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Model Identification and Attitude Control Methodology for the Flexible Body of a Satellite

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Abstract

The controller of a model reference adaptive control monitors the plant's inputs and outputs to acknowledge its characteristics. It then adapts itself to the characteristics it encounters instead of behaving in a fixed manner. An important part of every adaptive scheme is the adaptive law for estimating the unknown parameters on line. A more precise model is required to improve performance and to stabilize a given dynamic system, such as a satellite in which performance varies over time and the coefficients change due to disturbances, etc. After model identification, the robust controller (H∞) is designed to stabilize the rigid body and flexible body of a satellite, which can be perturbed due to disturbance. The result obtained by the H∞ controller is compared with that of the proportional and integration controller which is commonly used for stabilizing a satellite.

Key words: Satellite attitude control, Adaptive control, Model identification, Proportional and integration controller, Robust controller (H∞)

1. Introduction

A precise dynamic model, such as a satellite that varies with attitude and speed, is required for attitude control. The controller monitors the plant's inputs and outputs to acknowledge its characteristics by a model reference adaptive control (MRAC). An important part of every adaptive scheme is the adaptive law for estimating the unknown parameters on line. The adaptive law (Ioannou and Datta, 1991) is designed by first developing a parameterization of the unknown plant in terms of the unknown vector ψ^*, which has to be estimated on-line. The general problem of the on-line constant parameter vector ψ^* of a certain class of dynamic systems is described by

$$z(t) = f(\psi^*, t, \tau), t \geq \tau \geq 0 \tag{1}$$

where at each time t, the response z(t) with z≤t can be observed and is some function whose form may be known. If we consider $\psi(t)$ as an estimate of ψ^* at time t, then the estimate $\hat{z}(t)=\hat{z}(t, \psi)$ of z(t) can be constructed as

$$\hat{z}(t,\psi) = \hat{f}(\psi, t, \tau) \tag{2}$$

for some function. This estimation process is a mean through which the adjustment law for ψ may be designed so that $\hat{z}(t, \psi)$ is as close as possible to z(t). Standard options for ensuring the quality of the estimation might be:

i) $|\hat{z}(t,\psi) - z(t)| \to 0$ as $t \to \infty$

ii) $|\hat{z}(t,\psi) - z(t)| \in L_2 \cap L_\infty$

iii) $|\psi(t) - \psi^*(t)| \to 0$ as $t \to \infty$

In particular, method i) is more frequently used than ii) or iii).

After achieving precise model identification, the robust controller(H∞) was designed for the rigid body (Jin et al., 1994; Lho et al., 1998) and for the flexible body in order to attain stabilization and the desired performance. It was assumed that the plant model comprised pitch dynamics, an earth sensor, and momentum wheel dynamics. In order to design a control system, we obtained a simplified mathematical model which described the actual plant as being controlled with a reasonable degree of accuracy over the operating range of interest. While a simple model leads

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to a simpler control design, such a design must possess a sufficient degree of robustness or sensitivity with respect to the unmodeled plant characteristics. The plant with uncertainty is represented as

$$P_1(s) = P_o(s) + \delta P(s) \tag{3}$$

where $P_o(s)$ is the ideal plant dynamic model, and $\delta p(s)$ is the perturbation of uncertainty. Given a compensator C(s) which stabilizes $P_o(s)$, we established the conditions for C(s) to be a robust stabilizer for all the plants in the class $C(P_o(s), r(s))$. From the hypothesis that C(s) stabilizes $P_o(s)$ we have

$$P_o(jw)C(jw) + 1 \neq 0 \quad \forall w \tag{4}$$

and $P_o(jw)C(jw)$ has the correct encirclements of -1 point to guarantee, from Nyquist's stability criterion, that the nominal closed-loop system is stable.

A sufficient condition (Dorato et al., 1989) for robust stability is then

$$\| (1 + P_o(s)C(s))^{-1} C(s)r(s) \|_\infty < 1 \tag{5}$$

The performance of the robust controller was compared to that of the proportional and integration (PI) controller, which has been applied to satellites such as Korea multi-purpose satellite (KOMPSAT) (KOMPSAT, 1996).

