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EIGENVALUE PROBLEMS FOR SYSTEMS OF NONLINEAR HIGHER ORDER BOUNDARY VALUE PROBLEMS

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ABSTRACT. Values of the parameter λ are determined for which there exist positive solutions of the system of boundary value problems, $u^{(n)} + \lambda p(t)f(v) = 0$, $v^{(n)} + \lambda q(t)g(u) = 0$, for $t \in [a,b]$, and satisfying, $u^{(i)}(a) = 0$, $u^{(\alpha)}(b) = 0$, $v^{(i)}(a) = 0$, $v^{(\alpha)}(b) = 0$, for $0 \le i \le n-2$ and $1 \le \alpha \le n-1$ (but fixed). A well-known Guo-Krasnosel'skii fixed point theorem is applied.

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1. Introduction

In this study we are concerned with determining values of λ (eigenvalues) for which there exist positive solutions for the system of n-th order differential equations

$$\begin{aligned} u^{(n)}(t)+\lambda p(t)f(v(t)) &= 0, \ t \in [a,b], \\ v^{(n)}(t)+\lambda q(t)g(u(t)) &= 0, \ t \in [a,b], \end{aligned} \tag{1}$$

satisfying the boundary conditions

$$\begin{aligned} &u^{(i)}(a) = 0, \ 0 \le i \le n-2, \ u^{(\alpha)}(b) = 0, \ (1 \le \alpha \le n-1, \text{ (but fixed)}, \\ &v^{(i)}(a) = 0, \ 0 \le i \le n-2, \ v^{(\alpha)}(b) = 0, \ (1 \le \alpha \le n-1, \text{ (but fixed)}, \end{aligned} \tag{2}$$

where

(A1) $f, g \in C([0,\infty),[0,\infty))$,
(A2) $p, q \in C([a,b],[0,\infty))$, and each does not vanish identically on any closed subinterval of $[a,b]$,

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(A3) All of

$$f_0 := \lim_{x\to 0^+} \frac{f(x)}{x},\ g_0 := \lim_{x\to 0^+} \frac{g(x)}{x},$$

$$f_\infty := \lim_{x\to\infty} \frac{f(x)}{x},\ \text{and } g_\infty := \lim_{x\to\infty} \frac{g(x)}{x},$$

exist as positive real numbers.

There has been much interest recently in this area of obtaining optimal eigenvalue intervals of boundary value problems, often using Krasnoselskii fixed point theorems to obtain intervals based on positive solutions in side a cone. On a larger scale, there has been a great deal of study focused on positive solutions of boundary value problems for ordinary differential equations. Interest in such solutions is high from a theoretical sense [8, 9, 12, 14, 20] and as applications for which only positive solutions are meaningful [1, 9, 10, 16]. These considerations are caste primarily for scalar problems, but good attention has been given to boundary value problems for systems of differential equations [12, 13, 21, 22, 23].

This paper is organized as follows. In Section 2, we state and prove some lemmas which are needed in our main results. In Section 3, we establish a criteria to determine eigenvalue intervals for which the boundary value problem (1)-(2) has at least one positive solution, by using Krasnosel'skii fixed point theorem. In Section 4, as an application we give examples to demonstrate our results.

2. Green's function and bounds

In this section, we construct the Green's function and estimate the bounds of the Green's function for the homogeneous two point boundary value problem corresponding to (1)-(2).

Let $G(t,s)$ be the Green's function for the boundary value problem

$$\begin{aligned} &-y^{(n)} = 0, \\ &y^{(i)}(a) = 0,\ 0 \le i \le n-2, \\ &y^{(\alpha)}(b) = 0,\ 1 \le \alpha \le n-1, \text{ but fixed.} \end{aligned} \tag{3}$$

By using Cauchy function concept we construct the $G(t,s)$ as

$$G(t,s) = \begin{cases} \frac{(t-a)^{n-1}(b-s)^{n-\alpha-1}}{(n-1)!(b-a)^{n-\alpha-1}}, & a \le t \le s \le b, \\ \\ \frac{(t-a)^{n-1}(b-s)^{n-\alpha-1}}{(n-1)!(b-a)^{n-\alpha-1}} - \frac{(t-s)^{n-1}}{(n-1)!}, & a \le s \le t \le b. \end{cases} \tag{4}$$

