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DELAY-DEPENDENT GLOBAL ASYMPTOTIC STABILITY ANALYSIS OF DELAYED CELLULAR NEURAL NETWORKS

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ABSTRACT. In this paper, the problem of delay-dependent stability analysis for cellular neural networks systems with time-varying delays was considered. By using a new Lyapunov-Krasovskii function, delay-dependant stability conditions of the delayed cellular neural networks systems are proposed in terms of linear matrix inequalities (LMIs). Examples are provided to demonstrate the reduced conservatism of the proposed stability results.
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1. Introduction

Over the past years, cellular neural networks(CNNs) (see [6]), have been widely investigated and have found applications in many areas such as image processing, pattern recognition, signal processing, solving nonlinear algebraic equations [7] - [8]. Such applications heavily depend on the dynamical behaviors. In recent years, the stability problem, which is one of the most important issues on the analysis of the dynamical behavior has received much attention, and many results on this problem have been reported; e.g., [9], and the references therein.

On the other hand, time delays are unavoidably in application of neural networks, and a time delay is often a source of instability and oscillations in dynamic system. So the stability analysis of delayed cellular networks(DCNNs) has become one of the most active research areas and has attracted much attention during the past years [1] - [5], [11] - [24]. For example, stability conditions for delayed Hopfield neural networks have been reported in the literature [21] and [23]. In [1], [4] and [15], the exponential stability of delayed neural networks are studied and sufficient conditions are obtained. Delay-dependent and Delay-independent global asymptotical stability criterions are given in [2] - [3], [5], [13]

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- [14], [17] - [20], [24], among them [3] and [24] considered the global asymptotical stability of DCNNs with uncertain parameters.

In this paper, we are concerned with the problem of global asymptotic stability analysis of DCNNs and DCNNs with uncertain parameters. A new Lyapunov-Krasovskii function, in which cross terms between the state and nonlinear function of the state as well as the delayed state and nonlinear function of the delayed are included, is introduced. By using the Leibniz-Newwton formula and some LMI techniques , delay-dependent global asymptotic stability criterions are derived based on the new Lyapunov-Krasovskii function. The stability results derived from the Lyapunov-Krasovskii function with cross terms are less conservative than those derived from Lyapunov-Krasovskii function without cross terms. Numerical examples are also provided to demonstrate the less conservativeness of the proposed approach.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (or $X > Y$) means that the matrix $X - Y$ is positive semi-definite (or positive definite). The notation X^T and X^{-1} mean the transpose of and the inverse of a square matrix X. $\|x\|$ denotes the Euclidean norm of a vector x. Matrices, if not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation

Consider the following DCNN:

$$\dot{u}(t) = -Au(t) + Bg(u(t)) + B_1 g(u(t - \tau(t))) + v \tag{1}$$

$$x(t) = \phi(t), \qquad t \in [-h, 0] \tag{2}$$

where $u(t) = [u_1(t), u_2(t), \dots, u_n]^T$ is the neural state vector, $A = diag(a_1, a_2, \dots, a_n)$ is a positive diagonal matrix, $g(u(t)) = [g_1(u_1(t)), g_2(u_2(t)), \dots, g_n(u_n(t))]^T$ denotes the neuron activation with $g(0) = 0$, and $v = [v_1, v_2, \dots, v_n]^T$ is a constant input vector, $B = (b_{ij})_{n\times n}$ and $B_1 = (b^1_{ij})_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neorons.The delay $\tau(t)$ is a time-varying differentiable function satisfying

$$\dot{\tau}(t) \leq \mu, \quad 0 < \tau(t) \leq h. \tag{3}$$

In addition, in the analysis of neural networks, it is usually assumed that the activation functions are continuous, differentiable, monotonically increasing and bounded. In this paper, we will assume that the activation functions are bounded and satisfy the following condition:

$$0 \leq \frac{g_j(x) - g_j(y)}{x - y} \leq \sigma_j, \quad \forall\, x, y \in \Re,\ x \neq y,\ j = 1, 2, \dots, n \tag{4}$$

where $\sigma_j,\ j = 1, 2, \dots, n$ are positive constants.

