geq r \right\}$$

contains $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}}$.

Then the $(X, \mathcal{M})$ is called a *fuzzy r-minimal space* (simply r-FMS) if $\mathcal{M}$ has a fuzzy r-minimal structure. Every member of $\mathcal{M}_r$ is called a *fuzzy r-minimal open* set. A fuzzy set A is called a *fuzzy r-minimal closed* set if the complement of A (simply, A^c) is a fuzzy r-minimal open set.

Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure and the fuzzy r-minimal interior of A [9], denoted by $mC(A,r)$ and $mI(A,r)$, respectively, are defined as

$$mC(A,r) = \bigcap \left\{ B \in I^X : B^c \in \mathcal{M}_r \text{ and } A \subseteq B \right\},$$

$$mI(A,r) = \bigcup \left\{ B \in I^X : B \in \mathcal{M}_r \text{ and } B \subseteq A \right\}.$$

Theorem 1 ([9]). *Let $(X, \mathcal{M})$ be an r-FMS and A, B in I^X.*

(1) *$mI(A,r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A,r) = A$.*

(2) *$A \subseteq mC(A,r)$ and if A is a fuzzy r-minimal closed set, then $mC(A,r) = A$.*

(3) *If $A \subseteq B$, then $mI(A,r) \subseteq mI(B,r)$ and $mC(A,r) \subseteq mC(B,r)$.*

(4) *$mI(A,r) \cap mI(B,r) \supseteq mI(A \cap B, r)$ and $mC(A,r) \cup mC(B,r) \subseteq mC(A \cup B, r)$.*

(5) *$mI\Big(mI(A,r), r\Big) = mI(A,r)$ and $mC(mC(A,r),r) = mC(A,r)$.*

(6) *$\tilde{\mathbf{1}} - mC(A,r) = mI\Big(\tilde{\mathbf{1}} - A, r\Big)$ and $\tilde{\mathbf{1}} - mI(A,r) = mC\Big(\tilde{\mathbf{1}} - A, r\Big)$.*

Definition 2 ([9]). Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two r-FMS's. Then a function $f : X \to Y$ is said to be

(1) *fuzzy r-M continuous* if for every fuzzy r-minimal open set A in Y, $f^{-1}(A)$ is fuzzy r-minimal open in X,

(2) *fuzzy r-M open* if for every fuzzy r-minimal open set G in X, $f(G)$ is fuzzy r-minimal open in Y.

3. Fuzzy weakly r-M continuous functions

Definition 3. Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then f is said to be *fuzzy weakly r-M continuous* if for fuzzy point x_α in X and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there is a fuzzy r-minimal open set U containing x_α such that $f(U) \subseteq mC(V,r)$.

Every fuzzy r-M continuous function f is clearly fuzzy weakly r-M continuous but the converse is not always true.

Example 1. Let $X = I$ and let A, B and C be fuzzy sets defined as follows

$$A(x) = \frac{1}{2}x, \quad x \in I;$$

$$B(x) = -\frac{1}{2}(x-1), \quad x \in I;$$

and

$$C(x) = \begin{cases} \frac{1}{2}(x+1), & \text{if } 0 \le x \le \frac{1}{2}, \\ -\frac{1}{2}(x-2), & \text{if } \frac{1}{2} < x \le 1. \end{cases}$$

Consider two fuzzy families $\mathcal{M}$, $\mathcal{N}$ defined as the following:

$$\mathcal{M}(\mu) = \begin{cases} \frac{2}{3}, & \text{if } \mu = \tilde{\mathbf{1}}, \\ \frac{1}{2}, & \text{if } \mu = \tilde{\mathbf{0}}, C, \\ 0, & \text{otherwise}; \end{cases}$$

$$\mathcal{N}(\mu) = \begin{cases} \frac{1}{2}, & \text{if } \mu = \tilde{\mathbf{0}}, \tilde{\mathbf{1}}, \\ \frac{2}{3}, & \text{if } \mu = A, B, \\ 0, & \text{otherwise}. \end{cases}$$

Then the identity function $f : (X, \mathcal{M}) \to (X, \mathcal{N})$ is fuzzy weakly $\frac{1}{2}$-M continuous but not fuzzy $\frac{1}{2}$-M continuous.

