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Asymptotic Properties of the Disturbance Variance Estimator in a Spatial Panel Data Regression Model with a Measurement Error Component

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Abstract

The ordinary least squares based estimator of the disturbance variance in a regression model for spatial panel data is shown to be asymptotically unbiased and weakly consistent in the context of SAR(1), SMA(1) and SARMA(1,1)-disturbances when there is measurement error in the regressor matrix.

Keywords: Asymptotic unbiasedness, consistency, measurement error, spatial panel.

1. Introduction

When disturbances in the linear regression model are correlated or have unequal variances, it is well known that the ordinary least squares(OLS) based estimator of the disturbance variance, S^2, is generally biased and inconsistent. Also, spatial dependence models which are widely used in urban and environmental economics cannot avoid these disadvantages. Although spatial panel regression models are becoming increasingly attractive in empirical econometrics, the related literature mainly focuses on the maximum likelihood or generalized moments estimation and hypothesis testing.

In regression analysis, the (potential) presence of measurement error in the explanatory variables results in bias and inconsistency of the OLS estimates and thus severely affect the quality of regression analysis. In order to solve this problem, some extraneous information or additional assumptions are needed to identify the model parameters of interest. Griliches and Hausman (1986) showed that one can control the measurement error without the use of external instruments using panel data. However, they only considered the simplest situation of a single regressor. In addition, several steps need to be performed to obtain consistent estimators, which causes the individual effect to be wiped out.

Many articles have investigated the behavior of the bias of S^2 (Watson, 1955; Theil, 1971; Sathe and Vinod, 1974; Neudecker, 1977, 1978; Dufour, 1986, 1988; Krämer, 1991; Kiviet and Krämer, 1992). Especially, Krämer (1991) showed that the OLS estimator of S^2 in the linear regression model is asymptotically unbiased in the context of AR(1)-disturbances without any restrictions on the regressor matrix. In addition, Baltagi and Krämer (1994) showed the asymptotic unbiasedness and consistency of S^2 in the panel regression model with error component disturbances, and Song (1996) extended their work to the serially correlated error components regression model for panel data. Also, Song and Kim (2006) and Song and Lee (2008) studied the asymptotic properties of S^2 in a panel data regression model with measurement error and with spatially correlated error, respectively.

In this paper, we will investigate the asymptotic unbiasedness and weak consistency of S^2 in a panel data regression model with measurement error when the disturbances follow SAR(1), SMA(1) and SARMA(1,1) processes.

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2. Model

As a point of departure, consider the following panel data regression model (Hsiao, 1986; Baltagi, 2001),

$$y_{it} = \beta x_{it} + u_{it}, \quad i = 1, \dots, N; \ \ t = 1, \dots, T, \tag{2.1}$$

where y_{it} is the observation on a dependent variable for the i^{th} spatial unit (*e.g.*, country, census track) at the t^{th} time period, x_{it} denotes the $k \times 1$ vector of independent variables and u_{it} is the regression disturbance. Moreover, the x_{it} is observed with error,

$$x^*_{it} = x_{it} + \upsilon_{it}, \quad i = 1, \dots, N; \ \ t = 1, \dots, T,$$

where υ_{it} represents the measurement error, which is assumed to be $i.i.d.(0, \sigma^2_\upsilon)$. Song and Kim (2006) considered an additive measurement error components structure, which is related with an individual and remainder. However, we separately consider a single measurement error component as well as region effect as below.

The regression disturbance u_{it} follows an error components structure:

$$u_{it} = \mu_i + \varepsilon_{it}, \quad i = 1, \dots, N; \ \ t = 1, \dots, T,$$

where μ_i denotes the ith random spatial effect assumed to be $i.i.d.(0, \sigma^2_\mu)$ and ε_{it} is the remainder disturbance which is independent of μ_i. We let ε_{it} follows a first-order spatial autoregressive(SAR(1)), a first-order spatial moving average(SMA(1)), or a first-order spatial autoregressive moving average(SARMA(1,1)) error model as follows (for extensive technical discussion, see Anselin, 1988, 2003; Anselin and Bera, 1998, and references therein):

$$\varepsilon_{it} = \begin{cases} \phi \sum_{k=1}^{N} w_{ik}\varepsilon_{kt} + \nu_{it}, & : \text{SAR(1) process,} \\ \theta \sum_{k=1}^{N} w_{ik}\nu_{kt} + \nu_{it}, & : \text{SMA(1) process,} \\ \phi \sum_{k=1}^{N} w_{ik}\varepsilon_{kt} + \theta \sum_{k=1}^{N} w_{ik}\nu_{kt} + \nu_{it}, & : \text{SARMA(1,1) process} \end{cases}$$

