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On Choquet Integrals with Respect to a Fuzzy Complex Valued Fuzzy Measure of Fuzzy Complex Valued Functions

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Abstract

In this paper, using fuzzy complex valued functions and fuzzy complex valued fuzzy measures ([11]) and interval-valued Choquet integrals ([2-6]), we define Choquet integral with respect to a fuzzy complex valued fuzzy measure of a fuzzy complex valued function and investigate some basic properties of them.

Key Words : fuzzy numbers, comonotonic, fuzzy complex numbers, fuzzy complex valued function, fuzzy complex valued fuzzy measures, Choquet integrals.

1. Introduction

Buckley [1] first defined the concept of fuzzy complex numbers and have studied the theory of fuzzy complex numbers, the differentiability and integrability of fuzzy complex valued functions on a complex plane $\mathbb{C}$. Wang and Li [11] studied generalized Lebesgue integrals of fuzzy complex valued functions.

By using the method of establishing the basic framework for fuzzy complex analysis, we will define Choquet integrals with respect to a fuzzy complex valued fuzzy measure of fuzzy complex valued functions. We note that interval-valued Choquet inetgrals were defined by Jang [2-6].

Let (X, Ω) be a measurable space. A mapping $\mu : \Omega \to [0,\infty]$ on X is called a fuzzy measure if it is satisfying the following conditions;

(i) $\mu(\varnothing)=0$,

(ii) $\mu(A) \leq \mu(B)$,
whenever $A,B \in \Omega$, $A \subset B$.

(iii) for every increasing sequence $\{A_n\}$ of measurable sets, we have

$$\mu(\cup_{n=1}^{\infty} A_n) = \lim_{n\to\infty}\mu(A_n).$$

(iv) for every decreasing sequence $\{A_n\}$ of measurable sets and $\mu(A_1) < \infty$, we have

$$\mu(\cap_{n=1}^{\infty} A_n) = \lim_{n\to\infty}\mu(A_n).$$

In many papers, a fuzzy measure is satisfying the conditions (i) and (ii). In this paper, we assume that a fuzzy measure is satisfying the four conditions (i)-(iv).

Definition 1.1 ([2-6]) (1) The Choquet integral of a measurable function f with respect to a fuzzy measure μ on $A \in \Omega$ is defined by

$$(C)\int_A f d\mu = \int_0^\infty \mu(\{x|f(x)>r\}\cap A)dr$$

where the integral on the right-hand side is an ordinary one.

(2) A measurable function f is said to be integrable if the Choquet integral of f can be defined and its value is finite.

Instead of $(C)\int_X f d\mu$, we will write $(C)\int f d\mu$. Throughout this paper, $\mathbb{R}^+$ will denote the interval $[0,\infty)$.

Definition 1.2 ([2-6]) A set $N \in \Omega$ is called a null set with respect to μ if $\mu(A \cup N)=\mu(A)$, for all $A \in \Omega$.

We note that $[P(x)\ \ \mu-a.e\quad on\ \ A]$ means there exists a null set N such that $P(x)$ is true for all $x \in A-N$, where $P(x)$ is a proposition concerning the point of A.

Definition 1.3 ([2-8]) Let f,g be measurable nonnegative functions. We say that f is comonotonic to g, in symbol $f \sim g$ if and only if $f(x)<f(x') \Rightarrow g(x) \leq g(x')$ for all $x,x' \in X$.

Theorem 1.4 ([2-8]) Let f,g,h be measurable functions. Then we have

(1) $f \sim f$.

(2) If $f \sim g$, then $g \sim f$.

(3) For all $a \in R^+$, we have $f \sim a$.

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(4) If $f \sim g$ and $f \sim h$, then $f \sim g+h$.

Theorem 1.5 ([2-8]) Let f, g, h be measurable functions.

(1) If $f \leq g$, then $(C)\int f d\mu \leq (C)\int g d\mu$.

