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Complete Moment Convergence of Moving Average Processes Generated by Negatively Associated Sequences

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Abstract

Let $\{X_i, -\infty < i < \infty\}$ be a doubly infinite sequence of identically distributed and negatively associated random variables with mean zero and finite variance and $\{a_i, -\infty < i < \infty\}$ be an absolutely summable sequence of real numbers. Define a moving average process as $Y_n = \sum_{i=-\infty}^{\infty} a_{i+n} X_i, n \geq 1$ and $S_n = Y_1 + \cdots + Y_n$. In this paper we prove that $E|X_1|^r h(|X_1|^p) < \infty$ implies $\sum_{n=1}^{\infty} n^{r/p-2-q/p} h(n) E\{\max_{1\leq k\leq n} |S_k| - \epsilon n^{1/p}\}_+^q < \infty$ and $\sum_{n=1}^{\infty} n^{r/p-2} h(n) E\{\sup_{k\geq n} |k^{-1/p} S_k| - \epsilon\}_+^q < \infty$ for all $\epsilon > 0$ and all $q > 0$, where $h(x) > 0\ (x > 0)$ is a slowly varying function, $1 \leq p < 2$ and $r > 1 + p/2$.

Keywords: Moving average process, negatively associated, complete moment convergence, doubly infinite sequence.

1. Introduction

We assume that $\{X_i, -\infty < i < \infty\}$ is a doubly infinite sequence of identically distributed random variables with mean zero and finite variance and $\{a_i, -\infty < i < \infty\}$ is an absolutely summable sequence of real numbers. We define the moving average process $\{Y_n, n \geq 1\}$ as follows:

$$Y_k = \sum_{i=-\infty}^{\infty} a_{i+k} X_i, \quad k \geq 1. \tag{1.1}$$

Under some suitable conditions, many limiting results for the moving average process $\{Y_n, n \geq 1\}$ have been obtained. For example, Burton and Dehling (1990) have obtained a large deviation principle for $\{Y_n, n \geq 1\}$, Zhang (1996) has obtained the complete convergence and Kim and Ko (2008) proved the complete moment convergence under φ-mixing assumption, respectively, Kim and Baek (2001) established the central limit theorem under linearly positive quadrant dependence condition, Baek *et al.* (2003) obtained the complete convergence and Li and Zhang (2004) discussed the complete moment convergence under negative association assumption, respectively.

Recently, Chen *et al.* (2009) improved the result in Zhang (1996) and Zhou (2010) improved the result in Kim and Ko (2008). The family of random variables $\{X_i, 1 \leq i \leq n\}$ is said to be negatively associated(NA) if for every pair of disjoint subsets A and B of $\{1, 2, \ldots, n\}$, $\mathrm{Cov}\{f(X_i, i \in A),\ g(X_j, j \in B)\} \leq 0$ whenever f and g are coordinatewise non-decreasing and the covariance exists. An infinite family is NA if every finite subfamily is NA. This notion was introduced by Joag-Dev and Proschan (1983). As pointed out and proved by Joag-Dev and Proschan (1983), a number of well known multivariate distributions possess the NA property, such as (a) multinomial, (b) multivariate hypergeometric, (c) negatively correlated normal distribution and (d) random sampling without replacement.

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Because of its wide applications in multivariate statistical analysis and reliability, the notion of NA has received considerable attention recently.

We say that the sequence $\{U_n, n \geq 1\}$ satisfies the complete moment of order $q\,(> 0)$ convergence if

$$\sum_{n=1}^{\infty} E\{|U_n|\}_+^q < \infty, \quad \text{where } E\{|U_n|\}_+^q = \int_0^{\infty} P\{|U_n|^q > x\}dx \tag{1.2}$$

and a_+ means that $\max(a, 0)$.

Especially, when $q = 1$ if (1.2) holds we say that the sequence $\{U_n, n \geq 1\}$ satisfies the complete moment convergence.

Moreover, we say that the sequence $\{U_n, n \geq 1\}$ satisfies the complete convergence if $\sum_{n=1}^{\infty} P\{|U_n| > \epsilon\} < \infty$. Note that complete moment convergence implies complete convergence (see Li and Zhang, 2004).