Theorem 2. *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:*

(1) *f is fuzzy weakly r-M continuous.*

(2) *$f^{-1}(V) \subseteq mI\Big(f^{-1}(mC(V,r)), r\Big)$ for each fuzzy r-minimal open set V in Y.*

(3) *$mC\Big(f^{-1}(mI(B,r)), r\Big) \subseteq f^{-1}(B)$ for each fuzzy r-minimal closed set B in Y.*

(4) *$mC\Big(f^{-1}(V), r\Big) \subseteq f^{-1}\Big(mC(V,r)\Big)$ for each fuzzy r-minimal open set V in Y.*

Proof. (1) $\Rightarrow$ (2) For a fuzzy r-minimal open set V in Y and each fuzzy point $x_\alpha \in f^{-1}(V)$, by hypothesis, there exists a fuzzy r-minimal open set U containing x_α such that $f(U) \subseteq mC(V,r)$. So $x_\alpha \in U \subseteq f^{-1}(mC(V,r))$ and this implies $x_\alpha \in mI\Big(f^{-1}(mC(V,r)), r\Big)$. Hence we have $f^{-1}(V) \subseteq mI\Big(f^{-1}(mC(V,r)), r\Big)$.

(2) $\Rightarrow$ (3) For any fuzzy r-minimal closed set B in Y, by (2) and Theorem 1,

$$\begin{aligned} f^{-1}(Y - B) \quad &\subseteq mI\Big(f^{-1}(mC(Y - B, r)), r\Big) \\ &= mI\Big(f^{-1}(Y - mI(B,r)), r\Big) \\ &= mI\Big(X - f^{-1}(mI(B,r)), r\Big) \\ &= X - mC\Big(f^{-1}(mI(B,r)), r\Big). \end{aligned}$$

Hence $mC\Big(f^{-1}(mI(B,r)), r\Big) \subseteq f^{-1}(B)$.

Similarly, we can prove that (3) $\Rightarrow$ (2).

(2) $\Rightarrow$ (4) For a fuzzy r-minimal open set V in Y, suppose $x_\alpha \notin f^{-1}(mC(V,r))$. Then since $f(x_\alpha) \notin mC(V,r)$, there exists a fuzzy r-minimal open set U containing $f(x_\alpha)$ such that $U \cap V = \emptyset$, and so $mC(U,r) \cap V = \emptyset$. And for the fuzzy r-minimal open set U, by (2), $x_\alpha \in f^{-1}(U) \subseteq mI\Big(f^{-1}(mC(U,r)), r\Big)$. Thus there exists a fuzzy r-minimal open set G containing x_α such that $x_\alpha \in G \subseteq f^{-1}\Big(mC(U,r)\Big)$. Since $mC(U,r) \cap V = \emptyset$, it is $G \cap f^{-1}(V) = \emptyset$, and so $x_\alpha \notin mC(f^{-1}(V), r)$.

(4) $\Rightarrow$ (1) Let x_α be a fuzzy point in X and V a fuzzy r-minimal open set in Y containing $f(x_\alpha)$. For each $x_\alpha \in f^{-1}(V)$,

$$\begin{aligned} x_\alpha \in f^{-1}(V) &\subseteq f^{-1}\Big(mI(mC(V,r),r)\Big) \\ &= X - f^{-1}\Big(mC(Y - mC(V,r),r)\Big) \\ &\subseteq X - mC\Big(f^{-1}(Y - mC(V,r)),r\Big) \\ &= mI\Big(f^{-1}(mC(V,r)),r\Big). \end{aligned}$$