(2) If $A \subset B$ and $A, B \in \Omega$, then

$$(C)\int_A f d\mu \leq (C)\int_B f d\mu.$$

(3) If $f \sim g$ and $a, b \in R^+$, then

$$(C)\int (af+bg) d\mu = a(C)\int f d\mu + b(C)\int g d\mu.$$

(4) If $(f \vee g)(x) = max\{f(x), g(x)\}$ and $(f \wedge g)(x) = min\{f(x), g(x)\}$ for all $x \in X$, then

$$(C)\int (f \vee g) d\mu \geq (C)\int f d\mu \vee (C)\int g d\mu \text{ and}$$

$$(C)\int (f \wedge g) d\mu \leq (C)\int f d\mu \wedge (C)\int g d\mu.$$

Throughout this paper, $I(\mathbb{R}^+)$ is the class of all intervals in $\mathbb{R}^+$, that is,

$$I(\mathbb{R}^+) = \{[a^-, a^+] \mid a^-, a^+ \in \mathbb{R}^+ \text{and}\, a^- \leq a^+\}.$$

For any $a \in \mathbb{R}^+$, we define $a = [a, a]$. Obviously, $a \in I(\mathbb{R}^+)$.

Definition 1.6 ([5,6]) If $\overline{a}, \overline{b} \in I(\mathbb{R}^+)$ and $k \in \mathbb{R}^+$, then we define

(1) $\overline{a} + \overline{b} = [a^- + b^-, a^+ + b^+]$,

(2) $k\overline{a} = [ka^-, ka^+]$,

(3) $\overline{a}\overline{b} = [a^-b^-, a^+b^+]$,

(4) $\overline{a} \wedge \overline{b} = [a^- \wedge b^-, a^+ \wedge b^+]$,

(5) $\overline{a} \vee \overline{b} = [a^- \vee b^-, a^+ \vee b^+]$,

(6) $\overline{a} \leq \overline{b}$ if and only if

$$a^- \leq b^- \text{ and } a^+ \leq b^+,$$

(7) $\overline{a} < \overline{b}$ if and only if $\overline{a} \leq \overline{b}$ and $\overline{a} \neq \overline{b}$,

(8) $\overline{a} \subset \overline{b}$ if and only if

$$b^- \leq a^- \text{ and } a^+ \leq b^+.$$

Theorem 1.7 ([5,6]) Let $\overline{a}, \overline{b} \in I(\mathbb{R}^+)$. Then the followings hold.

(1) idempotent law: $\overline{a} \wedge \overline{a} = \overline{a}, \overline{a} \vee \overline{a} = \overline{a}$,

(2) commutative law:

$$\overline{a} \wedge \overline{b} = \overline{b} \wedge \overline{a},\ \overline{a} \vee \overline{b} = \overline{b} \vee \overline{a},$$

(3) associative law:

$$(\overline{a} \wedge \overline{b}) \wedge \overline{c} = \overline{a} \wedge (\overline{b} \wedge \overline{c}),$$
$$(\overline{a} \vee \overline{b}) \vee \overline{c} = \overline{a} \vee (\overline{b} \vee \overline{c}),$$

(4) absorption law:

$$\overline{a} \wedge (\overline{a} \vee \overline{b}) = \overline{a} \vee (\overline{a} \wedge \overline{b}) = \overline{a},$$

(5) distributive law:

$$\overline{a} \wedge (\overline{b} \vee \overline{c}) = (\overline{a} \wedge \overline{b}) \vee (\overline{a} \wedge \overline{c}),$$
$$\overline{a} \vee (\overline{b} \wedge \overline{c}) = (\overline{a} \vee \overline{b}) \wedge (\overline{a} \vee \overline{c}).$$

Definition 1.8 ([5,6]) A set function $d_H : I(\mathbb{R}^+) \times I(\mathbb{R}^+) \to [0, \infty]$ is called the Hausdorff metric if

$$d_H(A,B) = \max\{\sup_{x \in A} \inf_{y \in B} |x-y|,\ \sup_{y \in B} \inf_{x \in A} |x-y|\},$$

for all $A, B \in I(\mathbb{R}^+)$.

