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AN APPLICATION OF DARBO'S FIXED POINT THEOREM TO A NONLINEAR QUADRATIC INTEGRAL EQUATION OF VOLTERRA TYPE

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Abstract. Using Darbo's fixed point theorem, we establish the existence of monotone solutions for a nonlinear quadratic integral equation of Volterra type in the Banach space of real functions defined and continuous on a bounded and closed interval.

1. Introduction and Preliminaries

It is known that the theory of integral equations is an important part of nonlinear analysis and frequently applicable in other branches of mathematics and in mathematical physics, engineering, economics, biology as well in describing problems connected with real world. The theory is now very developed with help of various tools of functional analysis, topology and fixed point theory, etc. For details, we refer to [1],[2], [5]-[14], [16] and [17] and the references therein. Recently, Banaś and Martinon [5] studied the existence of monotone solutions for a nonlinear quadratic integral equation of Volterra type.

The goal of this paper is to investigate the following nonlinear quadratic integral equation of Volterra type

$$x(t) = a(t)+b(t)x(t)\int_0^t u(t,s,x(s))ds+c(t)x^2(t)\int_0^t v(t,s,x(s))ds,\ t\in[0,T], \tag{1.1}$$

where the functions a, b, c, u and v are given while $x = x(t)$ is an unknown function. If $b(t) = 1$, $c(t) = 0$, $\forall t \in [0,T]$, then equation (1.1) reduces to the equation

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discussed by Banaś and Martinon [5]. Using Darbo's fixed point theorem, we establish the existence of monotone solutions for equation (1.1) in the Banach space of real functions defined and continuous on the interval $[0,T]$. An example is given to illustrate the advantage of the result presented in this paper.

Throughout this paper, let $R=(-\infty,\infty)$, $R_+=[0,\infty)$, $I=[0,T]$, $(E,\|\cdot\|)$ denote an infinite-dimensional Banach space with the zero element θ and $B(\theta,r)$ stand for the closed ball centered at θ and with radius r. Let $B(E)$ denote the family of all nonempty bounded subsets of E and $C(I)$ represent the classical Banach space of all continuous functions acting from I into R with the standard norm

$$\|x\|=\max\{|x(t)|:t\in I\},\ x\in C(I).$$

For any nonempty bounded subset X of $C(I)$, $x\in X$ and $\varepsilon\ge 0$, put

$$\begin{aligned}\omega(x,\varepsilon)&=\sup\{|x(t)-x(s)|:t,s\in I,|t-s|\le\varepsilon\},\\ \omega(X,\varepsilon)&=\sup\{\omega(x,\varepsilon):x\in X\},\quad \omega_0(X)=\lim_{\varepsilon\to 0}\omega(X,\varepsilon),\\ d(x)&=\sup\{|x(s)-x(t)|-[x(s)-x(t)]:t,s\in I,t\le s\},\\ d(X)&=\sup\{d(x):x\in X\},\quad \mu(X)=\omega_0(X)+d(X).\end{aligned}$$

Remark 1.1. Banaś and Olszowy [4] proved that the function μ is a measure of noncompactness in the space $C(I)$ and the kernel ker μ of this measure includes nonempty bounded sets X such that functions from X are equicontinuous and nondecreasing on the interval I.

Theorem 1.1 ([3, 5])**.** *Let D be a nonempty bounded closed convex subset of the space E and let $f:D\to D$ be a continuous mapping such that $\mu(fA)\le k\mu(A)$ for any nonempty subset A of D, where $k\in[0,1)$ is a constant and μ is a measure of noncompactness on $B(E)$. Then f has a fixed point in D.*

2. Existence of Monotone Solutions

In this section, we will study equation (1.1) under the following assumptions:

(i) a,b and c are in $C(I)$ and are nondecreasing and nonnegative on the interval I;

(ii) u and $v:I\times I\times R\to R$ are continuous functions such that u and $v:I\times I\times R_+\to R_+$, and for any $s\in I$ and $x\in R_+$, $u(\cdot,s,x)$ and $v(\cdot,s,x)$ are nondecreasing with respect to the first argument in I;