2. Modeling of Satellite

The rotational motion equation of the satellite is represented by the moment equation (Jin et al., 1994) as

$$T = \dot{H} + w \times H \tag{6}$$

where T is the external torque, H is the linear angular momentum, and w is the angular velocity. The attitude angle of the satellite contains a roll (φ), pitch (θ), and yaw (δ). In Eq. (6), the angular velocity vector is composed of the attitude angle, the orbit angular velocity, and the momentum equation of the satellite under the assumption that the movement of pitch axis is unrelated to that of the other axis, and the small attitude angle.

2.1 The flexible model of satellite

Flexible characteristics of the satellite are attributed to the satellite's possession of a solar panel, an antenna, and the antenna's supporting body. The vibration mode of solar panel correlates to the attitude angle of satellite, and the twisted mode of the solar panel is related to the attitude angle of the pitch. The mathematical model of the flexible model (KOMPSAT, 1996) is shown below

$$I_{yy}\ddot{\theta} + \alpha\theta + D_y\ddot{q}_y = (T_s + T_c)_y - \dot{h} = T \tag{7}$$

$$\ddot{q}_y + 2\zeta\sigma_y\dot{q}_y + \sigma_y^2 q_y + D_y\ddot{\theta} = 0 \tag{8}$$

where, I_{yy} is the inertia moment about the pitch of the satellite, h is the angular momentum of the momentum wheel rotating toward the pitch axis, T_s is the disturbed torque, T_c is the control torque, T is the outside torque reacting to the satellite, D_y is the related coefficient between the vibration mode and the attitude angle of rigid body, q_y is the modal coordinate of the twisted mode of solar panel, σ_y is the number of vibration of the twisted mode of the solar panel, and τ is the passive attenuation coefficient of the vibration mode of the solar panel. The satellite body and attitude angle is shown in Fig. 1, and the modeling of the optimized flexible body of the satellite is in Fig. 2.

2.2 The rigid model of satellite

In the movement equation of the rigid model (Jin et al., 1994), the dynamic equation of the pitch under the assumption that no coefficients are related between the vibration mode and the attitude angle of the rigid body is written as

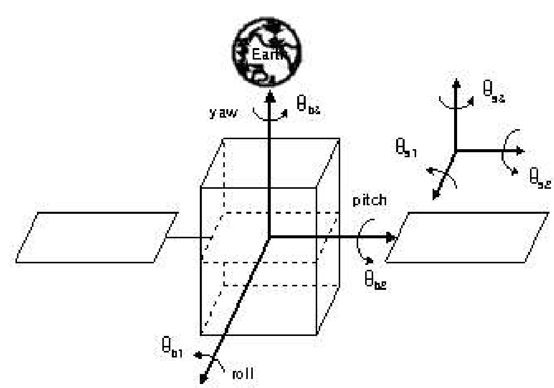

Fig. 1. Satellite body and attitude angle.

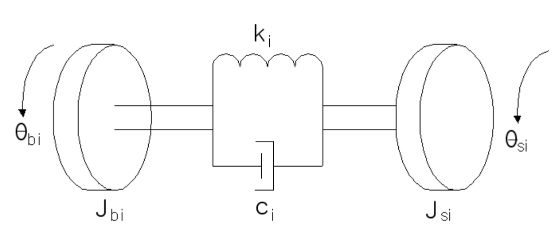

Fig. 2. Optimized flexible body of the satellite.

$$I_{yy}\ddot{\theta} + \alpha\theta = T \tag{9}$$

·α=0case)

Eq. (9) represents the dynamic equation of the pitch in the rigid model with no position angle.

When the inertia moment of the roll is almost same as that of the yaw, and the angular velocity is small, the angle θ can be neglected. In the rigid body, the pitch circuit contains pitch dynamics in which the moment equation incorporates the pitch axis, the earth sensor, and the controller. The remaining components, except for the controller, are considered as the plant. For the input signal u and the output torque T, the differential equation of the momentum wheel in Fig. 3 is

$$T_m\dot{T} + T = K_m\dot{u} \tag{10}$$

Substituting Eq. (7) into Eq. (10), we obtain

$$T_m I_{yy}\ddot{\theta} + T_m D_y \ddot{q}_y + I_{yy}\dot{\theta} + I_{yy}\dot{q}_y = K_m u \tag{11}$$

Assuming the output of the plant is y=θ, and the state variables are considered as $x=[\theta\ \dot{\theta}\ q_y\ \dot{q}_y]^T$, the state equation becomes a 4th order system (Lho et al., 1998).