Theorem 1.9 ([5,6]) If $d_H : I(\mathbb{R}^+) \times I(\mathbb{R}^+) \to [0, \infty]$ is the Hausdorff metric, then we have for $\overline{a} = [a^-, a^+]$, $\overline{b} = [b^-, b^+] \in I(\mathbb{R}^+)$,

$$d_H(\overline{a}, \overline{b}) = \max\{|a^- - b^-|, |a^+ - b^+|\}.$$

For a sequence of intervals $\{\overline{a_n}\} \subset I(\mathbb{R}^+)$ and $\overline{a} \in I(\mathbb{R}^+)$, we say that $\{\overline{a_n}\}$ converges to $\overline{a}$, in symbol, $d_H - \lim_{n\to\infty} \overline{a_n} = \overline{a}$ if $d_H(\overline{a}_n, \overline{a}) \to 0\ (n \to \infty)$.

Obviously, we obtain $d_H - \lim_{n\to\infty} \overline{a}_n = \overline{a}$ if and only if $a_n^- \to a^-$ and $a_n^+ \to a^+ (n \to \infty)$.

A fuzzy number $\tilde{u}$ on $\mathbb{R}^+$ is a fuzzy set satisfying the following conditions (see[6,9,11]);

(i)(normality) $\tilde{u}(x) = 1$ for some $x \in \mathbb{R}^+$,

(ii)(fuzzy convexity) for every $\lambda \in (0,1]$,

$$\overline{u_\lambda} = \{x \in \mathbb{R}^+ \mid \tilde{u}(x) \geq \lambda\} \in I(\mathbb{R}^+).$$

Let $FN(\mathbb{R}^+)$ denote the set of fuzzy numbers, we define basic arithmetic operations on $FN(\mathbb{R}^+)$ (see [6,9,11]); for each pair $\tilde{u}, \tilde{v} \in FN(\mathbb{R}^+)$ and $k \in \mathbb{R}^+$,

$$(\tilde{u} + \tilde{v})_\lambda = \overline{u_\lambda} + \overline{v_\lambda},\ (k\tilde{u})_\lambda = k\overline{u_\lambda},$$

$\tilde{u} \leq \tilde{v}$ if and only if $\overline{u_\lambda} \leq \overline{v_\lambda}$ for all $\lambda \in (0,1]$,

$\tilde{u} < \tilde{v}$ if and only if $\tilde{u} \leq \tilde{v}$ and $\tilde{u} \neq \tilde{v}$,

In section 2, we introduce fuzzy complex numbers and discuss their basic arithmetic properties of them. And also we consider an interval-valued fuzzy measure. In section 3, we consider fuzzy valued functions and fuzzy complex valued fuzzy measures. And also we define Choquet integrals with respect to a fuzzy complex valued fuzzy measure of fuzzy complex valued functions.

2. Fuzzy Complex Fuzzy Measures.

Definition 2.1 ([11]) Let $\tilde{a},\tilde{b}\in FN(\mathbb{R}^+)$. We define a ordered fuzzy numbers $(\tilde{a},\tilde{b})$ as follows:

$$(\tilde{a},\tilde{b}):\ \mathbb{C}^+\rightarrow[0,1],$$

$$z=x+yi\mapsto(\tilde{a},\tilde{b})(z)=\tilde{a}(x)\wedge\tilde{b}(y).$$

Then the mapping $(\tilde{a},\tilde{b})$ determines a fuzzy complex number, where $\tilde{a}$ and $\tilde{b}$ is called a real part and an imaginary part of $(\tilde{a},\tilde{b})$, respectively. Let $C=(\tilde{a},\tilde{b})$, then $\tilde{a}=Re\ C$, $\tilde{b}=Im\ C$.