where ϕ is the scalar spatial autoregressive coefficient with $|\phi| < 1$, θ is the scalar spatial moving average coefficient with $|\theta| < 1$, w_{ik} is the (i, k) th elements of $N \times N$ spatial weight matrix W, and ν_{it} is $i.i.d.(0, \sigma^2_\nu)$ and independent of μ_i. We assume that all error components are mutually independent. More specifically, each element in the weight matrix can be defined as

$$w_{ik} = \begin{cases} \dfrac{1}{N-1}, & \text{if } i \neq k, \\ 0, & \text{if } i = k. \end{cases} \tag{2.2}$$

as in Case (1992), Kelejian and Prucha (2002), Kelejian *et al.* (2006) and Lee (2004). As another example, we can consider the following popular specification for the weight matrix known as "one ahead and one behind:"

$$w_{12} = w_{1N} = w_{N1} = w_{N,N-1} = w_{ij} = 1, \quad \text{with } i = 2, \dots, N-1, \ j = 1, \dots, N, \ |i - j| = 1$$

and renormalize the rows such that the row sums are one (Kelejian and Prucha, 1999; Krämer and Donninger, 1987). We can also consider the orthogonal weight matrix, and its elements are defined by (Cliff and Ord, 1981)

$$w_{ik} = \begin{cases} > 0, & \text{if regions } i \text{ and } k \text{ are neighbors } (i \neq k), \\ = 0, & \text{otherwise.} \end{cases}$$

The model (2.1) can be rewritten in matrix notation as

$$y = X\beta + e, \tag{2.3}$$

where y is now of dimension $NT \times 1$, X is $NT \times k, \beta$ is $k \times 1$ and e is $NT \times 1$. The error term e can be written in vector form as

$$e = \upsilon + u = \begin{cases} \upsilon + (I_N \otimes i_T)\mu + \left(B^{-1} \otimes I_T\right)\nu, & : \text{SAR(1) process,} \\ \upsilon + (I_N \otimes i_T)\mu + (A \otimes I_T)\,\nu, & : \text{SMA(1) process,} \\ \upsilon + (I_N \otimes i_T)\mu + \left(B^{-1}A \otimes I_T\right)\nu, & : \text{SARMA(1,1) process,} \end{cases} \tag{2.4}$$

with $\upsilon' = (\upsilon_{11}, \ldots, \upsilon_{N1}, \ldots, \upsilon_{1T}, \ldots, \upsilon_{NT})$, $\mu' = (\mu_1, \ldots, \mu_N)$ and $\nu' = (\nu_{11}, \ldots, \nu_{N1}, \ldots, \nu_{1T}, \ldots, \nu_{NT})$, where i_T is a $T \times 1$ vector of ones, I_T is a $T \times T$ identity matrix, $\otimes$ denotes the Kronecker product, $B = I_N - \phi W$ is nonsingular for all $|\phi| < 1$ and $A = I_N + \theta W$.

From next section, we will focus on SARMA(1,1) process with the weight matrix of (2.2) since for other processes with any weight matrices mentioned above, we can similarly prove the asymptotic unbiasedness and weak consistency of S^2. We therefore see that our proof is applicable in many relevant spatial econometric specifications; however, note that our method of proof could be different if we use a different weight matrix from the matrices mentioned above.

3. Asymptotic Unbiasedness

Theorem 1. *In the panel regression model (2.3) with measurement error and SARMA(1,1) error terms and the equal weight matrix of (2.2), S^2 is asymptotically unbiased for σ^2 as N and T go to infinity.*

Proof: Under the assumptions in the previous section, the error covariance matrix is

$$E(ee') = \Omega = \sigma_\upsilon^2 I_{NT} + \sigma_\mu^2 (I_N \otimes J_T) + \sigma_\nu^2 \left\{ \left(B^{-1}A\right)\left(B^{-1}A\right)' \otimes I_T \right\},$$

where J_T is a $T \times T$ matrix of all ones. We can rewrite two matrices A and B^{-1} as $A = \alpha_{1N} J_N + \alpha_{2N} I_N$ and $B^{-1} = \alpha_{3N} J_N + \alpha_{4N} I_N$, respectively, where $\alpha_{1N} = \theta/(N-1)$, $\alpha_{2N} = (N-1-\theta)/(N-1)$, $\alpha_{3N} = \phi/\{(N-1+\phi)(1-\phi)\}$ and $\alpha_{4N} = (N-1)/(N-1+\phi)$ (Kelejian and Prucha, 2002). Then