· $\alpha = 3\ w_o^2(I_\varphi - I_\delta)$ case)

Using Eq. (7) and Eq. (8), we get Eq. (12) as

$$(I_{yy} - D_y^2)\ddot{\theta} + \alpha\theta - 2D_y\zeta\sigma_y\dot{q}_y - D_y\sigma_y^2 q_y = T \tag{12}$$

Combining Eq. (7) and Eq. (8) results in a fifth order state equation

$$\dot{x}=Ax+Bu$$
$$y=Cx \tag{13}$$

Table 1. Design data

Symbol	Name	Value	Unit	Remark
K_m	Motor gain	0.0792	N.m/V	Wheel loop
D_y	Related coefficient	3.015	-	Satellite body
I_{yy}	Inertia Moment	3,555	in·lb·sec²	Satellite body (Pitch axis)
ζ	Passive Attenuation Coefficient	0.005	-	Bending mode
σ_y	Number of vibration	3.618	-	Solar panel
J_{b1}	Moment of Inertia	100	kg.m²	Satellite body (Roll axis)
J_{s1}	Moment of Inertia	200	kg.m²	Solar array
w_1	Solar array first frequency	0.5	Hz	Bending mode
ζ_1	damping ratio	0.25	%	Bending mode

where $x_1=\theta$, $x_2=\theta$, x_3 ; the state variable of momentum wheel, $x_4=q_y$, $x_5=\dot{q}_y$, $x=[x_1\ x_2\ x_3\ x_4\ x_5]^T$, y= θ

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{\alpha T^2}{J} & 0 & \frac{K_m}{J} & -\frac{T_m^2 D_y \delta_y}{J} & -\frac{2D_y\zeta\delta_y T_m^2}{J} \\ 0 & 0 & -\frac{1}{T_m} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \frac{D_y\alpha T_m^2}{J} & 0 & -\frac{D_y K_m}{J} & \frac{T_m^2 I_{yy}\alpha_y^2}{J} & \frac{2I_{yy}\zeta\delta_y T_m^2}{J} \end{bmatrix}$$

$$B = [0\ \ -\frac{K_m}{J}\ \ 1\ \ 0\ \ \frac{D_y K_m T_m}{J}]^T,$$

$$C = [1\ \ 0\ \ 0\ \ 0\ \ 0],\ \ J = T_m^2(D_y^2 - I_{yy})$$

Eq. (12) and Eq. (13) do not share related coefficients. With an input of and an output of θ in Fig. 3, the transfer function (Phillips and Harbor, 2000) becomes

$$\frac{\theta}{u} = \frac{b_1 s}{s^3 + \alpha_1 s^2 - \alpha_2 s - \alpha_3} \tag{14}$$

where $\alpha_1=\frac{1}{T_m}$, $\alpha_2=\frac{\alpha}{I_\theta}$, $\alpha_3=\frac{\alpha}{T_m I_\theta}$, $b_1=\frac{K_m}{T_m I_\theta}$

Substituting the values in Table 1 into Eq. (14) leads to

α_1=0.6024, α_2=−4.8913E−9,

α_3=−2.9466E−9, b_1=1.3421E−5

3. MRAC and Robust Control of Satellite

3.1 Design of MRAC

The means by which the adaptive laws are established exist in two forms: a linear model and a bilinear model (Ioannou and Datta, 1991). An important class of parametric models for ψ^* that appears in the adaptive control (Kosut and Safonov, 2001; Tsao et al., 2003) and identification of linear plants is a class in which ψ^* appears in a linear form.

We considered the Lyapunov function when generating the parameter estimates previously defined as ψ(t). A certain Lyapunov-like function is then considered. The time derivative $\dot{V}$ along trajectories of the dynamic equation is made non-positive for $V \geq V_0$ and $V_0 \geq 0$.

The properties of V and establish stability properties of

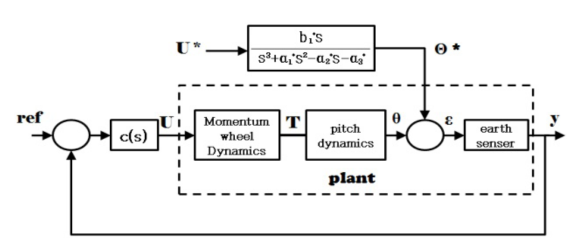

Fig. 3. Block diagram of attitude control and model identification in the satellite.