Let $\mathbb{C}^+=\{x+iy|\ x,y\in\mathbb{R}^+\}$ and $FCN(\mathbb{C}^+)$ be the set of fuzzy complex numbers on $\mathbb{C}^+$, writing

$$\tilde{c}=\tilde{a}+\tilde{b}i=(\tilde{a},\tilde{b}),\ \tilde{a},\tilde{b}\in FN(\mathbb{R}^+).$$

We note that if $c=a+bi$ is a complex number, then it's membership function is

$$c(z)=\begin{cases}1 & x=a,y=b\\ 0 & otherwise,\end{cases}$$

whenever $z=(x,y)\in\mathbb{C}^+$. If $C_1,C_2\in FCN(\mathbb{C}^+)$ and we define

$$C_1*C_2=(Re\ C_1*ReC_2,Im\ C_1*ImC_2)$$

for operation $*\in\{+,-,\cdot,\wedge,\vee\}$, then clearly we have C_1*C_2 belongs to $FCN(\mathbb{C}^+)$. Now we introduce some order relations and equality relation on $FCN(\mathbb{C}^+)$.

Definition 2.2 ([11]) Let $C_1,C_2\in FCN(\mathbb{C}^+)$.

(1) $C_1\leq C_2$ if and only if

$$Re\ C_1\leq ReC_2,\ Im\ C_1\leq ImC_2.$$

(2) $C_1<C_2$ if and only if $C_1\leq C_2$ and $Re\ C_1<ReC_2$ or $Im\ C_1<ImC_2$.

(3) $C_1=C_2$ if and only if

$$C_1\leq C_2,\ C_2\leq C_1.$$

We will define the new metric D on $FCN(\mathbb{C}^+)$ as follows.

Definition 2.3 (1) If $C\in FCN(\mathbb{C}^+)$, then C_λ is closed rectangle region on $\mathbb{C}^+$, for all $\lambda\in(0,1]$, defined as $C_\lambda=\{z\in\mathbb{C}^+|\ C(z)\geq\lambda\}$. Obviously, if $\tilde{a},\tilde{b}\in FN(\mathbb{R}^+)$, then $(\tilde{a},\tilde{b})_\lambda=(\tilde{a}_\lambda,\tilde{b}_\lambda)$ for all $\lambda\in(0,1]$.

(2) A mapping

$$D:FCN(\mathbb{C}^+)\times FCN(\mathbb{C}^+)\rightarrow[0,\infty]$$

is defined by

$$D(C_1,C_2)=\max\{\triangle(ReC_1,ReC_2),\triangle(ImC_1,ImC_2)\},$$

where

$$\triangle(ReC_1,ReC_2)=\vee_{\lambda\in(0,1]}d_H((ReC_1)_\lambda,(ReC_2)_\lambda)$$

and

$$\triangle(ImC_1,ImC_2)=\vee_{\lambda\in(0,1]}d_H((ImC_1)_\lambda,(ImC_2)_\lambda).$$

It is clearly to see that $(FCN(\mathbb{C}^+),D)$ is a metric space. By using this metric, we define the concept of convergence of a sequence in the metric space $(FCN(\mathbb{C}^+),D)$.

Definition 2.4 Let $\{C_n\}\subset FCN(\mathbb{C}^+)$ be a sequence of fuzzy complex valued numbers and $C\in FCN(\mathbb{C}^+)$. The sequence $\{C_n\}$ converges to C, in symbol, $D-\lim_{n\to\infty}C_n=C$ if

$$\lim_{n\to\infty}D(C_n,C)=0.$$

We also consider an interval-valued fuzzy measure as follows.

Definition 2.5 ([11]) Let (X,Ω) be a measurable space, a mapping

$\bar{\mu}:\Omega\rightarrow I(\mathbb{R}^+)$ is called an interval-valued fuzzy measure if it is satisfying

(i) $\bar{\mu}(\emptyset)=[0,0]$,

(ii) $\bar{\mu}(A)\leq\bar{\mu}(B)$, whenever $A,B\in\Omega$, $A\subset B$.