In the following, we always shift the equilibrium point u^* of system (1) to the origin by the transformation $x^* = u - u^*$, which changes system (1) to

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + B_1 f(x(t - \tau(t))) \tag{5}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$ is the state vector of the transformed system, and $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T$, $f_j(x_j) = g_j(x_j - u_j^*) - g_j(u_j^*)$. Note that the functions $f_j(\cdot), j = 1, 2, \ldots, n$ satisfy the following conditions:

$$0 \leq \frac{f_j(x_j)}{x_j} \leq \sigma_j,\ f_j(0) = 0, \quad \forall\ x_j \neq 0,\ j = 1, 2, \ldots, n. \tag{6}$$

The main purpose of this paper is to establish delay-dependent sufficient condition based on linear matrix inequalities(LMIs) for checking the global asymptotical stability of the delayed cellular neural networks. In Section 3, DCNNs and DCNNs with uncertain parameters are considered respectively and stability criterions are stated. Numerical examples are provided to demonstrate the reduced conservativeness of the proposed results in Section 4, Concluding remarks are given in Section 5.

3. Stability Criterion

At first we introduce the following lemma, which will be used in the proof of our main result.

Lemma 1 [10](Jensen Inequality) For a matrix $P > 0$, scalar $\tau(t) \leq h_M$, and any differentiable vector function $x(t)$ with appropriate dimension, we have

$$\left(\int_{t-\tau(t)}^{t} \dot{x}(s)ds\right)^T P \left(\int_{t-\tau(t)}^{t} \dot{x}(s)ds\right) \leq h_M \int_{t-\tau(t)}^{t} \dot{x}^T(s)P\dot{x}(s)ds \tag{7}$$

Now we are in a position to give an asymptotic stability condition for the delayed cellular neural network system (5).

Theorem 1 Under the assumption (6), the origin of the DCNN in (5) is globally asymptotically stable for any delay satisfying $0 < \tau(t) \leq h, \dot{\tau}(t) \leq \mu$ if there exist matrices $P = P^T > 0,\ Q = Q^T > 0, N_1,\ N_2,\ N_4$, and diagonal matrices $S > 0,\ H > 0,\ U > 0,\ N_3 > 0$ such that the following LMIs hold:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} \\ * & * & \Phi_{33} & \Phi_{34} & \Phi_{35} \\ * & * & * & \Phi_{44} & \Phi_{45} \\ * & * & * & * & \Phi_{55} \end{bmatrix} < 0 \tag{8}$$

$$\Psi = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0 \tag{9}$$

where
$\Phi_{11} = -PA - AP + P_{11} - N_1 - N_1^T + h^2 A^T Q A$,
$\Phi_{12} = N_1 - N_2^T,\ \Phi_{13} = PB + P_{12} + H\Sigma - h^2 A^T Q B$,
$\Phi_{14} = PB_1 - h^2 A^T Q B_1 - N_4^T,\ \Phi_{15} = N_1$,
$\Phi_{22} = -(1-\mu)P_{11} + N_2^T + N_2,\ \Phi_{23} = N_3^T$,
$\Phi_{24} = -(1-\mu)P_{12} + U\Sigma + N_4^T,\ \Phi_{25} = N_2$,
$\Phi_{33} = SB + B^T S + P_{22} - 2SA\Sigma^{-1} - 2H - 2N_3\Sigma^{-1}$

$+ h^2B^TQB$, $\Phi_{34} = h^2B^TQB_1 + SB_1$,
$\Phi_{35} = N_3, \Phi_{44} = -(1-\mu)P_{22} - 2U + h^2B_1^TQB_1$,
$\Phi_{45} = N_4$, $\Phi_{55} = -Q$, $\Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_n)$ and $*$ denotes the corresponding transposed block matrix due to symmetry.

Proof. We construct Lyapunov-Krasovskii function as following:

$$V(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)) \tag{10}$$

where

$$V_1(x(t)) = x^T(t)Px(t) + 2\textstyle\sum_{i=1}^n s_i \int_0^{x_i} f_i(s)ds \tag{11}$$

$$V_2(x(t)) = \textstyle\int_{t-\tau(t)}^t \eta^T(s)\Psi\eta(s)ds \tag{12}$$

$$V_3(x(t)) = h\textstyle\int_{-h}^0 \int_{t+\beta}^t \dot{x}^T(\alpha)Q\dot{x}(\alpha)d\alpha d\beta \tag{13}$$

where Ψ is defined in (9) and $\eta(s) = \begin{bmatrix} x^T(s) & f^T(x(s)) \end{bmatrix}^T$. The derivative of $V_1(x(t))$ along the solution of system (5) is given by