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the on-line estimation scheme. The chosen form is

$$V(\tilde{\psi}) = \frac{\tilde{\psi}^T \Gamma^{-1} \tilde{\psi}}{2} \quad (15)$$

where $\Gamma = \Gamma^T > 0$.

In order to derive the adaptive law in the model dynamic system with non-disturbance, the Gradient method was used. The method is based on the development of an algebraic error equation and the minimization of a certain cost function $J(\psi, t)$ in terms of the estimated parameter ψ for each time t using the steepest descent method as shown in Eq. (16). Since ψ^* is constant we can write

$$z=\psi^{*T}\phi \quad (16)$$

The estimate $\hat{z}$ of z at time t is given by $\hat{z} = \psi^T\phi$ and the estimation error is shown as

$$\varepsilon = \frac{z-\psi^{*T}\phi}{m^2} \quad (17)$$

where ε is the normalized estimation error, and $m^2 = 1 + n_s^2$ and n_s is the normalizing signal designed so that $\frac{\phi}{m} \in L_\infty$. The adaptive law for updating ψ is derived by minimizing various cost functions of $J(\psi, t)$ with respect to ψ. The cost function can be chosen by considering the instantaneous cost as

$$J(\psi, t) = \frac{\epsilon^2 m^2}{2} = \frac{((\psi^{*T} - \psi^T)\varphi)^2}{2m^2} \quad (18)$$

Using the gradient method we obtain

$$\dot{\psi} = \Gamma\epsilon\phi, \quad \Gamma = \Gamma^T > 0 \quad (19)$$

Three control structures have become very popular in the adaptive control literature: the model reference control structure, the pole placement control structure, and the linear quadratic control structure. The control structure was assumed as no disturbances. Let us first consider model reference control structure.

The plant is assumed as $y=G_0(s)[u]=k_p\frac{Z_o(s)}{R_0(s)}$, and the reference model

$$y_m = W_m(S)[r] = k_m \frac{Z_m(s)}{R_m(s)}[r] \quad (20)$$

The objective for designing the MRAC is to calculate the plant input u such that the closed loop plant stable and y(t) $\rightarrow$ $y_m(t)$ as t $\rightarrow \infty$ for any bounded piecewise continuous reference input r(t). The following assumptions are given

i) $Z_0(s)$ is a monic Hurwitz polynomial of degree m≤n-1.

ii) $R_o(s)$ is a monic polynomial of degree n.

iii) The sign of k_p is known.

iv) The relative degree of n-m is known.

v) $Z_o(s)$, $R_o(s)$ are coprime.

vi) $Z_m(s)$, $R_m(s)$ are monic Hurwitz polynomials of degree m_r, n_r, respectively.

The MRAC law is given by

$$u = \psi_1^{*T}\frac{a(s)}{\Lambda(s)}u + \psi_2^{*T}\frac{a(s)}{\Lambda(s)}y + \psi_3^* y + c_o^* r \quad (21)$$

where $a(s)=[s^{n-2}, s^{n-3}, \cdots, 1]^T$, and and $\wedge(s)=\wedge_0(s)Z_m(s)$ and $\wedge_0(s)$ is a monic Hurwitz polynomial of degree $n\text{-}m_r\text{-}1$ and ψ_i^*, i=1, 2, 3, c_o^* are the constant controller parameters to be determined so that the control objective is achieved for the modeled part of the plant $P_o(s)$. There exists ψ_1^*, ψ_2^*, ψ_3^*, c_o^* so that the control objective is achieved for the nominal plant $P_o(s)$.