(iii) If $(A_n)\subset\Omega$ and $A_n\nearrow A$ or $A_n\searrow A$ implies $\bar{\mu}(A_n)\rightarrow\bar{\mu}(A)$.

We note that for any $A\in\Omega$, denote $\bar{\mu}(A)=[\mu^-(A),\mu^+(A)]$ or simply write as $\bar{\mu}=[\mu^-,\mu^+]$.

Proposition 2.6 ([11]) A mapping $\bar{\mu}:\Omega\rightarrow I(\mathbb{R}^+)$ is an interval-valued fuzzy measure if and only if μ^- and μ^+ are fuzzy measure under Sugeno's sense.

3. Choquet Integrals of Fuzzy Complex Valued Functions

In this section, we define a fuzzy complex valued fuzzy measure and Choquet integral with respect to a fuzzy complex valued fuzzy measure of fuzzy complex valued functions. Let $\mathbb{C}^+=\{x+yi\,|\,x,y\in\mathbb{R}^+\}$ and $(\mathbb{C}^+,\Omega)$ be a measurable space. We consider a fuzzy complex valued function as follows.

Definition 3.1 ([11]) If a mapping $\tilde{f}:\mathbb{C}^+\rightarrow FCN(\mathbb{C}^+)$ is defined by

$$z = x + yi \rightarrow \tilde{f}(z) = (Re\tilde{f}, Im\tilde{f})(z) \equiv Re\tilde{f}(x) \wedge Im\tilde{f}(y),$$

then $\tilde{f}$ is called a fuzzy complex valued function on $\mathbb{C}^+$.

We note that $Re\tilde{f}(x) \in FN(\mathbb{R}^+)$, $Im\tilde{f}(y) \in F(\mathbb{R}^+)$. For any $\lambda \in (0,1]$, let

$$\tilde{f}_\lambda(z) = (\tilde{f}(z))_\lambda = (Re\tilde{f}_\lambda(z), Im\tilde{f}_\lambda(z)),$$
$(Re\tilde{f})_\lambda = Re\tilde{f}_\lambda = (Re\tilde{f}_\lambda^{\,-}, Re\tilde{f}_\lambda^{\,+})$, and
$$(Im\tilde{f})_\lambda = Im\tilde{f}_\lambda = (Im\tilde{f}_\lambda^{\,-}, Im\tilde{f}_\lambda^{\,+}).$$

Theorem 3.2 ([11]) Let $\tilde{f}_1, \tilde{f}_2$ be fuzzy complex valued measurable functions on $(\mathbb{C}^+, \Omega)$, then $\tilde{f}_1 \pm \tilde{f}_2$ and $\tilde{f}_1 \cdot \tilde{f}_2$ are fuzzy complex valued measurable functions.

Definition 3.3 ([11]) Let $(\mathbb{C}^+, \Omega)$ be a measurable space, a mapping $\tilde{\mu}: \Omega \rightarrow FCN(\mathbb{C}^+)$ is called a fuzzy complex valued fuzzy measure, if the following conditions are satisfied:

(1) $\tilde{\mu}(\varnothing) = [\tilde{0}, \tilde{0}]$, where $\tilde{0} \in FCN(\mathbb{C}^+)$,

(2) $\tilde{\mu}(A) \leq \tilde{\mu}(B)$,
whenever $A, B \in \Omega, A \subset B$, and

(3) if $(A_n) \subset \Omega$ and $A_n \nearrow A$ or $A_n \searrow A$, then $\tilde{\mu}(A_n) \rightarrow \tilde{\mu}(A)$ $(n \rightarrow \infty)$ in meaning of the metric D, in symbol,

$$D - \lim_{n \rightarrow \infty} \tilde{\mu}_n = \tilde{\mu}.$$

We note that $(\mathbb{C}^+, \Omega, \tilde{\mu})$ is called a fuzzy complex valued fuzzy measure space and denote that $\tilde{\mu}(A) = (\tilde{\mu}_R(A), \tilde{\mu}_I(A))$ or simply write as $\tilde{\mu} = (\tilde{\mu}_R, \tilde{\mu}_I)$ for any $A \in \Omega$. Now, we will define the Choquet integral with respect to a fuzzy complex fuzzy measure of a fuzzy complex valued function as follows. The idea of the following definition is similar to the idea of the generalized Lebesgue integral in Wang and Li [11].