$$\begin{aligned}
\dot{V}_1(x(t)) &= 2x^T(t)P\dot{x}(t) + 2\sum_{i=1}^n s_i f_i(x_i)\dot{x}_i \\
&= 2x^TP[-Ax(t) + Bf(x(t)) + B_1f(x(t-\tau(t)))] \\
&\quad +2f^T(x(T))S[-Ax(t) + Bf(x(t)) \\
&\quad +B_1f(x(t-\tau(t)))] \\
&= -\ 2x^T(t)PAx(t) + 2x^T(t)PBf(x(t)) \\
&\quad +\ 2x^T(t)PB_1f(x(t-\tau(t))) \\
&\quad -\ 2f^T(x(t))SAx(t) + 2f^T(x(t))SBf(x(t)) \\
&\quad +\ 2f^T(x(t))SB_1f(x(t-\tau(t)))
\end{aligned} \tag{14}$$

Considering the relationship in (6), for a diagonal matrix $H > 0$, we can deduce

$$2f^T(x(t))Hf(x(t)) \le 2x^T(t)H\Sigma f(x(t)) \tag{15}$$

Then, by (14) and (15), it can be shown that

$$\begin{aligned}
\dot{V}_1(x(t)) \le &-2x^T(t)PAx(t) + 2x^T(t)(PB + H\Sigma)f(x(t)) \\
&+2x^T(t)PB_1f(x(t-\tau(t))) \\
&+f^T(x(t))(2SB - 2SA\Sigma^{-1} - 2H)f(x(t)) \\
&+2f^T(x(t))SB_1f(x(t-\tau(t)))
\end{aligned} \tag{16}$$

After some algebraic manipulations, the derivative of $V_2(x(t))$ along the solution of system (5) provides

$$\begin{aligned}\dot{V}_2(x(t)) &= \eta^T(t)\Psi\eta(t) - (1-\dot{\tau}(t))\eta^T(t-\tau(t))\Psi\eta(t-\tau(t)) \\ &= \begin{bmatrix} x^T(t) & f^T(x(t)) \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \\ &\quad -(1-\dot{\tau}(t)) \begin{bmatrix} x^T(t-\tau(t)) & f^T(x(t-\tau(t))) \end{bmatrix} \\ &\quad \times \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t))) \end{bmatrix} \\ &\le x^T(t)P_{11}x(t) + 2x^T(t)P_{12}f(x(t)) + f^T(x(t))P_{22} \\ &\quad \times f(x(t)) - (1-\mu)x^T(t-\tau(t))P_{11}x(t-\tau(t)) \\ &\quad -2(1-\mu)x^T(t-\tau(t))P_{12}f(x(t-\tau(t))) \\ &\quad -(1-\mu)f^T(x(t-\tau(t)))P_{22}f(x(t-\tau(t)))\end{aligned} \tag{17}$$

Similar to (15), for a diagonal matrix $U > 0$, we have

$$\begin{aligned}&2f^T(x(t-\tau(t)))Uf(x(t-\tau(t))) \\ &\qquad \le x^T(t-\tau(t))U\Sigma f(x(t-\tau(t)))\end{aligned} \tag{18}$$

By (17) and (18), we deduce

$$\begin{aligned}\dot{V}_2(x(t)) &\le x^T(t)P_{11}x(t) + 2x^T(t)P_{12}f(x(t)) \\ &\quad + f^T(x(t))P_{22}f(x(t)) \\ &\quad - (1-\mu)x^T(t-\tau(t))P_{11}x(t-\tau(t)) \\ &\quad + 2x^T(t-\tau(t))[U\Sigma - (1-\mu)P_{12}]f(x(t-\tau(t))) \\ &\quad - f^T(x(t-\tau(t)))[(1-\mu)P_{22} + 2U]f(x(t-\tau(t)))\end{aligned} \tag{19}$$

The derivative of $V_3(x(t))$ along the solution of system (5) yields

$$\begin{aligned}\dot{V}_3(x(t)) &= h\int_{-h}^{0} [\dot{x}^T(t)Q\dot{x}(t) - \dot{x}^T(t+\beta)Q\dot{x}(t+\beta)]d\beta \\ &= h^2\dot{x}^T(t)Q\dot{x}(t) - h\int_{-h}^{0} \dot{x}^T(t+\beta)Q\dot{x}(t+\beta)d\beta \\ &= h^2\dot{x}^T(t)Q\dot{x}(t) - h\int_{t-h}^{t} \dot{x}^T(\alpha)Q\dot{x}(\alpha)d\alpha \\ &\le h^2\dot{x}^T(t)Q\dot{x}(t) - h\int_{t-\tau(t)}^{t} \dot{x}^T(\alpha)Q\dot{x}(\alpha)d\alpha\end{aligned} \tag{20}$$