For the case of α=0, the model has already been obtained in the paper (Jin et al., 1994) However, for $\alpha=3w_o^2(I_\varphi\text{-}I_6)$, the transfer function for filtering both sides by $\frac{1}{(s+0.1)^3}$ is

$$\frac{s^3}{(s+0.1)^3}[\psi] + \alpha_1\frac{s^2}{(s+0.1)^3}[\psi] - \alpha_2\frac{s}{(s+0.1)^3}[\psi] - \alpha_3\frac{1}{(s+0.1)^3}[\psi] =$$

$$b_1\frac{s}{(s+0.1)^3}[u] \quad (22)$$

Here, the parameter z, ψ^*, and φ in the linear parametric model is given by

$$z = \psi^{*T}\phi \quad (23)$$

where $z=\frac{s^3}{(s+0.1)^3}[\psi]$, $\psi^*=[\alpha_1\ \alpha_2\ \alpha_3\ b_1]^T$,

$$\varphi=[\frac{-s^2}{(s+0.1)^3}[\psi], \frac{s}{(s+0.1)^3}[\psi], \frac{1}{(s+0.1)^3}[\psi], \frac{s}{(s+0.1)^3}[u]]^T$$

3.2 Design of robust controller

In designing robust controller, the q parameter (Dorato et al., 1989) is first introduced as

$$q(s) = \frac{c(s)}{1+P_o(s)c(s)} \quad (24)$$

The condition of q(s) necessary for c(s) to guarantee internal stability is

$$q(s) = B(s)\tilde{q}(s) \quad (25)$$

where B(s) is the Blaschke product of poles of $P_o(s)$ in right half plane (RHP) and $\tilde{q}(s)$ has to satisfy the interpolation conditions

$$\tilde{q}(\alpha_1) = \frac{1}{\tilde{P_o}(\alpha_1)} \quad (26)$$

The robust stability condition can then be written

$$\| \tilde{q}(s) r_m(s) \|_\infty < 1 \quad (27)$$

where $r_m(s)$ is a minimum phase H_∞ function.
The unit function is now introduced as

$$u(s) = \tilde{q}(s)r_m(s) \quad (28)$$

The robust stability condition becomes

$$\|u(s)\|_\infty < 1 \quad (29)$$

The above condition implies that u(s) must be an strictly bounded real (SBR) function (Dorato et al., 1989; Kosut and Safonov, 2001) since $r_m(s)$ and $\tilde{q}(s)$ are H_∞. The interpolation conditions on u(s) are

$$u(\alpha_1) = \tilde{q}(\alpha_1)r_m(\alpha_1) = \frac{r_m(\alpha_1)}{\widetilde{P_o}(\alpha_0)} = \beta_1 \quad (30)$$

The robust stability problem is reduced to an equivalent interpolation problem. The problem that presents itself comprises finding an SBR function u(s) which interpolates given points in the RHP. In mathematical literature, this problem is known as the Nevanlinna-Pick interpolation problem (Kimura, 1984). The controller is represented by the Blaschke product (Giarre et al., 1997), the uncertainty boundary r(s), and the function q(s) with unit function u(s) as shown below.

$$C(s) = \frac{q(s)}{1-P_o(s)q(s)} \quad (31)$$

In the flexible body model as shown in Eq. (13), the value of the inertia moment for roll axis and yaw axis is almost equivalent, and the angular velocity can be neglected for the small value. For the case in which the plant contains one pole at the right half of s-plane, the controller is designed by the interpolation theory (Dorato et al., 1989) where

$$P_o(s) = \frac{as(s+\beta_1)(s+\beta_2)}{(s+\alpha_1)(s+\alpha_2)(s+\alpha_3)(s-\alpha_4)(s+\alpha_5)} \quad (32)$$

where
$\alpha_1 = 0.0181 - 3.6226i$, $\alpha_2 = 0.0181 + 3.6226i$,
$\alpha_3 = 0.6024$,
$\alpha_4 = \alpha_5 = 0.0001$, $a = 1.3455E-5$,
$\beta_1 = 0.0181 - 3.6180i$, $\beta_2 = 0.0181 + 3.6180i$
Design procedure followed was:
i) Choose the upper bound of uncertainty,

$$r(s) = \frac{k}{s+0.0001}, \; k = 0.05.$$

ii) Obtain $\widetilde{P}_0(s)$ as $\widetilde{P}_0(s)=B(S)Po(s)$ where $B(s)=\frac{(s-\alpha_4)}{(s+\alpha_4)}$

iii) Compute the unit function u(s) with the pole at the right half s-plane and the ∞ value since r(s) is the strictly proper.

$$u(\alpha_4) = \frac{r(\alpha_4)}{\widetilde{P_o}(\alpha_4)} = 3695.0215 \text{ Then, } u_1(s) = \frac{0.6279}{s+0.0001}$$

iv) Get the proper function as

$$q(s) = \frac{B(s)}{r(s)}u_1(s) = \frac{12.5585(s-\alpha_4)}{(s+\alpha_4)}$$

v) The controller C(s) is obtained by pole and zero cancellation.

vi) Check the stability of the closed loop system using the characteristic equation.