Since $x_\alpha \in mI\Big(f^{-1}(mC(V,r)),r\Big)$, there exists a fuzzy r-minimal open set U containing x_α such that $U \subseteq f^{-1}(mC(V,r))$. Hence f is fuzzy weakly r-M continuous. □

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy family $\mathcal{M}$ is said to have the property $(\mathcal{U})$ [9] if for $A_i \in \mathcal{M}$ $(i \in J)$,

$$\mathcal{M}(\cup A_i) \geq \wedge \mathcal{M}(A_i).$$

Theorem 3 ([9]). *Let $(X, \mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$. Then*
(1) *$mI(A,r) = A$ if and only if $A \in \mathcal{M}_r$ for $A \in I^X$.*
(2) *$mC(A,r) = A$ if and only if $A^c \in \mathcal{M}_r$ for $A \in I^X$.*

Corollary 1. *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If $\mathcal{M}_X$ has the property $(\mathcal{U})$, then the following statements are equivalent:*

(1) *f is fuzzy weakly r-M continuous.*
(2) *$mC\Big(f^{-1}(mI(F,r)),r\Big) \subseteq f^{-1}(F)$ for each fuzzy r-minimal closed set F in Y.*
(3) *$mC\Big(f^{-1}(mI(mC(B,r),r)),r\Big) \subseteq f^{-1}\Big(mC(B,r)\Big)$ for each $B \in I^Y$.*
(4) *$f^{-1}(mI(B,r)) \subseteq mI\Big(f^{-1}(mC(mI(B,r),r)),r\Big)$ for each $B \in I^Y$.*
(5) *$mC(f^{-1}(V),r) \subseteq f^{-1}\Big(mC(V,r)\Big)$ for a fuzzy r-minimal open set V in Y.*

Theorem 4. *Let $f : X \to Y$ be a function on r-FMS's $(X, \mathcal{M}_X)$, $(Y, \mathcal{M}_Y)$ and $A \in I^Y$. If f is fuzzy weakly r-M continuous, then the following things are hold:*
(1) *$f^{-1}(A) \subseteq mI\Big(f^{-1}(mC(A,r)),r\Big)$ for $A = mI(A,r)$.*
(2) *$mC\Big(f^{-1}(mI(A,r)),r\Big) \subseteq f^{-1}(A)$ for $A = mC(A,r)$.*

Proof. (1) Let A be a fuzzy set in Y such that $A = mI(A,r)$. Then for each $x_\alpha \in f^{-1}(A)$, since $f(x_\alpha) \in mI(A,r)$, there exists a fuzzy r-minimal open set V containing $f(x_\alpha)$ such that $f(x_\alpha) \in V \subseteq A$. For the fuzzy r-minimal open set V containing $f(x_\alpha)$, from definition of fuzzy weakly r-M continuity, there exists a fuzzy r-minimal open set U containing x_α such that $f(U) \subseteq$

$mC(V,r)$. It implies $x_\alpha \in mI\Big(f^{-1}(mC(V,r)), r\Big) \subseteq mI\Big(f^{-1}(mC(A,r)), r\Big)$. Hence $f^{-1}(A) \subseteq mI\Big(f^{-1}(mC(A,r)), r\Big)$.

(2) It is similar to the proof of (1) □

Corollary 2. *Let $f : X \to Y$ be a function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ and $\mathcal{M}_X$ have property $(\mathcal{U})$. Then f is fuzzy weakly r-M continuous if and only if $f^{-1}(A) \subseteq mI\Big(f^{-1}(mC(A,r)), r\Big)$ for $A = mI(A,r)$ in Y.*

Proof. For $A \in I^Y$, if $A = mI(A,r)$, then A is a fuzzy r-minimal open set and so it is obtained from Theorem 2 (2). □

Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is said to be *fuzzy r-minimal semiopen* [7] if $A \subseteq mC\Big(mI(A,r), r\Big)$. A fuzzy set A is called a *fuzzy r-minimal semiclosed* set if the complement of A is a fuzzy r-minimal semiopen set.