Definition 3.4 Let $\tilde{\mu} = (\tilde{\mu}_R, \tilde{\mu}_I)$ be a fuzzy complex valued fuzzy measure and $\tilde{f} = (Re\tilde{f}, Im\tilde{f})$ a fuzzy complex valued measurable function.

(1) For any $A \in \Omega$, the Choquet integral with respect to $\tilde{\mu}$ of $\tilde{f}$ is defined by

$$\left((C)\int_A \tilde{f}\, d\tilde{\mu}\right)_\lambda \equiv \left((C)\int_A Re\tilde{f}_\lambda d(\tilde{\mu}_R)_\lambda,\ (C)\int_A Im\tilde{f}_\lambda d(\tilde{\mu}_I)_\lambda\right)$$

for all $\lambda \in (0,1]$, where

$$(C)\int_A Re\tilde{f}_\lambda d(\tilde{\mu}_R)_\lambda = \left[(C)\int_A (Re\tilde{f})_\lambda^- d(\tilde{\mu}_R)_\lambda^-,\ (C)\int_A (Re\tilde{f})_\lambda^+ d(\tilde{\mu}_R)_\lambda^+\right]$$

and

$$(C)\int_A Im\tilde{f}_\lambda d(\tilde{\mu}_I)_\lambda = \left[(C)\int_A (Im\tilde{f})_\lambda^- d(\tilde{\mu}_I)_\lambda^-, (C)\int_A (Im\tilde{f})_\lambda^+ d(\tilde{\mu}_I)_\lambda^+\right].$$

(2) If there exists $\tilde{u} \in FCN(\mathbb{C}^+)$ such that $(\tilde{u})_\lambda = \left((C)\int_A \tilde{f}\, d\tilde{\mu}\right)_\lambda$ for all $\lambda \in (0,1]$, then $\tilde{f}$ is said to be Choquet integrable on A.

(3) $\tilde{f}$ is said to be Choquet integrably bounded if $Re\tilde{f}$ and $Im\tilde{f}$ are Choquet integrably bounded.

Instead of $(C)\int_X \tilde{f} d\tilde{\mu}$, we will write $(C)\int \tilde{f} d\tilde{\mu}$.

Remark 3.5 $Re\tilde{f}$ and $Im\tilde{f}$ are Choquet integrably bounded if and only if for all $\lambda \in (0,1]$, interval-valued measurable functions $(Re\tilde{f})_\lambda$ and $(Im\tilde{f})_\lambda$ are Choquet integrably bounded (see [5,6]). And we also see that

$$(C)\int_A (Re\tilde{f})_\lambda^- d(\tilde{\mu}_R)_\lambda^-,\ (C)\int_A (Re\tilde{f})_\lambda^+ d(\tilde{\mu}_R)_\lambda^+,$$
$$(C)\int_A (Im\tilde{f})_\lambda^- d(\tilde{\mu}_I)_\lambda^- \text{ and } (C)\int_A (Im\tilde{f})_\lambda^+ d(\tilde{\mu}_I)_\lambda^+$$

are finite, that is, they are well-defined (see [2-4]).