$$\begin{aligned} B^{-1}A &= (N\alpha_{1N}\alpha_{3N} + \alpha_{2N}\alpha_{3N} + \alpha_{1N}\alpha_{4N})J_N + \alpha_{2N}\alpha_{4N} I_N \\ &= \pi_{1N} J_N + \pi_{2N} I_N \end{aligned} \tag{3.1}$$

and

$$\begin{aligned} \left(B^{-1}A\right)\left(B^{-1}A\right)' &= \left[N(\pi_{1N})^2 + 2\pi_{1N}\pi_{2N}\right] J_N + (\pi_{2N})^2 I_N \\ &= \pi_N^* J_N + (\pi_{2N})^2 I_N, \end{aligned} \tag{3.2}$$

where $\pi_{1N} = N\alpha_{1N}\alpha_{3N} + \alpha_{2N}\alpha_{3N} + \alpha_{1N}\alpha_{4N}$, $\pi_{2N} = \alpha_{2N}\alpha_{4N}$ and $\pi_N^* = N(\pi_{1N})^2 + 2\pi_{1N}\pi_{2N}$. Moreover,

$$\begin{aligned}\mathrm{Var}(e_{it}) &= \sigma_\upsilon^2 + \sigma_\mu^2 + \sigma_\nu^2\left[\pi_N^* + (\pi_{2N})^2\right]\\ &= \sigma_\upsilon^2 + \sigma_\mu^2 + \sigma_\nu^2\,\gamma_N,\end{aligned}$$

where $\gamma_N = \pi_N^* + (\pi_{2N})^2$.

From Watson (1955), Sathe and Vinod (1974), Neudecker (1977), Dufour (1986, 1988), Krämer (1991) and Kiviet and Krämer (1992), we have the inequalities

$$0 \le \frac{\text{mean of } NT-k}{\text{smallest characteristic roots of } \Omega} \le E\left(S^2\right) \le \frac{\text{mean of } NT-k}{\text{greatest characteristic roots of } \Omega} \le \frac{tr(\Omega)}{NT-k},$$

which imply that the upper bound for $E(S^2)$ tends to σ^2 $(= \sigma_\upsilon^2 + \sigma_\mu^2 + \sigma_\nu^2)$, as N and T go to infinity since trace$(\Omega) = NT\{\sigma_\upsilon^2 + \sigma_\mu^2 + \sigma_\nu^2\,\gamma_N\}$ and γ_N tends to 1. Furthermore, the mean of the $NT-k$ smallest characteristic roots of Ω is as $N, T \to \infty$

$$\begin{aligned}\frac{1}{NT-k}\sum_{l=1}^{NT-k}\lambda_{l+k}(\Omega) &= \frac{1}{NT-k}\left\{\sum_{l=1}^{NT}\lambda_l(\Omega) - \sum_{l=1}^{k}\lambda_l(\Omega)\right\}\\ &\ge \sigma_\upsilon^2 + \frac{NT-kT}{NT-k}\sigma_\mu^2 + \frac{NT}{NT-k}\gamma_N\sigma_\nu^2 - \frac{k}{NT-k}\left(\frac{1+\theta}{1-\phi}\right)^2\sigma_\nu^2\\ &\to \sigma_\upsilon^2 + \sigma_\mu^2 + \sigma_\nu^2 = \sigma^2,\end{aligned}$$

since from the inequality of Horn and Johnson (1985, p.181) we have

$$\begin{aligned}\lambda_l(\Omega) &\le \lambda_l\left\{\sigma_\upsilon^2 I_{NT} + \sigma_\nu^2\left[\left(B^{-1}A\right)\left(B^{-1}A\right)' \otimes I_T\right]\right\} + \lambda_{\max}\left\{\sigma_\mu^2\left(I_N \otimes J_T\right)\right\}\\ &= \lambda_l\left\{\sigma_\upsilon^2 I_{NT}\right\} + \lambda_i\left\{\sigma_\nu^2\left(B^{-1}A\right)\left(B^{-1}A\right)'\right\}\lambda_t(I_T) + T\sigma_\mu^2\\ &= \sigma_\upsilon^2 + \left(\frac{1+\theta\lambda_i(W)}{1-\phi\lambda_i(W)}\right)^2\sigma_\nu^2 + T\sigma_\mu^2\\ &\le \sigma_\upsilon^2 + \left(\frac{1+\theta\lambda_{\max}(W)}{1-\phi\lambda_{\max}(W)}\right)^2\sigma_\nu^2 + T\sigma_\mu^2\\ &\le \sigma_\upsilon^2 + \left(\frac{1+\theta}{1-\phi}\right)^2\sigma_\nu^2 + T\sigma_\mu^2, \quad l = 1, \ldots, NT,\end{aligned} \tag{3.3}$$

where $\lambda(\cdot)$ denotes characteristic roots of $\cdot$. Therefore, the lower bound for $E(S^2)$ also goes to σ^2. This completes the proof. □

4. Consistency

Krämer and Berghoff (1991) provided that a sufficient condition for consistency of the standard OLS-based estimator of the disturbance variance in the linear regression model with correlated disturbances. Based on their result, we establish the consistency of S^2 in the model (2.3) with measurement error and SARMA(1,1) process error terms in (2.4).