Theorem 5. *Let $f : X \to Y$ be a function on r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If $\mathcal{M}_Y$ has the property $(\mathcal{U})$, then f is fuzzy weakly r-M continuous if and only if $mC\Big(f^{-1}(mI(mC(B,r), r)), r\Big) \subseteq f^{-1}\Big(mC(B,r)\Big)$ for each fuzzy r-minimal semiopen set B in Y.*

Proof. Suppose f is fuzzy weakly r-M continuous. For a fuzzy r-minimal semi - open set B in Y, since $\mathcal{M}_Y$ has the property $(\mathcal{U})$, $mC\Big(mI(B,r), r\Big)$ is fuzzy r-minimal closed. So by Theorem 2 (3) and $mC(B,r) = mC(mI(B,r), r)$,

$$\begin{aligned} mC\Big(f^{-1}(mI(mC(B,r), r)), r\Big) &= mC\Big(f^{-1}(mI(mC(mI(B,r), r), r)), r\Big) \\ &\subseteq f^{-1}\Big(mC(mI(B,r), r)\Big) \\ &\subseteq f^{-1}(mC(B,r)). \end{aligned}$$

Hence $mC\Big(f^{-1}(mI(mC(B,r), r)), r\Big) \subseteq f^{-1}(mC(B,r))$.

For the converse, let V be a fuzzy r-minimal open set in Y. Then V is also r-minimal semiopen, and from hypothesis and $V \subseteq mI\Big(mC(V,r), r\Big)$, it follows

$$mC\Big(f^{-1}(V), r\Big) \subseteq mC\Big(f^{-1}(mI(mC(V,r), r)), r\Big) \subseteq f^{-1}\Big(mC(V,r)\Big).$$

Hence by Theorem 2 (4), f is fuzzy weakly r-M continuous. □

We recall that the following notions introduced in [10]: Let $(X, \mathcal{M})$ be an r-FMS and $\mathcal{A} = \{A_i \in I^X : i \in J\}$. $\mathcal{A}$ is called a *fuzzy r-minimal cover* if $\cup\{A_i : i \in J\} = \tilde{1}$. It is a *fuzzy r-minimal open cover* if each A_i is a fuzzy

r-minimal open set. A subcover of a fuzzy r-minimal cover $\mathcal{A}$ is a subfamily of it which also is a fuzzy r-minimal cover. A fuzzy set A in X is said to be *fuzzy r-minimal compact* (resp. *almost fuzzy r-minimal compact*, *nearly fuzzy r-minimal compact*) if every fuzzy r-minimal open cover $\mathcal{A} = \{A_i \in \mathcal{M}_r : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \bigcup_{i \in J_0} A_i$ $\Big($resp. $A \subseteq \bigcup_{i \in J_0} mC(A_i, r)$, $A \subseteq \bigcup_{i \in J_0} mI(mC(A_i, r), r)\Big)$.

Theorem 6. *Let $f : X \to Y$ be a fuzzy weakly r-M continuous function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If A is a fuzzy r-minimal compact set in X and $\mathcal{M}_X$ has property $(\mathcal{U})$, then $f(A)$ is an almost fuzzy r-minimal compact set.*

Proof. Let $\Big\{B_i \in I^Y : i \in J\Big\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then from fuzzy weak r-M continuity, $f^{-1}(B_i) \subseteq mI\Big(f^{-1}(mC(B_i, r)), r\Big)$ for each $i \in J$. And by Theorem 3 and the property $(\mathcal{U})$ of $\mathcal{M}_X$, $\Big\{mI(f^{-1}(mC(B_i))) : i \in J\Big\}$ is a fuzzy r-minimal open cover of A in X. By the fuzzy r-minimal compactness, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that

$$A \subseteq \bigcup_{i \in J_0} mI\Big(f^{-1}(mC(B_i, r)), r\Big) \subseteq f^{-1}\Big(mC(B_i, r)\Big).$$