Lemma 3.6 ([6,9]) Let $\{[a_\lambda, b_\lambda] | \lambda \in (0,1]\}$ be given a family of nonempty intervals in $I(\mathbb{R}^+)$. If (i) for all $0 < \lambda_1 \leq \lambda_2$, $[a_{\lambda_1}, b_{\lambda_1}] \supset [a_{\lambda_2}, b_{\lambda_2}]$ and (ii) for any nonincreasing sequence $\{\lambda_k\}$ in $(0,1]$ converging to λ, $[a_\lambda, b_\lambda] = \bigcap_{k=1}^{\infty} [a_{\lambda_k}, b_{\lambda_k}]$. Then there exists a unique fuzzy number $\tilde{u} \in FN(\mathbb{R}^+)$ such that the family $[a_\lambda, b_\lambda]$ represents the λ-level sets of a fuzzy number $\tilde{u} \in FN(\mathbb{R}^+)$. Conversely, if $[a_\lambda, b_\lambda]$ are the λ-level set of a fuzzy number $\tilde{u} \in FN(\mathbb{R}^+)$, then the conditions (i) and (ii) are satisfied.

By using the definition of a fuzzy complex valued fuzzy measure with condition (iii), we easily obtain the following lemma.

Lemma 3.7 Let $\{\lambda_k\}$ be a nonincreasing sequence in $(0,1]$ converging to λ. If we put

$$g_{k,m}^R(\alpha) = (\tilde{\mu}_R)_{\lambda_k}^-\left(\left\{x \mid (Re\tilde{f})_{\lambda_m}^-(x) > \alpha\right\}\right),$$
$$h_{k,m}^R(\alpha) = (\tilde{\mu}_R)_{\lambda_k}^+\left(\left\{x \mid (Re\tilde{f})_{\lambda_m}^+(x) > \alpha\right\}\right), \text{ and}$$

$$g^I_{k,m}(\alpha) = (\tilde{\mu}_I)^-_{\lambda_k}\Big(\Big\{x \,|\, (Im\tilde{f})^-_{\lambda_m}(x) > \alpha\Big\}\Big),$$

$$h^I_{k,m}(\alpha) = (\tilde{\mu}_I)^+_{\lambda_k}\Big(\Big\{x \,|\, (Im\tilde{f})^+_{\lambda_m}(x) > \alpha\Big\}\Big), \text{ and}$$

$$g^R_{\lambda}(\alpha) = (\tilde{\mu}_R)^-_{\lambda}\Big(\Big\{x \,|\, (Re\tilde{f})^-_{\lambda}(x) > \alpha\Big\}\Big),$$

$$h^R_{\lambda}(\alpha) = (\tilde{\mu}_I)^+_{\lambda}\Big(\Big\{x \,|\, (Re\tilde{f})^+_{\lambda}(x) > \alpha\Big\}\Big), \text{ and}$$

$$g^I_{\lambda}(\alpha) = (\tilde{\mu}_I)^-_{\lambda}\Big(\Big\{x \,|\, (Im\tilde{f})^-_{\lambda}(x) > \alpha\Big\}\Big),$$

$$h^I_{\lambda}(\alpha) = (\tilde{\mu}_I)^+_{\lambda}\Big(\Big\{x \,|\, (Im\tilde{f})^+_{\lambda}(x) > \alpha\Big\}\Big),$$

for all $\alpha \in \mathbb{R}^+$ and $k,m \in \mathbb{N}$, then we have $g^R_{k,m}(\alpha) \searrow g^R_{\lambda}(\alpha)$, $h^R_{k,m}(\alpha) \searrow h^R_{\lambda}(\alpha)$, $g^I_{k,m}(\alpha) \searrow g^I_{\lambda}(\alpha)$, and $h^I_{k,m}(\alpha) \searrow h^I_{\lambda}(\alpha)$.