The purpose of this paper is to show the complete moment of order q convergence for maximum and supremum of the partial sums of moving average processes generated by negatively associated sequences.

2. Main Result and Lemmas

The following theorem is the main result of this paper and the proof will appear in Section3.

Theorem 1. *Suppose that $\{Y_n, n \geq 1\}$ is defined as (1.1), where $\{a_i, -\infty < i < \infty\}$ is a sequence of real numbers with $\sum_{i=-\infty}^{\infty} |a_i| < \infty$ and $\{X_i, -\infty < i < \infty\}$ is a sequence of identically distributed NA random variables with mean zero and finite second moment. Let $h(x) > 0 (x > 0)$ be a slowly varying function and $1 \leq p < 2$ and $r > 1 + p/2$. If $E|X_1|^r h(|X_1|^p) < \infty$ then, for all $\epsilon > 0$ and all $q > 0$ we obtain*

(i)
$$\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) E\left\{\max_{1\leq k\leq n} |S_k| - \epsilon n^{\frac{1}{p}}\right\}_+^q < \infty$$

and

(ii)
$$\sum_{n=1}^{\infty} n^{\frac{r}{p}-2} h(n) E\left\{\sup_{k\geq n} \left|k^{-\frac{1}{p}} S_k\right| - \epsilon\right\}_+^q < \infty.$$

Remark 1.

(1) Theorem 1.1 in Li and Zhang (2004) is the special case of (i) in Theorem 1 for $q = 1$.

(2) From (i) the complete moment convergence for supremum of the partial sum of moving average process under NA assumption is obtained.

Next we state some lemmas which have important roles in proving our main result.

Lemma 1. **(Burton and Dehling, 1990)** *Let $\sum_{i=-\infty}^{\infty} a_i$ be an absolutely convergent series of real numbers with $a = \sum_{i=-\infty}^{\infty} a_i$ and $k \geq 1$. Then*

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=-\infty}^{\infty} \left|\sum_{j=i+1}^{i+n} a_j\right|^k = |a|^k.$$

Lemma 2. **(Shao, 2000)** *Let $\{X_i, i \geq 1\}$ be a sequence of negatively associated random variables with mean zero and finite second moment. Let $S_n = \sum_{i=1}^n X_i$ and $B_n = \sum_{i=1}^n EX_i^2$. Then for all $x > 0$ and $y > 0$,*

$$P\left(\max_{1\leq k\leq n} |S_k| \geq x\right) \leq 2P\left(\max_{1\leq k\leq n} |X_k| \geq y\right) + 4\exp\left(-\frac{x^2}{8B_n}\right) + 4\left(\frac{B_n}{4(xy + B_n)}\right)^{\frac{x}{12y}}.$$

In the proof of Theorem 1.1 and Remark 1.2 of Li and Zhang (2004), by taking $\max_{1\leq k\leq n} |S_k|$ instead of $|S_n|$ we obtain the following complete convergence for the maximum of the partial sum of the moving average process based on the negatively associated sequence.

Lemma 3. *Suppose that $\{Y_n, n \geq 1\}$ is a moving average process defined as (1.1), where $\{a_i, -\infty < i < \infty\}$ is a sequence of real numbers with $\sum_{i=-\infty}^{\infty} |a_i| < \infty$ and $\{X_i, -\infty < i < \infty\}$ is a sequence of identically distributed NA random variables with mean zero and finite second moment. Let $h(x) > 0\,(x > 0)$ be a slowly varying function and $1 \leq p < 2,\ r > 1 + p/2$. Then, $E|X_1|^r h(|X_1|^p) < \infty$ implies*

$$\sum_{n=1}^{\infty} n^{\frac{r}{p}-2} h(n) P\left\{\max_{1\leq k\leq n} |S_k| > \epsilon n^{\frac{1}{p}}\right\} < \infty, \quad \textit{for all } \epsilon > 0. \tag{2.1}$$

3. Proof of Theorem 1

Proof: Proof of (i)