Hence $f(A) \subseteq \bigcup_{i \in J_0} mC(B_i, r)$. □

Theorem 7 ([9]). *Let $f : X \to Y$ be a function on two r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then*

(1) *f is fuzzy r-M open.*

(2) $f\Big(mI(A, r)\Big) \subseteq mI\Big(f(A), r\Big)$ *for* $A \in I^X$.

(3) $mI\Big(f^{-1}(B), r\Big) \subseteq f^{-1}\Big(mI(B, r)\Big)$ *for* $B \in I^Y$.

Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Theorem 8. *Let $f : X \to Y$ be a fuzzy weakly r-M continuous and fuzzy r-M open function between r-FMS's $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If A is an almost fuzzy r-minimal compact set and $\mathcal{M}_X$ has property $(\mathcal{U})$, then $f(A)$ is an almost fuzzy r-minimal compact set.*

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then by the property $(\mathcal{U})$, $\Big\{mI(f^{-1}(mC(B_i, r)), r) : i \in J\Big\}$ is a fuzzy r-minimal open cover of A in X. So there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\}$ of J such that $A \subseteq \bigcup_{i \in J_0} mC\Big(mI(f^{-1}(mC(B_i, r)), r), r\Big)$. From Theorem 4 and Theorem

7, it follows

$$\begin{aligned} A &\subseteq \bigcup_{i\in J_0} mC\Big(mI(f^{-1}(mC(B_i,r)),r),r\Big) \\ &\subseteq \bigcup_{i\in J_0} mC\Big(f^{-1}(mI(mC(B_i,r),r)),r\Big) \\ &\subseteq \bigcup_{i\in J_0} f^{-1}\Big(mC(B_i,r)\Big). \end{aligned}$$

Hence $f(A) \subseteq \bigcup_{i\in J_0} mC(B_i,r)$. □

Theorem 9. *Let $f : X \to Y$ be a fuzzy weakly r-M continuous and fuzzy r-M open function between r-FMS's $(X,\mathcal{M}_X)$ and $(Y,\mathcal{M}_Y)$. If A is a nearly fuzzy r-minimal compact set and $\mathcal{M}_X$ has property $(\mathcal{U})$, then $f(A)$ is a nearly fuzzy r-minimal compact set.*

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal open cover of $f(A)$ in Y. Then $\Big\{mI(f^{-1}(mC(B_i,r)),r) : i \in J\Big\}$ is a fuzzy r-minimal open cover of A in X. So there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\}$ of J such that $A \subseteq \bigcup_{i\in J_0} mI\Big(mC(mI(f^{-1}(mC(B_i,r)),r),r),r\Big)$ by the nearly fuzzy r-minimal compactness. From Theorem 4 and Theorem 7, it follows,

$$\begin{aligned} A &\subseteq \bigcup_{i\in J_0} mI\Big(mC(mI(f^{-1}(mC(B_i,r)),r),r),r\Big) \\ &\subseteq \bigcup_{i\in J_0} mI\Big(mC(f^{-1}(mI(mC(B_i,r),r)),r),r\Big) \\ &\subseteq \bigcup_{i\in J_0} mI\Big(f^{-1}(mC(mI(mC(B_i,r),r),r)),r\Big) \\ &\subseteq \bigcup_{i\in J_0} mI\Big(f^{-1}(mC(B_i,r)),r\Big) \\ &\subseteq \bigcup_{i\in J_0} f^{-1}\Big(mI(mC(B_i,r),r)\Big). \end{aligned}$$

Hence $f(A) \subseteq \bigcup_{i\in J_0} mI\Big(mC(B_i,r),r\Big)$. □

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