Applying Lemma 1 to (20) leads to

$$\dot{V}_3(x(t)) \le h^2\dot{x}^T(t)Q\dot{x}(t) - \left(\int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha\right)^T Q \left(\int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha\right) \tag{21}$$

By the Leibniz-Newwton formula, one has

$$0 = 2\zeta^T(t)N\left[-x(t) + x(t-\tau(t)) + \int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha\right]$$
$$+2f^T(x(t))N_3\left[-x(t) + x(t-\tau(t)) + \int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha\right] \qquad (22)$$

where $N = [N_1^T \;\; N_2^T \;\; N_4^T]^T, \zeta(t) = [x^T(t)\; x^T(t-\tau(t))\; f^T(x(t-\tau(t)))]^T$, diagonal matrix N_3 is positive definite and $N_i,\; i = 1, 2, 4$ are any matrices with appropriate dimensions. Similar to (15) and (18), we have

$$-2f^T(x(t))N_3x(t) \leq -2f^T(x(t))N_3\Sigma^{-1}f(x(t)) \qquad (23)$$

It is easy to see from (22) and (23) that

$$\begin{aligned}
0 \leq\; & 2\zeta^T(t)N\left[-x(t) + x(t-\tau(t)) + \int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha\right] \\
& -2f^T(x(t))N_3\Sigma^{-1}f(x(t)) + 2f^T(x(t))N_3x(t-\tau(t)) \\
& +2f^T(x(t))N_3\int_{t-\tau(t))}^{t} \dot{x}(\alpha)d\alpha \\
=\; & -2x^T(t)N_1x(t) + 2x^T(t)(N_1 - N_2^T)x(t-\tau(t)) \\
& 2x^T(t-\tau(t))N_2x(t-\tau(t)) - 2x^T(t)N_4^Tf(x(t-\tau(t))) \\
& +2x^T(t-\tau(t))N_4^Tf(x(t-\tau(t))) \\
& +2x^T(t)N_1\int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha \\
& +2x^T(t-\tau(t))N_2\int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha \\
& +2f^T(x(t-\tau(t)))N_4\int_{t-\tau(t)}^{t} \dot{x}(\alpha)d\alpha \\
& -2f^T(x(t))N_3\Sigma^{-1}f(x(t)) + 2f^T(x(t))N_3x(t-\tau(t)) \\
& +2f^T(x(t))N_3\int_{t-\tau(t))}^{t} \dot{x}(\alpha)d\alpha
\end{aligned} \qquad (24)$$

Then, by (14) - (24), it can be shown that $\dot{V}(x(t)) \leq \xi^T(t)\Phi\xi(t)$ where

$$\xi(t) = [x^T(t),\; x^T(t-\tau(t)),\; f^T(x(t)),\; f^T(x(t-\tau(t))),\; \int_{t-\tau(t)}^{t} \dot{x}^T(\alpha)d\alpha]^T$$

Finally, by (8), there exists a positive scalar a such that $\dot{V}(x(t)) < -a\|x(t)\|^2$, which guarantees the stability of the system. This completes the proof. □

Remark 1. The cross terms between $x(t)$ and $f(x(t))$, as well as cross terms between $x(t-\tau(t))$ and $f(x(t-\tau(t)))$ are included in the new Lyapunov-Krasovskii function (10). Therefore, the result of theorem 1 derived from this kind of Lyapunov-Krasovskii function is less conservative than the ones derived from Lyapunov-Krasovskii function not including these cross terms.

Remark 2. In [20], time-varying delay system are considered and delay - independent exponential stability criterions are proposed. Generally speaking, delay-dependent criteria is less conservative than delay-independent criteria when the delay is small.

Using Schur complement lemma to (8), the following corollary is obtained immediately.