Lemma 1. *For $(t,s) \in [a,b]\times[a,b]$, we have*

$$G(t,s) \le G(b,s). \tag{5}$$

Proof. For $a \leq t \leq s \leq b$, we have

$$G(t,s) = \frac{(t-a)^{n-1}(b-s)^{n-\alpha-1}}{(n-1)!(b-a)^{n-\alpha-1}} \leq \frac{(b-a)^{n-1}(b-s)^{n-\alpha-1}}{(n-1)!(b-a)^{n-\alpha-1}} = G(b,s).$$

Similarly, for $a \leq s \leq t \leq b$, we have $G(t,s) \leq G(b,s)$. Thus, we have

$$G(t,s) \leq G(b,s), \text{ for all } (t,s) \in [a,b] \times [a,b].$$

□

Lemma 2. *Let $I = [\frac{3a+b}{4}, \frac{a+3b}{4}]$. For $(t,s) \in I \times [a,b]$, we have*

$$G(t,s) \geq kG(b,s). \tag{6}$$

Proof. The Green's function $G(t,s)$ for the BVP (3) is clearly shows that

$$G(t,s) > 0 \text{ on } (a,b) \times (a,b). \tag{7}$$

For $a \leq t \leq s < b$ and $t \in I$, we have

$$\frac{G(t,s)}{G(b,s)} = \left(\frac{t-a}{b-a}\right)^{n-1} \geq \frac{1}{4^{n-1}}.$$

Similarly, for $a \leq s \leq t < b$ and $t \in I$ we have

$$\begin{aligned}\frac{G(t,s)}{G(b,s)} &= \frac{(t-a)^{n-1}(b-s)^{n-\alpha-1} - (t-s)^{n-1}(b-a)^{n-\alpha-1}}{(b-a)^{n-1}(b-s)^{n-\alpha-1} - (b-s)^{n-1}(b-a)^{n-\alpha-1}} \\ &\geq \frac{(t-a)^{n-\alpha-1}(b-s)^{n-\alpha-1}[(t-a)^{\alpha} - (t-s)^{\alpha}]}{(b-a)^{n-1}(b-s)^{n-\alpha-1} - (b-s)^{n-1}(b-a)^{n-\alpha-1}} \\ &= \frac{1}{\alpha}\left(\frac{t-a}{b-a}\right)^{n-2} \geq \frac{1}{\alpha}\left(\frac{t-a}{b-a}\right)^{n-1} \geq \frac{1}{\alpha.4^{n-1}}.\end{aligned}$$

Therefore $G(t,s) \geq kG(b,s)$, where

$$\gamma = \min\left\{\frac{1}{4^{n-1}}, \frac{1}{\alpha.4^{n-1}}\right\}. \tag{8}$$

□

We note that a pair $(u(t), v(t))$ is a solution of the eigenvalue problem (1), (2) if, and only if,

$$u(t) = \lambda \int_a^b G(t,s)p(s)f\left(\lambda \int_a^b G(s,r)q(r)g(u(r))dr\right) ds, \; a \leq t \leq b,$$

where

$$v(t) = \lambda \int_a^b G(t,s)q(s)g(u(s))ds, \; a \leq t \leq b.$$

Values of λ for which there are positive solutions (positive with respect to a cone) of (1), (2) will be determined via applications of the following fixed point-theorem, which is now commonly called the Guo-Krasnosel'skii fixed point theorem.

Theorem 1. *Let X be a Banach space, and let $X \subset \kappa$ be a cone in X. Assume that Ω_1 and Ω_2 are open subsets of X with $0 \in \Omega_1 \subset \overline{\Omega}_1 \subset \Omega_2$, and let*

$$T : \kappa \cap (\overline{\Omega}_2 \backslash \Omega_1) \to \kappa$$

be a completely continuous operator such that, either

(i) $\| Tu \| \leq \| u \|$, $u \in \kappa \cap \partial\Omega_1$, *and* $\| Tu \| \geq \| u \|$, $u \in \kappa \cap \partial\Omega_2$, *or*
(ii) $\| Tu \| \geq \| u \|$, $u \in \kappa \cap \partial\Omega_1$, *and* $\| Tu \| \leq \| u \|$, $u \in \kappa \cap \partial\Omega_2$.

Then T has a fixed point in $\kappa \cap (\overline{\Omega}_2 \backslash \Omega_1)$.

3. Positive solutions in a cone

In this section, we apply Theorem 1 to obtain solutions in a cone (that is, positive solutions) of (1), (2).