(iii) there exist two nondecreasing functions f and $g:R_+\to R_+$ satisfying

$$|u(t,s,x)| \leq f(|x|), |v(t,s,x)| \leq g(|x|), \text{for all } t,s \in I \text{ and for each } x \in R;$$

(iv) there exists a positive number r satisfying

$$\|a\| + T\|b\|rf(r) + T\|c\|r^2 g(r) \leq r \quad \text{and} \quad T[\|b\|f(r) + 2\|c\|rg(r)] < 1.$$

Theorem 2.1. *Under Assumptions* (i)-(iv)*, Equation* (1.1) *possesses at least one solution* $x = x(t)$ *which belongs to the space* $C(I)$ *and is nondecreasing on the interval* I.

Proof. Let G be the operator defined on the space $C(I)$ by the formula

$$\begin{aligned}(Gx)(t) &= a(t) + b(t)x(t)\int_0^t u(t,s,x(s))ds \\ &\quad + c(t)x^2(t)\int_0^t v(t,s,x(s))ds, \ \forall x \in C(I), t \in I. \qquad (2.1)\end{aligned}$$

It follows from Assumptions (i) and (ii) that Gx is continuous on I for any $x \in C(I)$. That is, $G : C(I) \to C(I)$. In view of (2.1) and Assumptions (i) and (iii), we deduce that for any $x \in C(I)$ and $t \in I$

$$\begin{aligned}|(Gx)(t)| &\leq a(t) + b(t)|x(t)|\int_0^t |u(t,s,x(s))|ds + c(t)x^2(t)\int_0^t |v(t,s,x(s))|ds \\ &\leq \|a\| + \|b\|\|x\|\int_0^t f(|x(s)|)ds + \|c\|\|x\|^2\int_0^t g(|x(s)|)ds \\ &\leq \|a\| + T\|b\|\|x\|f(\|x\|) + T\|c\|\|x\|^2 g(\|x\|),\end{aligned}$$

which implies that

$$\|Gx\| \leq \|a\| + T\|b\|\|x\|f(\|x\|) + T\|c\|\|x\|^2 g(\|x\|), \ \forall x \in C(I). \qquad (2.2)$$

Put

$$B_r^+ = \{x \in B(\theta, r) : x(t) \geq 0, \ \forall t \in I\}.$$

Obviously, B_r^+ is a nonempty closed bounded and convex set. Using Assumptions (i)-(iv) and (2.2), we easily conclude that G maps not only $B(\theta, r)$ into itself, but also transforms B_r^+ into itself. For any $\varepsilon \geq 0$, put

$$\begin{aligned}\alpha(\varepsilon, r) &= \sup\{|u(t,s,x) - u(t,s,y)| : \forall t,s \in I, x,y \in [0,r], |x-y| \leq \varepsilon\}, \\ \beta(\varepsilon, r) &= \sup\{|v(t,s,x) - v(t,s,y)| : \forall t,s \in I, x,y \in [0,r], |x-y| \leq \varepsilon\}, \\ \gamma(\varepsilon, r) &= \sup\{|u(t,s,x) - u(p,s,x)| : \forall x \in [0,r], t,p,s \in I, |t-p| \leq \varepsilon\}, \\ \delta(\varepsilon, r) &= \sup\{|v(t,s,x) - v(p,s,x)| : \forall x \in [0,r], t,p,s \in I, |t-p| \leq \varepsilon\}. \qquad (2.3)\end{aligned}$$

Now we show that G is continuous on B_r^+. Let $\varepsilon > 0$, $x, y \in B_r^+$, $\|x - y\| \le \varepsilon$ and $t \in I$. By virtue of Assumptions (i) and (iii), and (2.1) and (2.3), we infer that