Corollary 1. *Under the assumption (6), the origin of the DCNN in (5) is globally asymptotically stable for any delay satisfying* $0 < \tau(t) \leq h, \dot{\tau}(t) \leq \mu$ *if there exist matrices* $P = P^T > 0,\ Q = Q^T > 0, N_1,\ N_2,\ N_4,$ *and diagonal matrices* $S > 0,\ H > 0,\ U > 0,\ N_3 > 0$ *such that the following LMIs hold:*

$$\widehat{\Phi} = \begin{bmatrix} \widehat{\Phi}_{11} & \Phi_{12} & \widehat{\Phi}_{13} & \widehat{\Phi}_{14} & \Phi_{15} & \Phi_{16} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} \\ * & * & \widehat{\Phi}_{33} & \widehat{\Phi}_{34} & \Phi_{35} & \Phi_{36} \\ * & * & * & \widehat{\Phi}_{44} & \Phi_{45} & \Phi_{46} \\ * & * & * & * & \Phi_{55} & \Phi_{56} \\ * & * & * & * & * & \Phi_{66} \end{bmatrix} < 0 \tag{25}$$

$$\Psi = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0 \tag{26}$$

where
$\widehat{\Phi}_{11} = -PA - AP + P_{11} - N_1 - N_1^T,$
$\Phi_{12} = N_1 - N_2^T,\ \widehat{\Phi}_{13} = PB + P_{12} + H\Sigma,$
$\widehat{\Phi}_{14} = PB_1 - N_4^T,\ \Phi_{15} = N_1, \Phi_{16} = -hA^TQ,$
$\Phi_{22} = -(1-\mu)P_{11} + N_2^T + N_2,\ \Phi_{23} = N_3^T,$
$\Phi_{24} = -(1-\mu)P_{12} + U\Sigma + N_4^T,\ \Phi_{25} = N_2,\ \Phi_{26} = 0,$
$\widehat{\Phi}_{33} = SB + B^TS + P_{22} - 2SA\Sigma^{-1} - 2H - 2N_3\Sigma^{-1},$
$\widehat{\Phi}_{34} = SB_1,\ \Phi_{35} = N_3,\ \Phi_{36} = hB^TQ,$
$\widehat{\Phi}_{44} = -(1-\mu)P_{22} - 2U,\ \Phi_{45} = N_4,\ \Phi_{46} = hB_1^TQ,$
$\Phi_{55} = -Q,\ \Phi_{56} = 0,\ \Phi_{66} = -Q,$
$\Sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_n)$ *and* $*$ *denotes the corresponding transposed block matrix due to symmetry.*

Remark 3. The upper bound of the derivative of the delay μ is assumed to be less than one in [3], [20], [24] while our results are also applicable when $\mu \geq 1$ due to the introducing of the matrices N_2 and U.

Next, we will consider the DCNN with uncertain parameters:

$$\dot{x}(t) = -\mathcal{A}x(t) + \mathcal{B}f(x(t)) + \mathcal{B}_1 f(x(t - \tau(t))) \tag{27}$$

where $\mathcal{A} = A + \Delta A(t)$, $\mathcal{B} = B + \Delta B(t)$, $\mathcal{B}_1 = B_1 + \Delta B_1(t)$, the perturbed $\Delta A(t), \Delta B(t), \Delta B_1(t)$ are time-varying uncertain matrices satisfying

$$\Delta A(t) = M_1 F_1(t) N_A \tag{28}$$

$$\Delta B(t) = M_2 F_2(t) N_B \tag{29}$$

$$\Delta B_1(t) = M_3 F_3(t) N_{B_1} \tag{30}$$

where $\{M_i\}_{i=1}^{3}$, N_A, N_B, N_{B_1} are some given constant matrices, $F_i(t)$ are unknown real matrices satisfying

$$F_i^T(t) F_i(t) \le I, \quad \forall\, t \ge 0,\ i = 1, 2, 3 \tag{31}$$

The following lemma will be used to derive the result for uncertain DCNN.

Lemma 2. *Let U, V, W and M be real matrices of appropriate dimensions with M satisfying $M = M^T$, then*

$$M + UVW + W^T V^T U^T < 0, \quad \textit{for all} \quad V^T V \le I,$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$M + \varepsilon^{-1} U U^T + \varepsilon W^T W < 0.$$

Now, we have the following result. It is easy to derived from Lemma 2 and Corollary 1.