For our construction, let $X = \{x : [a,b] \to \mathbb{R}\}$ with supremum norm $\| x \| = \sup\{| x(t) | : t \in [a,b]\}$ and define a cone $\kappa \subset X$ by

$$\kappa = \left\{ x \in X \mid x(t) \geq 0 \text{ on } [a,b], \text{ and } \min_{t \in I} x(t) \geq \gamma \| x \| \right\}.$$

For our first result, let $\tau \in [a,b]$ be defined by

$$\max_{t \in [a,b]} \int_{s \in I} G(t,s)p(s)ds = \int_{s \in I} G(\tau,s)p(s)ds,$$

and define positive numbers L_1 and L_2 by

$$L_1 := \max\left\{ \left[\gamma \int_{s \in I} G(\tau,s)p(s)ds f_\infty\right]^{-1}, \left[\gamma \int_{s \in I} G(\tau,s)q(s)ds g_\infty\right]^{-1} \right\},$$

$$L_2 := \min\left\{ \left[\int_a^b G(b,s)p(s)ds f_0\right]^{-1}, \left[\int_a^b G(b,s)q(s)ds g_0\right]^{-1} \right\}.$$

Theorem 2. *Assume conditions $(A1) - (A3)$ are satisfied. Then, for each λ satisfying*

$$L_1 < \lambda < L_2, \tag{9}$$

there exists a pair (u,v) satisfying (1), (2) such that $u(x) > 0$ and $v(x) > 0$ on (a,b).

Proof. Let λ be as in (9), and let $\epsilon > 0$ be chosen such that

$$\max\left\{\left[\gamma\int_{s\in I} G(\tau,s)p(s)ds(f_\infty-\epsilon)\right]^{-1},\left[\gamma\int_{s\in I} G(\tau,s)q(s)ds(g_\infty-\epsilon)\right]^{-1}\right\}\le\lambda,$$

$$\lambda\le\min\left\{\left[\int_a^b G(b,s)p(s)ds(f_0+\epsilon)\right]^{-1},\left[\int_a^b G(b,s)q(s)ds(g_0+\epsilon)\right]^{-1}\right\}.$$

Define an integral operator $T:\kappa\to X$ by

$$Tu(t):=\lambda\int_a^b G(t,s)p(s)f\left(\lambda\int_a^b G(s,r)q(r)g(u(r))dr\right)ds,\ u\in\kappa. \tag{10}$$

By the remarks in Section 2, it suffices to exhibit fixed points of T in the cone κ.

First, from $(A1)$, $(A2)$, and (7), for $u\in\kappa$, $Tu(t)\ge 0$ on $[a,b]$. Also, for $u\in\kappa$, we have from (5) that

$$\begin{aligned}Tu(t)&=\lambda\int_a^b G(t,s)p(s)f\left(\lambda\int_a^b G(s,r)q(r)g(u(r))dr\right)ds\\&\le\lambda\int_a^b G(b,s)p(s)f\left(\lambda\int_a^b G(s,r)q(r)g(u(r))dr\right)ds,\end{aligned}$$

and so

$$\| Tu \|\le\lambda\int_a^b G(b,s)p(s)f\left(\lambda\int_a^b G(s,r)q(r)g(u(r))dr\right)ds.$$

Next, if $u\in\kappa$, we have from (6) and (10),

$$\begin{aligned}\min_{t\in I}Tu(t)&=\min_{t\in I}\lambda\int_a^b G(t,s)p(s)f\left(\lambda\int_a^b G(s,r)q(r)g(u(r))dr\right)ds\\&\ge\lambda\gamma\int_a^b G(b,s)p(s)f\left(\lambda\int_a^b G(s,r)q(r)g(u(r))dr\right)ds\\&\ge\gamma\| Tu \|.\end{aligned}$$

Consequently, $T:\kappa\to\kappa$. In addition, standard arguments show that T is completely continuous.