Remark 3.8 (1) For all $0 < \lambda_1 \le \lambda_2$, we obtain

$$(C)\int_A (Re\,\tilde{f})^-_{\lambda_1} d(\tilde{\mu}_R)^-_{\lambda_1} \ \ge\ (C)\int_A (Re\,\tilde{f})^-_{\lambda_2} d(\tilde{\mu}_R)^-_{\lambda_2},$$

$$(C)\int_A (Re\,\tilde{f})^+_{\lambda_1} d(\tilde{\mu}_R)^+_{\lambda_1} \ \ge\ (C)\int_A (Re\,\tilde{f})^+_{\lambda_2} d(\tilde{\mu}_R)^+_{\lambda_2},$$

$$(C)\int_A (Im\,\tilde{f})^-_{\lambda_1} d(\tilde{\mu}_I)^-_{\lambda_1} \ \ge\ (C)\int_A (Im\,\tilde{f})^-_{\lambda_2} d(\tilde{\mu}_I)^-_{\lambda_2},$$

and

$$(C)\int_A (Im\,\tilde{f})^+_{\lambda_1} d(\tilde{\mu}_I)^+_{\lambda_1} \ \ge\ (C)\int_A (Im\,\tilde{f})^+_{\lambda_2} d(\tilde{\mu}_I)^+_{\lambda_2}.$$

(2) If we take $k = m$, then we obtain the followings: For any nonincreasing sequence $\{\lambda_k\}$ in $(0,1]$ conversing to λ,

$$(C)\int_A (Re\tilde{f})^-_{\lambda} d(\tilde{\mu}_R)^-_{\lambda} = \bigcap_{k=1}^{\infty}\left((C)\int_A (Re\tilde{f})^-_{\lambda_k} d(\tilde{\mu}_R)^-_{\lambda_k}\right),$$

$$(C)\int_A (Re\tilde{f})^+_{\lambda} d(\tilde{\mu}_R)^+_{\lambda} = \bigcap_{k=1}^{\infty}\left((C)\int_A (Re\tilde{f})^+_{\lambda_k} d(\tilde{\mu}_R)^+_{\lambda_k}\right),$$

$$(C)\int_A (Im\tilde{f})^-_{\lambda} d(\tilde{\mu}_I)^-_{\lambda} = \bigcap_{k=1}^{\infty}\left((C)\int_A (Im\tilde{f})^-_{\lambda_k} d(\tilde{\mu}_I)^-_{\lambda_k}\right), \quad \text{and}$$

$$(C)\int_A (Im\tilde{f})^+_{\lambda} d(\tilde{\mu}_I)^+_{\lambda} = \bigcap_{k=1}^{\infty}\left((C)\int_A (Im\tilde{f})^+_{\lambda_k} d(\tilde{\mu}_I)^+_{\lambda_k}\right).$$

From Definition 3.4, Remark 3.5, Lemma 3.7 and Remark 3.8, we obtain the following theorem.

Theorem 3.9 Let $(\mathbb{C}^+, \Omega, \tilde{\mu})$ be fuzzy complex valued fuzzy measure space. If $\tilde{f}$ is a fuzzy complex valued integrably bounded function, then we have

(i) for all $0 < \lambda_1 \le \lambda_2$,

$$\left((C)\int_A \tilde{f}\, d\tilde{\mu}\right)_{\lambda_1} \supset \left((C)\int_A \tilde{f}\, d\tilde{\mu}\right)_{\lambda_2},$$

(ii) for any nonincreasing sequence $\{\lambda_k\}$ in $(0,1]$ conversing to λ,

$$\left((C)\int_A \tilde{f}\, d\tilde{\mu}\right)_{\lambda} = \bigcap_{k=1}^{\infty}\left((C)\int_A \tilde{f}\, d\tilde{\mu}\right)_{\lambda_k}.$$

Remark 3.10 From Theorem 3.9 and Lemma 3.6, there exists a fuzzy complex number $\tilde{u} \in FCN(\mathbb{C}^+)$ such that

$$(\tilde{u})_{\lambda} = \left((C)\int_A \tilde{f}\, d\tilde{\mu}\right)_{\lambda}$$

for all $\lambda \in (0,1]$. That is, if a fuzzy complex valued function $\tilde{f}$ is integrably bounded, then it is Choquet integrable.

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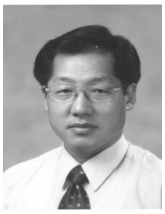

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