4. Simulation

For the model identification of the satellite, the parameter identification of the dynamic model was conducted by the MRAC method. Additionally, the PI and robust controller (H_∞) were next designed for the desired performance. In the study conducted by the paper (Lho et al., 1998) three parameters of the pitch dynamics were proven to converge. In order to show convergence of the 4 parameters describing

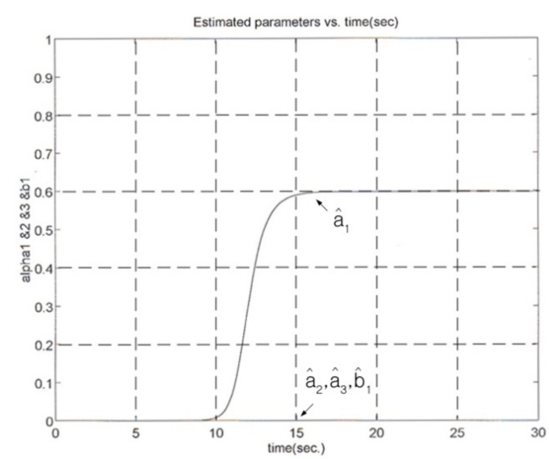

Fig. 4. Estimations of 4 parameters of α_1, α_2, α_3, and b_1.

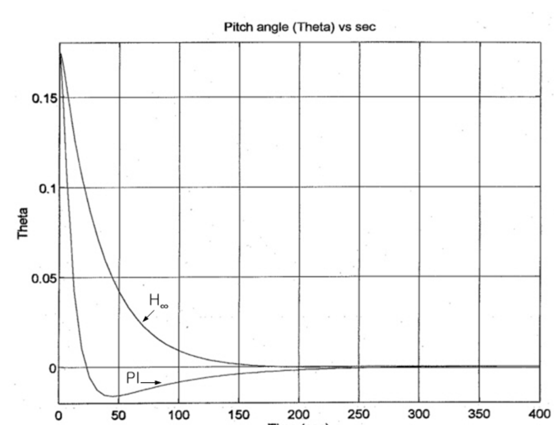

Fig. 5. Simulations of robust controller and proportional and integration.

ψ^*in Eq. (23), an input with three different frequencies for Eq. (33) is applied.

$$u(t) = 1 + \sin(t) + \sin(3t) + \sin(5t) \tag{33}$$

The true values are $\alpha_1 = 0.6024$, $\alpha_2 = -4.8913E-9$, $\alpha_3 = -2.9466E-9$, $b_1 = 1.3421E-5$.

The simulation results converge to $\tilde{\alpha}_1 \cong 0.6$, $\hat{\alpha}_1 = \hat{\alpha}_2 = \hat{b}_1 \cong 0$ which are the same as the true values as shown in Fig. 4.

For the pitch dynamic model of the satellite, the simulation was accomplished by Matlab/Simulink software. The initial value applied for x_o is 0.1745 rad. The proportional gain (K_p) and the integration gain (K_i) were 4,308.6 (V/rad) and 53.9 (V/rad/sec), respectively. The robust controller attained by using of the above design procedure encompassed a second order proper function as

$$C(s) = \frac{12.55848\ s^2 + 7.56623\ s + 0.00056}{s^2 + 0.60262\ s - 0.00004} \tag{34}$$

As Fig. 5 displays, the convergence time of the robust controller is 160 seconds. The convergence time for the PI controller was 250 seconds. The robust controller converges to the steady state in shorter time than PI controller.

5. Conclusions

The rigid and flexible body of satellite was implemented. With the MRAC, the parameters of the dynamic model of plant in satellite were identified for the desired performance. After model identification, the robust controller was successfully designed to stabilize the satellite. With simulation, it was shown that the convergence time of the robust controller performed better than the PI controller in the attitude stabilization technique of KOMPSAT.

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