We will use the standard method. Observe that $\sum_{k=1}^n Y_k = \sum_{i=-\infty}^{\infty} \sum_{k=1}^n a_{k+i} X_i = \sum_{i=-\infty}^{\infty} a_{ni} X_i$, where $a_{ni} = \sum_{k=1}^n a_{k+i}$. From Lemma 1, we can assume, without loss of generality, that $\sum_{i=-\infty}^{\infty} |a_i| \leq n,\ n \geq 1$ and $\tilde{a} = \sum_{i=-\infty}^{\infty} |a_{ni}| \leq 1$. Let $\lambda = (EX_1^2)^{-1/2}$. Then $B_n = \sum_{i=-\infty}^{\infty} a_{ni}^2 EX_i^2 \leq n\lambda^{-2}$. Using Lemma 2 with $y = \beta x$ (where $\beta > 0$ whose value will be specified later), we have

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) E\left\{\max_{1\leq k\leq n} |S_k| - \epsilon n^{\frac{1}{p}}\right\}_+^q \qquad (3.1)\\
&= \sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_0^{\infty} P\left\{\max_{1\leq k\leq n} |S_k| - \epsilon n^{\frac{1}{p}} > x^{\frac{1}{q}}\right\} dx\\
&= \sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_0^{n^{\frac{q}{p}}} P\left\{\max_{1\leq k\leq n} |S_k| > \epsilon n^{\frac{1}{p}} + x^{\frac{1}{q}}\right\} dx + \sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{n^{\frac{q}{p}}}^{\infty} P\left\{\max_{1\leq k\leq n} |S_k| > \epsilon n^{\frac{1}{p}} + x^{\frac{1}{q}}\right\} dx\\
&= I + J.
\end{aligned}$$

$$\begin{aligned}
I &= \sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_0^{n^{\frac{q}{p}}} P\left\{\max_{1\leq k\leq n} |S_k| > \epsilon n^{\frac{1}{p}} + x^{\frac{1}{q}}\right\} dx \qquad (3.2)\\
&\quad \left(\text{letting } y = x^{\frac{1}{q}}\right)\\
&= q\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_0^{n^{\frac{1}{p}}} y^{q-1} P\left\{\max_{1\leq k\leq n} |S_k| > \epsilon n^{\frac{1}{p}} + y\right\} dy\\
&\leq q\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_0^{n^{\frac{1}{p}}} y^{q-1} P\left\{\max_{1\leq k\leq n} |S_k| > \epsilon n^{\frac{1}{p}}\right\} dy
\end{aligned}$$

$$\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}+\frac{q}{p}} h(n) P\left\{\max_{1\leq k\leq n}|S_k| > \epsilon n^{\frac{1}{p}}\right\}$$
$$\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-2} h(n) P\left\{\max_{1\leq k\leq n}|S_k| > \epsilon n^{\frac{1}{p}}\right\} < \infty, \quad \text{by Lemma 3.}$$

By modifying the proof of Theorem 1.1 in Li and Zhang (2004) we can prove $J < \infty$. For the completeness we repeat it here.

$$\begin{aligned} J &\leq \sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{n^{\frac{q}{p}}}^{\infty} P\left\{\max_{1\leq k\leq n}|S_k| > x^{\frac{1}{q}}\right\} dx \qquad (3.3)\\ &\left(\text{letting } y = x^{\frac{1}{q}}\right)\\ &= q\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{n^{\frac{1}{p}}}^{\infty} P\left\{\max_{1\leq k\leq n}|S_k| > y\right\} y^{q-1} dy\\ &\leq q\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{n^{\frac{1}{p}}}^{\infty}\left[2P\left\{\max_{-\infty<i<\infty}|a_{ni}X_i| > \beta y\right\} + 4\exp\left(-\frac{y^2 n^{-1}\lambda^2}{8}\right) + 4\left(\frac{1}{4(\beta y^2 n^{-1}\lambda^2+1)}\right)^{\frac{1}{12\beta}}\right] y^{q-1} dy\\ &= J_{11} + J_{22} + J_{23}. \end{aligned}$$