Theorem 2. *The origin of the uncertain DCNN in (27) is globally asymptotically stable for all admissible uncertainties and any time-delay satisfying $0 < \tau(t) \le h$, $\dot{\tau}(t) \le \mu$ if there exist matrices $P = P^T > 0$, $Q = Q^T > 0, N_1$, N_2, N_4, diagonal matrices $S > 0$, $H > 0$, $U > 0$, $N_3 > 0$, scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, and $\varepsilon_4 > 0$, such that the following LMIs conditions hold:*

$$\Pi = \begin{bmatrix} \widetilde{\Phi} & \Lambda \\ \Lambda^T & \Theta \end{bmatrix} < 0 \tag{32}$$

$$\Psi = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \ge 0 \tag{33}$$

where
$\widetilde{\Phi} = (\widetilde{\Phi}_{ij}), i, j = 1, \dots, 6.$
$\widetilde{\Phi}_{11} = \widehat{\Phi}_{11} + \varepsilon_1 N_A^T N_A, \widetilde{\Phi}_{12} = \Phi_{12},$
$\widetilde{\Phi}_{13} = \widehat{\Phi}_{13},\ \widetilde{\Phi}_{14} = \widehat{\Phi}_{14},\ \widetilde{\Phi}_{15} = \Phi_{15}, \widetilde{\Phi}_{16} = \Phi_{16},$
$\widetilde{\Phi}_{22} = \Phi_{22},\ \widetilde{\Phi}_{23} = \Phi_{23},\ \widetilde{\Phi}_{24} = \Phi_{24},\ \widetilde{\Phi}_{25} = \Phi_{25},$
$\widetilde{\Phi}_{26} = \Phi_{26},\ \widetilde{\Phi}_{33} = \widehat{\Phi}_{33} + \varepsilon_2 N_B^T N_B + \varepsilon_4 \Sigma^{-T} N_A^T N_A \Sigma^{-1},$
$\widetilde{\Phi}_{34} = \widehat{\Phi}_{34},\ \widetilde{\Phi}_{35} = \Phi_{35},\ \widetilde{\Phi}_{36} = \Phi_{36},$
$\widetilde{\Phi}_{44} = \widehat{\Phi}_{44} + \varepsilon_3 N_{B_1}^T N_{B_1},\ \widetilde{\Phi}_{45} = \Phi_{45},\ \widetilde{\Phi}_{46} = \Phi_{46},$
$\widetilde{\Phi}_{55} = \Phi_{55},\ \widetilde{\Phi}_{56} = \Phi_{56},\ \widetilde{\Phi}_{66} = \Phi_{66},$

$$\Lambda = \begin{bmatrix} -PM_1 & PM_2 & PM_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & SM_2 & SM_3 & SM_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -hQ^TM_1 & hQ^TM_2 & hQ^TM_3 & 0 \end{bmatrix},$$

$$\Theta = diag(-\varepsilon_1 I \quad -\varepsilon_2 I \quad -\varepsilon_3 I \quad -\varepsilon_4 I).$$

Proof. By corollary 1, the uncertain DCNN (27) is global asymptotically stable if the following LMIs hold:

$$\widehat{\Phi} + \Delta\Phi < 0 \tag{34}$$

$$\Psi = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \geq 0 \tag{35}$$

where

$$\Delta\Phi = \begin{bmatrix} -2P\Delta A & 0 & 0 & 0 & 0 & -h\Delta A^TQ \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & 2S\Delta B - 2S\Delta A\Sigma^{-1} & S\Delta B_1 & 0 & h\Delta B^TQ \\ * & * & * & 0 & 0 & h\Delta B_1^TQ \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix}. \tag{36}$$

On the other hand, by lemma 2, one has

$$\begin{aligned} \Delta\Phi &= \Upsilon_1F_1(t)\Xi_1 + \Xi_1^TF_1^T(t)\Upsilon_1^T + \Upsilon_2F_2(t)\Xi_2 \\ &\quad + \Xi_2^TF_2^T(t)\Upsilon_2^T + \Upsilon_3F_3(t)\Xi_3 + \Xi_3^TF_3^T(t)\Upsilon_3^T \\ &\quad + \Upsilon_4F_1(t)\Xi_4 + \Xi_4^TF_1^T(t)\Upsilon_4^T \\ &\leq \varepsilon_1^{-1}\Upsilon_1\Upsilon_1^T + \varepsilon_1\Xi_1^T\Xi_1 + \varepsilon_2^{-1}\Upsilon_2\Upsilon_2^T + \varepsilon_2\Xi_2^T\Xi_2 \\ &\quad + \varepsilon_3^{-1}\Upsilon_3\Upsilon_3^T + \varepsilon_3\Xi_3^T\Xi_3 + \varepsilon_4^{-1}\Upsilon_4\Upsilon_4^T \\ &\quad + \varepsilon_4\Xi_4^T\Xi_4 \end{aligned} \tag{37}$$