$$\begin{aligned}
&|(Gx)(t) - (Gy)(t)| \\
&\le b(t)\Big|x(t)\int_0^t u(t,s,x(s))ds - y(t)\int_0^t u(t,s,y(s))ds\Big| \\
&\quad + c(t)\Big|x^2(t)\int_0^t v(t,s,x(s))ds - y^2(t)\int_0^t v(t,s,y(s))ds\Big| \\
&\le \|b\|[x(t)\int_0^t |u(t,s,x(s)) - u(t,s,y(s))|ds + |x(t) - y(t)|\int_0^t u(t,s,y(s))ds] \\
&\quad + \|c\|[x^2(t)\int_0^t |v(t,s,x(s)) - v(t,s,y(s))|ds + |x^2(t) - y^2(t)|\int_0^t v(t,s,y(s))ds] \\
&\le \|b\|[r\int_0^t \alpha(\varepsilon,r)ds + \varepsilon\int_0^t f(y(s))ds] + \|c\|[r^2\int_0^t \beta(\varepsilon,r)ds + 2r\varepsilon\int_0^t g(y(s))ds] \\
&\le T\|b\|[r\alpha(\varepsilon,r) + \varepsilon f(r)] + T\|c\|[r^2\beta(\varepsilon,r) + 2r\varepsilon g(r)],
\end{aligned}$$

which yields that

$$\|Gx - Gy\| \le T\|b\|[r\alpha(\varepsilon,r) + \varepsilon f(r)] + T\|c\|[r^2\beta(\varepsilon,r) + 2r\varepsilon g(r)]. \tag{2.4}$$

Since u and v are uniformly continuous on $I \times I \times [0, r]$, it follows from (2.3) and (2.4) that G is continuous on B_r^+.

Let X be a nonempty subset of B_r^+. For any $\varepsilon > 0$, $x \in X$ and $t, p \in I$ with $|t - p| \le \varepsilon$, we may, without loss of generality, assume that $t \le p$. On account of Assumptions (i)-(iii), (2.1) and (2.3), we arrive at the following estimates

$$\begin{aligned}
&|(Gx)(p) - (Gx)(t)| \\
&\le |a(p) - a(t)| + \Big|b(p)x(p)\int_0^p u(p,s,x(s))ds - b(t)x(t)\int_0^t u(t,s,x(s))ds\Big| \\
&\quad + \Big|c(p)x^2(p)\int_0^p v(p,s,x(s))ds - c(t)x^2(t)\int_0^t v(t,s,x(s))ds\Big| \\
&\le \omega(a,\varepsilon) + |b(p)x(p) - b(t)x(t)|\int_0^p u(p,s,x(s))ds \\
&\quad + b(t)x(t)\int_0^p |u(p,s,x(s)) - u(t,s,x(s))|ds \\
&\quad + b(t)x(t)\int_t^p u(t,s,x(s))ds + |c(p)x^2(p) - c(t)x^2(t)|\int_0^p v(t,s,x(s))ds \\
&\quad + c(t)x^2(t)\int_0^p |v(p,s,x(s)) - v(t,s,x(s))|ds + c(t)x^2(t)\int_t^p v(t,s,x(s))ds
\end{aligned}$$

$$
\begin{aligned}
&\leq \omega(a,\varepsilon)+T[\|x\|\omega(b,\varepsilon)+\|b\|\omega(x,\varepsilon)]f(\|x\|)+\|b\|\|x\|[T\gamma(\varepsilon,r)+2(p-t)f(\|x\|)]\\
&\quad+T\|x\|[\|x\|\omega(c,\varepsilon)+2\|c\|\omega(x,\varepsilon)]g(\|x\|)+\|c\|\|x\|^2[T\delta(\varepsilon,r)+(p-t)g(\|x\|)]\\
&\leq \omega(a,\varepsilon)+T[r\omega(b,\varepsilon)+\|b\|\omega(x,\varepsilon)]f(r)+r\|b\|[T\gamma(\varepsilon,r)+2\varepsilon f(r)]\\
&\quad+rT[r\omega(c,\varepsilon)+2\|c\|\omega(x,\varepsilon)]g(r)+r^2\|c\|[T\delta(\varepsilon,r)+\varepsilon g(r)],
\end{aligned}
$$

which means that

$$
\begin{aligned}
\omega(GX,\varepsilon)&\leq \omega(a,\varepsilon)+T[r\omega(b,\varepsilon)+\|b\|\omega(X,\varepsilon)]f(r)+r\|b\|[T\gamma(\varepsilon,r)+2\varepsilon f(r)]\\
&\quad+rT[r\omega(c,\varepsilon)+2\|c\|\omega(X,\varepsilon)]g(r)+r^2\|c\|[T\delta(\varepsilon,r)+\varepsilon g(r)].
\end{aligned}\tag{2.5}
$$