Set $I_{nj} = \{i \in \mathbb{Z},\ (j+1)^{-1/p} < |a_{ni}| \leq j^{-1/p}\},\ j = 1, 2, \ldots$. Then we have $\sum_{j=1}^{k} \sharp I_{nj} \leq n(k+1)^{1/p}$, where $\sharp I_{nj}$ means the number of elements in I_{nj} (*cf.* Li *et al.*, 1992). For J_{21} and $1 \leq p \leq 2,\ r \geq p$, we get

$$\begin{aligned} J_{21} &\leq 2\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) q \int_{n^{\frac{1}{p}}}^{\infty} \sum_{i=-\infty}^{\infty} P\{|a_{ni}X_i| > \beta y\} y^{q-1} dy\\ &\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{\beta n^{\frac{1}{p}}}^{\infty} \sum_{i=-\infty}^{\infty} P\{|a_{ni}X_i| > y\} y^{q-1} dy\\ &\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{\beta n^{\frac{1}{p}}}^{\infty} \sum_{j=1}^{\infty}\sum_{i\in I_{nj}} P\left\{|X_1| \geq j^{\frac{1}{p}} y\right\} y^{q-1} dy\\ &\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{\beta n^{\frac{1}{p}}}^{\infty} \sum_{j=1}^{\infty}(\sharp I_{nj}) \sum_{k\geq jy^p} P\{k \leq |X_1|^p \leq k+1\} y^{q-1} dy\\ &\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{\beta n^{\frac{1}{p}}}^{\infty} \sum_{k=[y^p]}^{\infty}\sum_{j=1}^{[k/y^p]} (\sharp I_{nj}) P\{k \leq |X_1|^p \leq k+1\} y^{q-1} dy\\ &\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{\beta n^{\frac{1}{p}}}^{\infty} \sum_{k=[y^p]}^{\infty} n\left(\frac{k}{y^p}+1\right)^{\frac{1}{p}} P\{k \leq |X_1|^p \leq k+1\} y^{q-1} dy\\ &\leq C\sum_{n=1}^{\infty} n^{\frac{r}{p}-1-\frac{q}{p}} h(n) \int_{\beta n^{\frac{1}{p}}}^{\infty} \sum_{k=[y^p]}^{\infty} k^{\frac{1}{p}} y^{-1} P\{k \leq |X_1|^p \leq k+1\} y^{q-1} dy\\ &\leq C\int_{1}^{\infty} t^{\frac{r}{p}-1-\frac{q}{p}} h(t) \int_{\beta t^{\frac{1}{p}}}^{\infty} \sum_{k=[y^p]}^{\infty} k^{\frac{1}{p}} y^{-1} P\{k \leq |X_1|^p \leq k+1\} y^{q-1} dy dt \end{aligned}$$

$$
\begin{aligned}
&\left(\text{letting } \ x = \beta t^{\frac{1}{p}}\right)\\
&\le C \int_1^\infty x^{r-1-q} h(x^p) \int_x^\infty \sum_{k=[y^p]}^\infty k^{\frac{1}{p}} y^{-1} P\{k \le |X_1|^p \le k+1\} y^{q-1} dy dx\\
&\le C \int_1^\infty \left(\int_1^y x^{r-1-q} h(x^p) dx \right) \sum_{k=[y^p]}^\infty k^{\frac{1}{p}} y^{-1} P\{k \le |X_1|^p \le k+1\} y^{q-1} dy\\
&\le C \int_1^\infty y^{r-2} h(y^p) \sum_{k=[y^p]}^\infty k^{\frac{1}{p}} P\{k \le |X_1|^p \le k+1\} dy \qquad (3.4)\\
&\le C \sum_{k=0}^\infty k^{\frac{1}{p}} P\{k \le |X_1|^p \le k+1\} \int_1^{(k+1)^{\frac{1}{p}}} y^{r-2} h(y^p) dy\\
&\le C \sum_{k=0}^\infty k^{\frac{1}{p}} P\{k \le |X_1|^p \le k+1\} (k+1)^{\frac{r-1}{p}} h(k+1)\\
&\le C \sum_{k=0}^\infty (k+1)^{\frac{r}{p}} h(k+1) P\{k \le |X_1|^p \le k+1\}\\
&\le C E|X_1|^r h(|X_1|^p) + 1 < \infty.
\end{aligned}
$$

Now we estimate J_{22} for $r > 1 + p/2$ and $1 \le p < 2$.