where

$$\Upsilon_1 = \begin{bmatrix} -M_1^TP & 0 & 0 & 0 & 0 & -hM_1^TQ \end{bmatrix}^T,$$
$$\Xi_1 = \begin{bmatrix} N_A & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} M_2^TP & 0 & M_2^TS & 0 & 0 & hM_2^TQ \end{bmatrix}^T,$$
$$\Xi_2 = \begin{bmatrix} 0 & 0 & N_B & 0 & 0 & 0 \end{bmatrix},$$
$$\Upsilon_3 = \begin{bmatrix} M_3^TP & 0 & M_3^TS & 0 & 0 & hM_3^TQ \end{bmatrix}^T,$$
$$\Xi_3 = \begin{bmatrix} 0 & 0 & 0 & N_{B_1} & 0 & 0 \end{bmatrix},$$
$$\Upsilon_4 = \begin{bmatrix} 0 & 0 & M_1^TS & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\Xi_4 = \begin{bmatrix} 0 & 0 & N_A\Sigma^{-1} & 0 & 0 & 0 \end{bmatrix}.$$

Then, applying Schur complement lemma to (37), conditions (34) and (32) are equivalent, This completes the proof. □

4. Examples

In this section, three examples are provided to demonstrate the validity of these new stability criterions.

Example 1. ([12],[16],[19]) Consider the forth-order DCNN with the following parameters:

$$A = \begin{bmatrix} 1.2769 & 0 & 0 & 0 \\ 0 & 0.6231 & 0 & 0 \\ 0 & 0 & 0.9230 & 0 \\ 0 & 0 & 0 & 0.4480 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix}$$

In this example we suppose that

$$\sigma_1 = 0.1137 \qquad \sigma_2 = 0.1279\sigma_3 = 0.7994 \qquad \sigma_4 = 0.2386$$

For this DCNN, it can be checked that the asymptotic stability conditions Theorem 1 in [19], Theorem 1 in [2] and Theorem 2 in [13] are not satisfied. Therefore, they fail to conclude whether this delay system is asymptotically stable or not. If we use Theorem 2 in [19], Theorem 1 in [16] and Theorem 1 in [12], we can calculate the upper bound of a constant delay, h, are $1.4224, 1.9231$ and 3.5841, respectively, while in this paper, Theorem 1 yields a larger $h = 161.4906$. when $h = 30$, We can obtain the solutions as follows :

$$P = \begin{bmatrix} 78.2668 & -16.3744 & -8.2658 & 1.1349 \\ -16.3744 & 13.4131 & 0.1364 & -3.3490 \\ -8.2658 & 0.1364 & 46.4084 & 6.4431 \\ 1.1349 & -3.3490 & 6.4431 & 5.0694 \end{bmatrix}$$

$$S = \begin{bmatrix} 58.0944 & 0 & 0 & 0 \\ 0 & 169.6731 & 0 & 0 \\ 0 & 0 & 1.6585 & 0 \\ 0 & 0 & 0 & 31.1488 \end{bmatrix}$$

$$P_{11} = \begin{bmatrix} 66.4113 & -13.8309 & -5.3006 & 2.2119 \\ -13.8309 & 8.1706 & -4.3563 & -2.3356 \\ -5.3006 & -4.3563 & 30.1048 & 2.8668 \\ 2.2119 & -2.3356 & 2.8668 & 1.8479 \end{bmatrix}$$

$$P_{12} = \begin{bmatrix} -0.9012 & -3.9157 & 9.2368 & 7.9518 \\ 2.2743 & -7.9543 & -2.0684 & -8.5582 \\ -24.6333 & -2.5832 & -2.7152 & 28.3621 \\ -3.0023 & 2.4648 & 3.1584 & 4.2401 \end{bmatrix}$$

$$P_{22} = \begin{bmatrix} 387.8438 & 178.2290 & -65.9394 & -9.2152 \\ 178.2290 & 557.4651 & -3.0665 & 50.6310 \\ -65.9394 & -3.0665 & 42.7247 & 12.5003 \\ -9.2152 & 50.6310 & 12.5003 & 103.3454 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} 0.1087 & -0.0572 & -0.0906 & -0.1539 \\ 0.0623 & 0.0135 & -0.0744 & -0.0015 \\ 0.3635 & -0.0209 & -0.4791 & -0.1862 \\ 0.0066 & -0.0140 & -0.0180 & -0.0211 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} -0.0449 & 0.0154 & 0.0468 & -0.0507 \\ 0.1219 & -0.0525 & -0.1837 & -0.0060 \\ -0.3797 & 0.1632 & 0.4586 & 0.2234 \\ 0.0029 & 0.0033 & -0.0006 & -0.0192 \end{bmatrix}$$