Now, from the definitions of f_0 and g_0, there exists an $H_1>0$ such that

$$f(x)\le(f_0+\epsilon)x \text{ and } g(x)\le(g_0+\epsilon)x,\ 0<x\le H_1.$$

Let $u \in \kappa$ with $\| u \| = H_1$. First, from (5) and the choice of ϵ, we have

$$\begin{aligned}\lambda \int_a^b G(s,r)q(r)g(u(r))dr &\le \lambda \int_a^b G(b,r)q(r)g(u(r))dr \\ &\le \lambda \int_a^b G(b,r)q(r)(g_0+\epsilon)u(r)dr \\ &\le \lambda \int_a^b G(b,r)q(r)dr(g_0+\epsilon) \| u \| \\ &\le \| u \| = H_1.\end{aligned}$$

As a consequence, in view of (6) and the choice of ϵ, we obtain

$$\begin{aligned}Tu(t) &= \lambda \int_a^b G(t,s)p(s)f\left(\lambda \int_a^b G(s,r)q(r)g(u(r))dr\right)ds \\ &\le \lambda \int_a^b G(b,s)p(s)(f_0+\epsilon)\lambda \int_a^b G(s,r)q(r)g(u(r))drds \\ &\le \lambda \int_a^b G(b,s)p(s)(f_0+\epsilon)H_1 ds \le H_1 \\ &= \| u \| .\end{aligned}$$

So, $\| Tu \| \le \| u \|$, for every $u \in \kappa$ with $\| u \| = H_1$. Hence if we set

$$\Omega_1 = \{x \in X \mid \| x \| < H_1\},$$

then

$$\| Tu \| \le \| u \|, \text{ for } u \in \kappa \cap \partial\Omega_1. \tag{11}$$

Next, by the definitions of f_∞ and g_∞, there exists an $\overline{H}_2 > 0$ such that

$$f(x) \ge (f_\infty - \epsilon)x \text{ and } g(x) \ge (g_\infty - \epsilon)x,\ x \ge \overline{H}_2.$$

Let $H_2 = \max\left\{2H_1, \frac{\overline{H}_2}{\gamma}\right\}$. Then, for $u \in \kappa$ and $\| u \| = H_2$,

$$\min_{t \in I} u(t) \ge \gamma \| u \| \ge \overline{H}_2.$$

Consequently, from (6) and the choice of ϵ, we find

$$\begin{aligned}\lambda \int_a^b G(s,r)q(r)g(u(r))dr &\ge \lambda \int_{r \in I} G(s,r)q(r)g(u(r))dr \\ &\ge \lambda \int_{r \in I} G(\tau,r)q(r)g(u(r))dr\end{aligned}$$

$$\geq \lambda \int_{r\in I} G(\tau,r)q(r)(g_\infty - \epsilon)u(r)dr$$
$$\geq \gamma\lambda \int_{r\in I} G(\tau,r)q(r)(g_\infty - \epsilon)dr \parallel u \parallel$$
$$\geq \parallel u \parallel = H_2.$$

And so, we have from (6) and the choice of ϵ,

$$\begin{aligned} Tu(\tau) &= \lambda \int_a^b G(\tau,s)p(s)f\left(\lambda \int_a^b G(s,r)q(r)g(u(r))dr\right)ds \\ &\geq \lambda \int_a^b G(\tau,s)p(s)(f_\infty - \epsilon)\lambda \int_a^b G(s,r)q(r)g(u(r))drds \\ &\geq \lambda \int_a^b G(\tau,s)p(s)(f_\infty - \epsilon)H_2 ds \\ &\geq \gamma H_2 \\ &> H_2 = \parallel u \parallel . \end{aligned}$$

Hence, $\parallel Tu \parallel \geq \parallel u \parallel$. So if we set $\Omega_2 = \{x \in X \mid \parallel x \parallel < H_2\}$, then

$$\parallel Tu \parallel \geq \parallel u \parallel, \text{ for } u \in \kappa \cap \partial\Omega_2. \tag{12}$$

In view of (11) and (12), applying Theorem 1, we obtain that T has a fixed point $u \in \kappa \cap (\overline{\Omega}_2 \backslash \Omega_1)$. As such, and with v defined by

$$v(t) = \lambda \int_a^b G(t,s)q(s)g(u(s))ds,$$

the pair (u,v) is a desired solution of (1), (2) for the given λ. This completes the proof. □

Prior to our next result, we introduce another hypothesis. $(A4)$ $g(0) = 0$, and f is an increasing function.