$$
\begin{aligned}
J_{22} &= \sum_{n=1}^\infty n^{\frac{r}{p}-2-\frac{q}{p}} h(n) 4q \int_{n^{\frac{1}{p}}}^\infty \exp\left(-\frac{y^2 n^{-1} \lambda^2}{8}\right) y^{q-1} dy\\
&\quad \left(\text{letting } t = y^2 x^{-1}\right) \qquad (3.5)\\
&\le C \int_1^\infty x^{\frac{r}{p}-2-\frac{q}{p}} h(x) \int_{x^{\frac{1}{p}}}^\infty \exp\left(-\frac{y^2 x^{-1} \lambda^2}{8}\right) y^{q-1} dy dx\\
&\le C \int_1^\infty x^{\frac{r}{p}-2-\frac{q}{p}+\frac{q}{2}} h(x) x^{\frac{q}{2}} \int_{x^{\frac{2}{p}-1}}^\infty t^{\frac{q}{2}-1} \exp\left(-\frac{t\lambda^2}{8}\right) dt dx\\
&\le C \int_1^\infty \left(\int_1^{t^{\frac{p}{2-p}}} x^{\frac{r}{p}-2-\frac{q}{p}+\frac{q}{2}} h(x) dx \right) t^{\frac{q}{2}-1} \exp\left(-\frac{t\lambda^2}{8}\right) dt\\
&\le C \left(\frac{r}{p} - 1 - \frac{q}{p} + \frac{q}{2}\right)^{-1} \int_1^\infty t^{\frac{r-2}{2-p}} h\left(t^{\frac{p}{2-p}}\right) \exp\left(-\frac{t\lambda^2}{8}\right) dt < \infty.
\end{aligned}
$$

For J_{23}, under assumption $r > 1 + p/2$ and $1 \le p < 2$, which means that $(2-p)/12(r-p) < 1/6$. So we choose β such that $0 < \beta < (2-p)/(12(r-p))$. Then

$$
\begin{aligned}
J_{23} &= \sum_{n=1}^\infty n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_{n^{\frac{1}{p}}}^\infty 4q \left(\frac{1}{4(\beta y^2 n^{-1} \lambda^2 + 1)}\right)^{\frac{1}{12\beta}} y^{q-1} dy\\
&\le C \int_1^\infty x^{\frac{r}{p}-2-\frac{q}{p}} h(x) \int_{x^{\frac{1}{p}}}^\infty \left(\frac{1}{4(\beta y^2 x^{-1} \lambda^2 + 1)}\right)^{\frac{1}{12\beta}} y^{q-1} dy dx \qquad (3.6)
\end{aligned}
$$

$$
\begin{aligned}
&\le C\int_1^\infty x^{\frac{r}{p}-2-\frac{q}{p}}h(x)\int_{x^{\frac{1}{p}}}^\infty\left(\frac{x}{4y^2\lambda^2}\right)^{\frac{1}{12\beta}}y^{q-1}dydx\\
&\le C\int_1^\infty x^{\frac{r}{p}-2-\frac{q}{p}+\frac{1}{12\beta}}h(x)\int_{x^{\frac{1}{p}}}^\infty y^{-\frac{1}{6\beta}+q-1}dydx\\
&\le C\int_1^\infty x^{\frac{r}{p}-2-\frac{q}{p}+\frac{1}{12\beta}}h(x)\,y^{-\frac{1}{6\beta}+q-1}\Big|_{x^{\frac{1}{p}}}^\infty dx\\
&\le C\int_1^\infty x^{\frac{r}{p}-2-\frac{q}{p}+\frac{1}{12\beta}-\frac{1}{6\beta p}+\frac{q}{p}}h(x)dx\\
&\le C\int_1^\infty x^{\frac{r-2p}{p}-\frac{2-p}{12\beta p}}h(x)dx<\infty.
\end{aligned}
$$

Hence, by combining (3.1) ~ (3.6) for $1 \le p < 2$ and $r > 1 + p/2$, we have

$$
\sum_{n=1}^\infty n^{\frac{r}{p}-2-\frac{q}{p}}h(n)E\left\{\max_{1\le k\le n}|S_k|-\epsilon n^{\frac{1}{p}}\right\}_+^q<\infty.
$$