In the light of Assumptions (i) and (ii), (2.3) and (2.5), we derive that

$$
\omega_0(GX)=\lim_{\varepsilon\to 0}\omega(GX,\varepsilon)\leq T[\|b\|f(r)+2r\|c\|g(r)]\omega_0(X).\tag{2.6}
$$

In view of Assumptions (i)-(iii), we deduce that for any $x\in X$ and $t,p\in I$ with $t\leq p$, the following estimates can be derived,

$$
\begin{aligned}
&|(Gx)(p)-(Gx)(t)|-[(Gx)(p)-(Gx)(t)]\\
&=\Big|a(p)+b(p)x(p)\int_0^p u(p,s,x(s))ds+c(p)x^2(p)\int_0^p v(p,s,x(s))ds\\
&\quad-a(t)-b(t)x(t)\int_0^t u(t,s,x(s))ds-c(t)x^2(t)\int_0^t v(t,s,x(s))ds\Big|\\
&\quad-[a(p)+b(p)x(p)\int_0^p u(p,s,x(s))ds+c(p)x^2(p)\int_0^p v(p,s,x(s))ds\\
&\quad-a(t)-b(t)x(t)\int_0^t u(t,s,x(s))ds-c(t)x^2(t)\int_0^t v(t,s,x(s))ds]\\
&\leq |a(p)-a(t)|-[a(p)-a(t)]\\
&\quad+\Big|b(p)x(p)\int_0^p u(p,s,x(s))ds-b(t)x(t)\int_0^t u(t,s,x(s))ds\Big|\\
&\quad-[b(p)x(p)\int_0^p u(p,s,x(s))ds-b(t)x(t)\int_0^t u(t,s,x(s))ds]\\
&\quad+\Big|c(p)x^2(p)\int_0^p v(p,s,x(s))ds-c(t)x^2(t)\int_0^t v(t,s,x(s))ds\Big|\\
&\quad-[c(p)x^2(p)\int_0^p v(p,s,x(s))ds-c(t)x^2(t)\int_0^t v(t,s,x(s))ds]\\
&\leq |b(p)x(p)-b(t)x(t)|\int_0^p u(p,s,x(s))ds
\end{aligned}
$$

$$
\begin{aligned}
&+ b(t)x(t)\{\int_t^p u(p,s,x(s))ds + \int_0^t [u(p,s,x(s)) - u(t,s,x(s))]ds\} \\
&- [b(p)x(p) - b(t)x(t)] \int_0^p u(p,s,x(s))ds \\
&- b(t)x(t)\{\int_t^p u(p,s,x(s))ds + \int_0^t [u(p,s,x(s)) - u(t,s,x(s))]ds\} \\
&+ |c(p)x^2(p) - c(t)x^2(t)| \int_0^p v(p,s,x(s))ds \\
&+ c(t)x^2(t)\{\int_t^p v(p,s,x(s))ds + \int_0^t [v(p,s,x(s)) - v(t,s,x(s))]ds\} \\
&- [c(p)x^2(p) - c(t)x^2(t)] \int_0^p v(p,s,x(s))ds \\
&- c(t)x^2(t)\{\int_t^p v(p,s,x(s))ds + \int_0^t [v(p,s,x(s)) - v(t,s,x(s))]ds\} \\
\le\ & \{|b(p)x(p) - b(t)x(t)| - [b(p)x(p) - b(t)x(t)]\} \int_0^p u(p,s,x(s))ds \\
&+ \{|c(p)x^2(p) - c(t)x^2(t)| - [c(p)x^2(p) - c(t)x^2(t)]\} \int_0^p v(p,s,x(s))ds \\
\le\ & \{b(p)|x(p) - x(t)| + x(t)|b(p) - b(t)| \\
&- b(p)[x(p) - x(t)] - x(t)[b(p) - b(t)]\} \int_0^p f(\|x\|)ds \\
&+ \{c(p)|x^2(p) - x^2(t)| + x^2(t)|c(p) - c(t)| \\
&- c(p)[x^2(p) - x^2(t)] - x^2(t)[c(p) - c(t)]\} \int_0^p g(\|x\|)ds \\
\le\ & T\|b\|f(r)\{|x(p) - x(t)| - [x(p) - x(t)]\} \\
&+ T\|c\|g(r)\{|x^2(p) - x^2(t)| - [x^2(p) - x^2(t)]\} \\
\le\ & T[\|b\|f(r) + 2r\|c\|g(r)]d(x).
\end{aligned}
$$