Theorem 2. *In the panel regression model (2.3), S^2 is weakly consistent as N and T go to infinity if the disturbances υ's, μ's and ν's in measurement error and SARMA(1,1) process error terms of*

(2.4) with the equal weight matrix of (2.2) have finite fourth moments; that is, $E(\upsilon_{it}^4) = \tau < \infty$, $E(\mu_i^4) = \eta < \infty$ *and* $E(\nu_{it}^4) = \xi < \infty$.

Proof: According to Krämer and Berghoff (1991), S^2 is weakly consistent, if the following holds:

$$\frac{e'e}{NT-k} \xrightarrow{p} \sigma^2 \quad \text{and} \quad \lambda_{\max}(\Omega) = o(NT). \tag{4.1}$$

First, we have

$$\begin{aligned}\frac{e'e}{NT-k} = \frac{1}{NT-k}\Big\{&\upsilon'\upsilon + T\mu'\mu + \upsilon'(I_N \otimes i_T)\mu + \mu'(I_N \otimes i_T')\upsilon \\ &+ \upsilon'\left(B^{-1}A \otimes I_T\right)\nu + \mu'\left(B^{-1}A \otimes i_T'\right)\nu + \nu'\left(\left(B^{-1}A\right)' \otimes I_T\right)\upsilon + \nu'\left(\left(B^{-1}A\right)' \otimes i_T\right)\mu \\ &+ \nu'\left(\left(B^{-1}A\right)'\left(B^{-1}A\right) \otimes I_T\right)\nu\Big\}.\end{aligned}$$

We now examine the mean and variance of each term in the equation above.

(i) The mean and variance of the first term are

$$E\left(\frac{\upsilon'\upsilon}{NT-k}\right) = \frac{\operatorname{trace}(E(\upsilon\upsilon'))}{NT-k} = \frac{NT\sigma_\upsilon^2}{NT-k} \to \sigma_\upsilon^2 \quad \text{and}$$

$$\begin{aligned}\operatorname{Var}\left(\frac{\upsilon'\upsilon}{NT-k}\right) &= \left(\frac{1}{NT-k}\right)^2 \left\{\operatorname{Var}\left(\upsilon_1^2\right) + \cdots + \operatorname{Var}\left(\upsilon_{NT}^2\right)\right\} \\ &= \left(\frac{1}{NT-k}\right)^2 \left\{E\left(\upsilon_1^4\right) + \cdots + E\left(\upsilon_{NT}^4\right) - NT\sigma_\upsilon^4\right\} \\ &= \left(\frac{1}{NT-k}\right)^2 \left(NT\tau - NT\sigma_\mu^4\right) \to 0, \quad \text{respectively.}\end{aligned}$$

Thus, $(\upsilon'\upsilon)/(NT-k) \xrightarrow{p} \sigma_\upsilon^2$, as $N, T \to \infty$.

Likewise, $(T\mu'\mu)/(NT-k) \xrightarrow{p} \sigma_\mu^2$, as $N, T \to \infty$. Note again that υ, μ and ν are all independent of each other and their means are zeros.

(ii) $\dfrac{\upsilon'(I_N \otimes i_T)\mu}{NT-k} \xrightarrow{p} 0$, since $E\left\{\dfrac{\upsilon'(I_N \otimes i_T)\mu}{NT-k}\right\} = 0$ and $\operatorname{Var}\left\{\dfrac{\upsilon'(I_N \otimes i_T)\mu}{NT-k}\right\} \to 0$, as $N, T \to \infty$.

In the same way, it can be easily shown that $\{\mu'(I_N \otimes i_T')\upsilon\}/(NT-k)$ tends to zero.