$$N_3 = \begin{bmatrix} 2.4035 & 0 & 0 & 0 \\ 0 & 0.7466 & 0 & 0 \\ 0 & 0 & 0.2159 & 0 \\ 0 & 0 & 0 & 0.2776 \end{bmatrix}$$

$$H = \begin{bmatrix} 47.4234 & 0 & 0 & 0 \\ 0 & 37.5683 & 0 & 0 \\ 0 & 0 & 32.2447 & 0 \\ 0 & 0 & 0 & 18.7876 \end{bmatrix}$$

$$N_4 = \begin{bmatrix} -0.1236 & 0.0253 & -0.0185 & 0.5602 \\ -0.5701 & 0.4997 & 1.3126 & 0.6231 \\ 0.2635 & -0.1630 & -0.1275 & -0.4786 \\ -0.9764 & 0.0873 & 1.2899 & 0.2441 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.0485 & -0.0124 & -0.0155 & -0.0031 \\ -0.0124 & 0.0104 & 0.0033 & 0.0002 \\ -0.0155 & 0.0033 & 0.0260 & 0.0070 \\ -0.0031 & 0.0002 & 0.0070 & 0.0077 \end{bmatrix}$$

$$U = \begin{bmatrix} 77.2179 & 0 & 0 & 0 \\ 0 & 93.1918 & 0 & 0 \\ 0 & 0 & 13.4023 & 0 \\ 0 & 0 & 0 & 17.0531 \end{bmatrix}.$$

Example 2. ([12],[19],[21]) Consider the delayed neural network (5) with

$$A = \begin{bmatrix} 4.1989 & 0 & 0 \\ 0 & 0.7160 & 0 \\ 0 & 0 & 1.9985 \end{bmatrix}$$
$$B = 0$$
$$B_1 = \begin{bmatrix} -0.1052 & -0.5069 & -0.1121 \\ -0.0257 & -0.2808 & 0.0212 \\ 0.1205 & -0.2153 & 0.1315 \end{bmatrix}$$

We suppose that

$$\sigma_1 = 0.4219 \qquad \sigma_2 = 3.8993 \qquad \sigma_3 = 1.0160$$

The obtained upper bounds of a constant delay, h, which ensures that the system is asymptotically stable in [19], [21] and [12] are 1.7484, 1.7644 and 2.1423, respectively. On the contrary, Theorem 1 yield a larger $h = 61$. Obviously it's much larger than the previous results. When $\mu \geq 1$, we can obtain the upper bound of $h = 1.5021$ by Theorem 1, while in [12] it is $h = 0.8969$, which is 40.29% smaller than that obtained by our method. This shows that the condition given in Theorem 1 is less conservative.

Example 3. Consider the second-order uncertain DCNN (27) with the following parameters:

$$A = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.1 \end{bmatrix}, \ B = \begin{bmatrix} 1.1 & 1 \\ -0.2 & 0.1 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.9 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, \ M_1 = \begin{bmatrix} -0.2 & 0.2 \\ 0.2 & 0.2 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0.2 & 0.5 \\ 0.1 & -0.3 \end{bmatrix}, \ M_3 = \begin{bmatrix} -0.4 & 0.3 \\ 0.3 & 0.4 \end{bmatrix},$$
$$N_A = M_1, \ N_B = M_2, \ N_{B_1} = M_3, \ \mu = 0.4.$$

We choose $\sigma_1 = 0.5$, $\sigma_2 = 0.5$. It can be checked that Theorem 1 in [24] is not satisfied. It means that it fails to conclude whether this system is asymptotically stable or not. On the contrary, the obtained upper bound of h , which ensures that the system is global asymptotically stable is $h = 1.1241$ by Theorem 2 in this letter, which is much larger than $h = 0.5436$ by Theorem 1 in [3]. This also shows that our criterion is less conservative.

5. Conclusion

In this paper, the problem of global asymptotical stability has been considered for DCNNs and DCNNs with uncertainties. A new Lyapunov-Krasovskii function was introduced to derive the stability results. Delay-dependent sufficient conditions had been derived in terms of LMIs. Illustrative examples had been included to show the effectiveness of the results.

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