This yields that

$$
d(GX) \le T[\|b\|f(r) + 2r\|c\|g(r)]d(X). \tag{2.7}
$$

In view of (2.6) and (2.7), we see that

$$
\mu(GX) = \omega_0(GX) + d(GX) \le T[\|b\|f(r) + 2r\|c\|g(r)]\mu(X). \tag{2.8}
$$

Thus Theorem 2.1 follows from Assumption (iv), (2.8), Theorem 1.1 and Remark 1.1. This completes the proof. □

Remark 2.1. It follows from Theorem 2.1 that each solution of equation (1.1) is positive provided that $a(0) > 0$.

Remark 2.2. In the case $b(t) = 1$ and $c(t) = 0$ for all $t \in I$, Theorem 2.1 reduces to Theorem 3.1 of Banaś and Martinon [5].

The following example reveals that Theorem 2.1 generalizes essentially the result of Banaś and Martinon [5].

Example 2.1. Consider the following nonlinear quadratic integral equation

$$x(t) = 1 + 0.25\sqrt{t}x(t)\int_0^t \sqrt{\frac{(t+s)|x(s)|}{1+x^2(s)}}ds + 0.5t^2x^2(t)\int_0^t \frac{tx(s)}{1+s+|x(s)|}ds. \tag{2.9}$$

Let

$$0 < T < \frac{1}{\sqrt{0.25+\sqrt{2}}},$$

$$\frac{1-0.25T^2-\sqrt{(1-0.25T^2)^2-2T^4}}{T^4} \leq r < \frac{1-0.25T^2}{T^4},$$

$$a(t) = 1,\ b(t) = 0.25\sqrt{t},\ c(t) = 0.5t^2,\ \forall t \in I,$$

$$u(t,s,x) = \sqrt{\frac{(t+s)|x|}{1+x^2}},\ v(t,s,x) = \frac{tx}{1+s+|x|},\ \forall (t,s,x) \in I \times I \times R,$$

$$f(x) = \sqrt{T},\ g(x) = T,\ \forall x \in R_+.$$

It is easy to verify that the conditions of Theorem 2.1 are satisfied. Thus Theorem 2.1 and Remark 2.1 ensure that Equation (2.9) possesses at least one solution $x \in C(I)$. Moreover, it is positive and nondecreasing on the interval I. However, Theorem 3.1 of Banaś and Martinon [5] is not applicable for equation (2.9).

The following two examples show that condition (iv) in Theorem 2.1 can not be omitted.

Example 2.2. Consider the following integral equation

$$x(t) = 1 + \frac{1}{2}x(t)\int_0^t (2s+1)ds + \frac{1}{2}x^2(t)\int_0^t (2s+1)ds, \quad t \in [0,1]. \tag{2.10}$$

Let

$$T = 1, a(t) = 1, b(t) = c(t) = \frac{1}{2}, \quad \forall t \in I,$$

$$u(t,s,x) = v(t,s,x) = 2s+1,\ \forall (t,s,x) \in I \times I \times R,$$

$$f(x) = g(x) = 4,\ \forall x \in R_+.$$

Then it is clear that above all conditions satisfy conditions in Theorem 2.1, except the condition (iv). By simple calculation, for any $r > 0$,

$$\|a\| + T\|b\|rf(r) + T\|c\|r^2g(r) > r$$

and

$$T[\|b\|f(r) + 2\|c\|rg(r)] \geq 1.$$

Thus condition (iv) in Theorem 2.1 is not satisfied. Moreover, $x(t)$ of Equation (2.10) does not exist in R for any $t \in \left(\frac{1}{2}\left\{-1+\sqrt{1+4\times(6-4\sqrt{2})}\right\}, 1\right]$. Therefore condition (iv) in Theorem 2.1 is essential.