□

Proof: Proof of (ii)

$$
\begin{aligned}
\sum_{n=1}^\infty n^{\frac{r}{p}-2}h(n)E\left\{\sup_{k\ge n}|k^{-\frac{1}{p}}S_k|-\epsilon\right\}_+^q &= \sum_{n=1}^\infty n^{\frac{r}{p}-2}h(n)\int_0^\infty P\left\{\sup_{k\ge n}\left|k^{-\frac{1}{p}}S_k\right|>\epsilon+x^{\frac{1}{q}}\right\}dx\\
&=\sum_{i=1}^\infty\sum_{n=2^{i-1}}^{2^i-1}n^{\frac{r}{p}-2}h(n)\int_0^\infty P\left\{\sup_{k\ge n}\left|k^{-\frac{1}{p}}S_k\right|>\epsilon+x^{\frac{1}{q}}\right\}dx\\
&\le C\sum_{i=1}^\infty\int_0^\infty P\left\{\sup_{k\ge 2^{i-1}}\left|k^{-\frac{1}{p}}S_k\right|>\epsilon+x^{\frac{1}{q}}\right\}dx\sum_{n=2^{i-1}}^{2^i-1}n^{-\frac{r}{p}-2}h(n)\\
&\le C\sum_{i=1}^\infty 2^{i\left(\frac{r}{p}-1\right)}h\left(2^i\right)\int_0^\infty P\left\{\sup_{k\ge 2^{i-1}}\left|k^{-\frac{1}{p}}S_k\right|>\epsilon+x^{\frac{1}{q}}\right\}dx\\
&\le C\sum_{i=1}^\infty 2^{i\left(\frac{r}{p}-1\right)}h\left(2^i\right)\sum_{l=i}^\infty\int_0^\infty P\left\{\max_{2^{l-1}\le k<2^l}\left|k^{-\frac{1}{p}}S_k\right|>\epsilon+x^{\frac{1}{q}}\right\}dx\\
&\le C\sum_{l=1}^\infty\int_0^\infty P\left\{\max_{2^{l-1}\le k<2^l}\left|k^{-\frac{1}{p}}S_k\right|>\epsilon+x^{\frac{1}{q}}\right\}dx\sum_{i=1}^{l}2^{i\left(\frac{r}{p}-1\right)}h\left(2^i\right)\\
&\le C\sum_{l=1}^\infty 2^{l\left(\frac{r}{p}-1\right)}h\left(2^l\right)\int_0^\infty P\left\{\max_{2^{l-1}\le k<2^l}|S_k|>\left(\epsilon+x^{\frac{1}{q}}\right)2^{\frac{(l-1)}{p}}\right\}dx\\
&\quad\left(\text{letting }\ y=2^{(l-1)\frac{q}{p}}x\right)\\
&\le C\sum_{l=1}^\infty 2^{l\left(\frac{r}{p}-1-\frac{q}{p}\right)}h\left(2^l\right)\int_0^\infty P\left\{\max_{1\le k<2^l}|S_k|>2^{\frac{(l-1)}{p}}\epsilon+y^{\frac{1}{q}}\right\}dy
\end{aligned}
$$

$$
\begin{aligned}
&\le C \sum_{l=1}^{\infty} \sum_{n=2^{l-1}}^{2^l-1} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_0^{\infty} P\left\{\max_{1\le k<2^l} |S_k| > 2^{\frac{(l-1)}{p}}\epsilon + y^{\frac{1}{q}}\right\} dy \\
&\le C \sum_{l=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) \int_0^{\infty} P\left\{\max_{1\le k<n} |S_k| > n^{\frac{1}{p}} 2^{-\frac{1}{p}}\epsilon + y^{\frac{1}{q}}\right\} dy \\
&\quad \left(\text{letting } \epsilon_0 = 2^{-\frac{1}{p}}\epsilon\right) \\
&= C \sum_{n=1}^{\infty} n^{\frac{r}{p}-2-\frac{q}{p}} h(n) E\left\{\max_{1\le k\le n} |S_k| - \epsilon_0 n^{\frac{1}{p}}\right\}_+^q < \infty, \quad \text{by (i) of Theorem 1.}
\end{aligned}
$$

□

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