We now define positive numbers L_3 and L_4 by

$$L_3 := \max\left\{\left[\gamma \int_{s\in I} G(\tau,s)p(s)dsf_0\right]^{-1}, \left[\gamma \int_{s\in I} G(\tau,s)q(s)dsg_0\right]^{-1}\right\},$$
$$L_4 := \min\left\{\left[\int_a^b G(b,s)p(s)dsf_\infty\right]^{-1}, \left[\int_a^b G(b,s)q(s)dsg_\infty\right]^{-1}\right\}.$$

Theorem 3. *Assume conditions $(A1) - (A4)$ are satisfied. Then, for each λ satisfying*

$$L_3 < \lambda < L_4, \tag{13}$$

there exists a pair (u,v) satisfying (1), (2) such that $u(x) > 0$ and $v(x) > 0$ on (a,b).

Proof. Let λ be as in (13), and let $\epsilon > 0$ be chosen such that

$$\max\left\{\left[\gamma\int_{s\in I} G(\tau,s)p(s)ds(f_0-\epsilon)\right]^{-1}, \left[\gamma\int_{s\in I} G(\tau,s)q(s)ds(g_0-\epsilon)\right]^{-1}\right\} \le \lambda,$$

$$\lambda \le \min\left\{\left[\int_a^b G(b,s)p(s)ds(f_\infty+\epsilon)\right]^{-1}, \left[\int_a^b G(b,s)q(s)ds(g_\infty+\epsilon)\right]^{-1}\right\}.$$

Let T be the cone preserving, completely continuous operator defined by (10).

By the definitions of f_0 and g_0, there exists an $H_1 > 0$ such that

$$f(x) \ge (f_0-\epsilon)x \text{ and } g(x) \ge (g_0-\epsilon)x,\ 0 < x \le H_1$$

Also, from the definition of g_0 it follows that $g(0) = 0$, and so there exists $0 < H_2 < H_1$ such that

$$\lambda g(x) \le \frac{H_1}{\int_a^b G(b,s)q(s)ds},\ 0 \le x \le H_2.$$

Let $u \in \kappa$ with $\| u \| = H_2$. Then,

$$\lambda\int_a^b G(s,r)q(r)g(u(r))dr \le \frac{\int_a^b G(s,r)q(r)H_1 dr}{\int_a^b G(b,s)q(s)ds} \le H_1.$$

Then,

$$\begin{aligned} Tu(\tau) &= \lambda\int_a^b G(\tau,s)p(s)f\left(\lambda\int_a^b G(s,r)q(r)g(u(r))dr\right)ds \\ &\ge \lambda\int_{s\in I} G(\tau,s)p(s)(f_0-\epsilon)\lambda\int_a^b G(s,r)q(r)g(u(r))drds \\ &\ge \lambda\int_{s\in I} G(\tau,s)p(s)(f_0-\epsilon)\lambda\int_{r\in I} G(\tau,r)q(r)g(u(r))drds \\ &\ge \lambda\int_{s\in I} G(\tau,s)p(s)(f_0-\epsilon)\lambda\gamma\int_{r\in I} G(\tau,r)q(r)(g_0-\epsilon)\| u \| drds \\ &\ge \lambda\int_{s\in I} G(\tau,s)p(s)(f_0-\epsilon)\| u \| ds \\ &\ge \lambda\gamma\int_{s\in I} G(\tau,s)p(s)(f_0-\epsilon)\| u \| ds \\ &\ge \| u \| . \end{aligned}$$

So, $\| Tu \| \ge \| u \|$. If we put $\Omega_1 = \{x \in X \mid \| x \| < H_2\}$, then

$$\| Tu \| \ge \| u \|, \quad \text{for } u \in \kappa \cap \partial\Omega_1. \tag{14}$$

Next, by the definitions of f_∞ and g_∞, there exists an $\overline{H}_1$ such that

$$f(x) \le (f_\infty+\epsilon)x \text{ and } g(x) \le (g_\infty+\epsilon)x,\ x \ge \overline{H}_1$$

There are two cases: (i) g is bounded, and (ii) g is unbounded.