Example 2.3. Consider the following integral equation

$$x(t) = 1 + \frac{1}{4}x(t)\int_0^t 4ds + \frac{1}{4}x^2(t)\int_0^t 4ds, \quad t \in [0, 4]. \tag{2.11}$$

Let

$$T = 4, a(t) = 1, b(t) = c(t) = \frac{1}{4}, \quad \forall t \in I,$$

$$u(t, s, x) = v(t, s, x) = 4, \ \forall (t, s, x) \in I \times I \times R,$$

$$f(x) = g(x) = 4, \ \forall x \in R_+.$$

Then it is clear that above all conditions satisfy conditions in Theorem 2.1, except the condition (iv). Also $x(t)$ of Equation (2.11) does not exist in R for any $t \in (3 - 2\sqrt{2}, 4]$. Therefore condition (iv) in Theorem 2.1 is essential.

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REFERENCES

1. R.P. Agarwal & D. O'Regan: Global existence for nonlinear operator inclusion. *Computers Math. Applic.* **38** (1999), 131-139.
2. R.P. Agarwal, D. O'Regan & P.J.Y. Wong: *Positive solutions of differential, difference and integral equations.* Kluwer Academic, Dordrecht, 1999.
3. J. Banaś & K. Goebel: *Measures of noncompactness in Banach spaces.* In Lecture Notes in Pure and Applied Math., Volume 60, Marcel Dekker, New York, 1980.
4. J. Banaś & L. Olszowy: Measures of noncompactness related to monotonicity. *Comment. Math.* **41** (2001), 13-23.
5. J. Banaś & A. Martinon: Monotonic solutions of a quadratic integral equation of Volterra type. *Computers Math. Applic.* **47** (2004), 271-279.

6. J. Banaś, M. Lecko & W.G. El-Sayed: Existence theorems for some quadratic integral equations. *J. Math. Anal. Appl.* **222** (1998), 276-285.
7. J. Banaś & K. Sadarangani: Solvability of Volterra-Stieltjes operator-integral equations and their applications. *Computers Math. Applic.* **41** (2001), 1535-1544.
8. J. Banaś, J.R. Rodriguez & K. Sadarangani: On a class of Urysohn-Stieltjes quadratic integral equations and their applications. *J. Comput. Appl. Math.* **113** (2000), 35-50.
9. J. Banaś, J. Rocha & K.B. Sadarangani: Solvability of a nonlinear integral equation of Volterra type. J. Comput. Appl. Math. **157** (2003), 31-48.
10. J. Banaś & B. Rzepka: On existence and asymptotic stability of solutions of a nonlinear integral equation. *J. Math. Anal. Appl.* **284** (2003), 165-173.
11. J. Banaś, J.R. Rodriquez & K. Sadarangani: On a nonlinear quadratic integral equation of Urysohn-Stieltjes type and its applications. *Nonlinear Anal.* **47** (2001), 1175-1186.
12. J. Banaś & B. Rzepka: *An application of a measure of noncompactness in the study of asymptotic stability,* Appl. Math. Lett. **16** (2003), 1-6.
13. B. Cahlon and M. Eskin: Existence theorems for an integral equation of the Chandrasekhar H-equation with perturbation. *J. Math. Anal. Appl.* **83** (1981), 159-171.
14. C. Corduneanu: *Integral equations and applications.* Cambridge University Press, Cambridge, MA, 1991.
15. G. Darbo: Punti uniti in transformazioni a condominio non compatto. *Rend. Sem. Mat. Univ. Padova*, **24** (1955), 84-92.
16. S. Hu, M. Khavani & W. Zhuang: Integral equations arising in the kinetic theory of gases. *Appl. Anal.* **34** (1989), 261-266.
17. D. O'Regan & M. Meehan: *Existence theory for nonlinear integral and integrodifferential equations.* Kluwer Academic, Dordrecht, 1998.

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