(iii) The mean and variance of the fifth term are

$$E\left\{\frac{\upsilon'(B^{-1}A \otimes I_T)\nu}{NT-k}\right\} = E\left\{\frac{\upsilon'(\pi_{1N}J_N \otimes I_T)\nu}{NT-k}\right\} + E\left\{\frac{\upsilon'(\pi_{2N}I_N \otimes I_T)\nu}{NT-k}\right\} = 0,$$

since $B^{-1}A = \pi_{1N}J_N + \pi_{2N}I_N$ as in (3.1), and

$$\begin{aligned}\operatorname{Var}\left\{\frac{\upsilon'(B^{-1}A \otimes I_T)\nu}{NT-k}\right\} &= \frac{1}{(NT-k)^2}\operatorname{Var}\left\{\upsilon'\left[(\pi_{1N}J_N + \pi_{2N}I_N) \otimes I_T\right]\nu\right\} \\ &= \frac{1}{(NT-k)^2}\left\{NT(\pi_{1N} + \pi_{2N})^2 + NT(N-1)(\pi_{1N})^2\right\}\sigma_\upsilon^2\sigma_\nu^2 \to 0,\end{aligned}$$

respectively.

Thus, $\{\upsilon'(B^{-1}A \otimes I_T)\nu\}/(NT-k) \xrightarrow{p} 0$.

Similarly, $\{\mu'(B^{-1}A \otimes i_T')\nu\}/(NT-k)$, $\{\nu'((B^{-1}A)' \otimes I_T)\upsilon\}/(NT-k)$, and $\{\nu'((B^{-1}A)' \otimes i_T)\mu\}/(NT-k)$ converge to zero in probability.

(iv) For the last term, from (3.2),

$$E\left\{\frac{\nu'\left[(B^{-1}A)'(B^{-1}A) \otimes I_T\right]\nu}{NT-k}\right\} = \frac{1}{(NT-k)}\left\{\pi_N^* E\left[\nu'(J_N \otimes I_T)\nu\right] + (\pi_{2N})^2 E(\nu'\nu)\right\}$$
$$= \frac{1}{(NT-k)}\left\{\pi_N^* NT\sigma_\nu^2 + (\pi_{2N})^2 NT\sigma_\nu^2\right\} \to \sigma_\nu^2$$

and

$$\begin{aligned}
&\text{Var}\left\{\frac{\nu'[(B^{-1}A)'(B^{-1}A) \otimes I_T]\nu}{NT-k}\right\} \\
&= \frac{1}{(NT-k)^2}\left\{(\pi_N^*)^2 \text{Var}\left[\nu'(J_N \otimes I_T)\nu\right] + (\pi_{2N})^4 \text{Var}(\nu'\nu) + 2\pi_N^*(\pi_{2N})^2 \text{Cov}\left[\nu'(J_N \otimes I_T)\nu, \nu'\nu\right]\right\} \\
&= \frac{1}{(NT-k)^2}\left\{(\pi_N^*)^2 NT\left(\xi + \sigma_\nu^4(2N-3)\right) + (\pi_{2N})^4 NT\left(\xi - \sigma_\nu^4\right) + 2\pi_N^*(\pi_{2N})^2 NT(\xi - \sigma_\nu^4)\right\} \\
&\to 0, \quad \text{as } N, T \to \infty.
\end{aligned}$$

Thus, $[\nu'\{(B^{-1}A)'(B^{-1}A) \otimes I_T\}\nu]/(NT-k) \xrightarrow{p} \sigma_\nu^2$.

Accordingly,

$$\frac{e'e}{NT-k} \xrightarrow{p} \sigma_\upsilon^2 + \sigma_\mu^2 + \sigma_\nu^2 = \sigma^2.$$

Moreover, by (3.3),

$$\lambda_{max}(\Omega) \leq \sigma_\upsilon^2 + \left(\frac{1+\theta}{1-\phi}\right)^2 \sigma_\nu^2 + T\sigma_\mu^2,$$

which means that $\lambda_{max}(\Omega) = o(NT)$. Hence the Theorem follows from the above arguments. □

5. Conclusions

This paper examines the asymptotic properties of the disturbance variance estimator in a a panel data regression model with spatial autocorrelation and measurement error. It is clear that the spatial autocorrelation and measurement error lead to bias and inconsistency of the OLS estimates. However, there are very few published studies on the problem concerning the bias and inconsistency of S^2. Griliches and Hausman (1986) and Kapoor *et al.* (2007) studied the consistency using the generalized method of moments in the spatial panel regression model and the panel regression model with measurement error, respectively. However, their methods need several assumptions and steps to establish suitable properties.

In this paper, the OLS-based estimator of the disturbance variance, S^2 is shown to be weakly consistent and asymptotically unbiased for a panel data regression model when measurement error exists in the regressor matrix and the disturbances follow SAR(1), SMA(1) and SARMA(1,1) structures, based on very simple assumption that the disturbances have finite fourth moments.

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