For case (i), suppose $N > 0$ is such that $g(x) \leq N$ for all $0 < x < \infty$. Then, for $a \leq s \leq b$ and $u \in \kappa$,

$$\lambda \int_a^b G(s,r)q(r)g(u(r))dr \leq N\lambda \int_a^b G(b,r)q(r)dr.$$

Let

$$M = \max\left\{ f(x) \mid 0 \leq x \leq N\lambda \int_a^b G(b,r)q(r)dr \right\},$$

and let

$$H_3 > \max\left\{ 2H_2, M\lambda \int_a^b G(b,s)p(s)ds \right\}.$$

Then, for $u \in \kappa$ with $\| u \| = H_3$,

$$\begin{aligned} Tu(t) &\leq \lambda \int_a^b G(b,s)p(s)Mds \\ &\leq H_3 = \| u \| \end{aligned}$$

so that $\| Tu \| \leq \| u \|$. If $\Omega_2 = \{x \in X \mid \| x \| < H_3\}$, then

$$\| Tu \| \leq \| u \|, \text{ for } u \in \kappa \cap \partial\Omega_2. \tag{15}$$

For case (ii), there exists $H_3 > \max\{2H_2, \overline{H}_1\}$ such that $g(x) \leq g(H_3)$, for $0 < x \leq H_3$. Similarly, there exists $H_4 > \max\{H_3, \lambda \int_a^b G(b,r)q(r)g(H_3)dr\}$ such that $f(x) \leq f(H_4)$, for $0 < x \leq H_4$. Choosing $u \in \kappa$ with $\| u \| = H_4$, we have by $(A4)$ that

$$\begin{aligned} Tu(t) &\leq \lambda \int_a^b G(t,s)p(s)f\left(\lambda \int_a^b G(b,r)q(r)g(H_3)dr\right)ds \\ &\leq \lambda \int_a^b G(t,s)p(s)f(H_4)ds \\ &\leq \lambda \int_a^b G(b,s)p(s)ds(f_\infty + \epsilon)H_4 \\ &\leq H_4 = \| u \|, \end{aligned}$$

and so $\| Tu \| \leq \| u \|$. For this case, if we set $\Omega_2 = \{x \in X \mid \| x \| < H_4\}$, then

$$\| Tu \| \leq \| u \|, \text{ for } u \in \kappa \cap \partial\Omega_2. \tag{16}$$

In either of the cases, application of part (ii) of Theorem 1 yields a fixed point u of T belonging to $\kappa \cap (\overline{\Omega}_2 \backslash \Omega_1)$, which in turn yields a pair (u, v) satisfying (1), (2) for the chosen value of λ. This completes the proof. □

4. Examples

In this section we give some examples illustrating our results. For the sake of simplicity we take $p(t) = q(t)$ and $f(t) = g(t)$.

Example 1. Consider the system of two-point boundary value problem

$$u''(t) + \frac{1}{10}\lambda t \frac{kve^{2v}}{c + e^v + e^{2v}} = 0, \; 0 < t < 1,$$
$$v''(t) + \frac{1}{10}\lambda t \frac{kue^{2u}}{c + e^u + e^{2u}} = 0, \; 0 < t < 1,$$
$$u(0) = 0 = u'(1),$$
$$v(0) = 0 = v'(1).$$

Here: $p(t) = q(t) = \frac{1}{10}t$, $k = 500$, $c = 1000$, $f(v) = \frac{kve^{2v}}{c+e^v+e^{2v}}$, $f(u) = \frac{kue^{2u}}{c+e^u+e^{2u}}$. By simple calculations we find: $\gamma = \frac{1}{4}$, $f_0 = g_0 = \frac{k}{c+2} = \frac{500}{1002}$, $f_\infty = g_\infty = k = 500$, $L_1 = 0.1969$, $L_2 = 6.012$. By Theorem 2 it follows that for every λ such that $0.1969 < \lambda < 6.012$, the two-point boundary value problem has at least one positive solution.

Example 2. Consider the system of two-point boundary value problem

$$u'''(t) + \lambda tv\left(1 + \frac{c}{1 + v^2}\right) = 0, \; 0 < t < 1,$$
$$v'''(t) + \lambda tu\left(1 + \frac{c}{1 + u^2}\right) = 0, \; 0 < t < 1,$$
$$u(0) = u'(0) = u'(1) = 0,$$
$$v(0) = v'(0) = v'(1) = 0.$$

Here: $p(t) = q(t) = t$, $c = 100$, $f(v) = v\left(1 + \frac{c}{1+v^2}\right)$, $f(u) = u\left(1 + \frac{c}{1+u^2}\right)$. By simple calculations we find: $\gamma = \frac{1}{16}$, $f_0 = g_0 = 1 + c = 101$, $f_\infty = g_\infty = 1$, $L_3 = 2.7645$, $L_4 = 12$. Therefore Theorem 3 holds for every λ such that $2.7645 < \lambda < 12$.

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