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The Study on the Mean Residual Life Estimation of Reliability Data under Random Censoring

Mi-Sook Lee[1*]
[1]**BK21, Graduate School of Business Administration, Kyungpook National University**

임의절단 하에서 신뢰성 자료의 평균잔여수명 추정에 대한 연구

이미숙[1*]
[1]경북대학교 경영학부 대학원 BK21

Abstract Mean Residual Life (MRL) function plays a very important role in the area of engineering, medical science, survival studies, social sciences, and many other fields. Specially, in the reliability study of technical systems, the MRL estimation of a component is very important because the sudden stop of a system brings a serious problem. So, many simulation studies of MRL estimation have been done considering various situation variables. In this paper, four estimators of MRL are proposed under random censoring and their performances are compared through bias and Mean Square Error (MSE) by Monte Carlo simulation.

요 약 평균잔여수명은 공학, 의학, 생존분석, 사회과학 등 많은 분야에서 중요한 역할을 하고 있다. 특히 시스템의 신뢰성연구에서 시스템의 갑작스런 중지는 심각한 문제를 초래하기 때문에, 부품에 대한 평균잔여수명 추정은 매우 중요하다. 그래서 많은 상황변수를 고려한 시뮬레이션 연구가 되어왔다. 본 연구에서는 임의절단(random censoring) 하에서 4가지 평균잔여수명 추정기법을 소개하고 3가지 와이블 수명분포와 6가지 절단분포의 조합에서 시뮬레이션 하였다. 또한 이들의 성과를 편의(bias)와 MSE측면에서 비교 분석하였다.

Key Words : Mean Residual Life(MRL), Reliability Study, Partial Moment Approximation, Random Censoring.

1. Introduction

In the last three decades, reliability engineers, statisticians, medical scientists and economists have shown intensified interest in estimating Mean Residual Life(MRL) and derived many useful results concerning it [3,14,11,8,10,5,2]. Since it was first developed, the MRL concept has had a tremendous range of applications, such as optimum burn-in time determination, setting rates and benefits analysis of a life insurance plan, survival analysis in biomedical studies, life-length prediction of wars and strikes in social sciences, optimum preventive replacement decision-making and renewal theory.

Especially, MRL function is the important function which can be used to characterize a lifetime in survival analysis and reliability.

In reliability studies, since the sudden stop of a system stemmed from a mechanical trouble brings a serious problem, estimating the residual life time of a device or a component is very important and MRL provides the engineer an idea of how long a device of any particular age can be expected to survive. Some reliability engineers showed that components and systems are frequently characterized as having increasing or decreasing MRL and the others have discussed MRL for components or systems whose lifetimes are measured in discrete units

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* Corresponding Author : Lee, Mi-Sook(leems@knu.ac.kr)
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(number of trials until failure). Embrechts et al. provided a detailed discussion and statistical applications for the MRL function in reliability theory [13]. Social scientists use MRL in studies of lengths of wars, duration of strikes, job mobility etc. Actuaries also apply MRL to setting rates and benefits for life insurance [7]. In economics, Bhattacharjee and Krishnaji presented applications of MRL for investigating landholding [9].

For a long time, many researchers also have interested in the estimation of MRL. Chaubey and Kochar proposed a modification of Chaubey–Sen method (Chaubey and Sen [18]) for estimating survival distributions constrained by stochastic ordering [17]. Hu et al. modified Ebrahimi's estimators to ensure that the resulting estimators are indeed MRL functions and studied the asymptotic and finite sample properties of their estimators [15]. Abdous and Berred used the local linear fitting technique to estimate the MRL function, given a sample from an unknown distribution function [1]. Chaubey and Sen proposed a smooth estimator of the MRL function based on randomly censored data. It was derived by smoothing the product-limit estimator using the Chaubey-Sen method [12].

Since reliability engineering deals with the study of reliability: the ability of a system or component to perform its required functions under stated conditions for a specified period of time, it is often reported as a probability. So, simulation in system reliability analysis is based on the Monte Carlo simulation method that generates random failure times from each component's failure distribution. Simulation is simple to apply and it can produce results that can be rather difficult to solve analytically. Especially, Monte Carlo simulation methods are useful for modeling phenomena with significant uncertainty in inputs, especially in studying systems. It is a widely successful method when compared with alternative methods because actual observations of failures are better predicted by the simulation than by human intuition.

The purpose of this paper is to provide four MRL estimators under random censoring model and to compare them through the mean square error using Monte Carlo simulation method.

Unlike the other papers, this paper analyzed the proposed four estimators considering many realistic situation variables and three failure rates(decreasing, constant, and increasing). So, this paper will be useful to apply its result to the various problem situations.

In section 2, it is proposed the approximation of the MRL by using the partial moment approximation. In section 3, two MRL estimators based on Kaplan-Meier estimator are proposed and in section 4, two MRL estimators based on the B-spline function are proposed. Finally, in section 5, it is investigated the performance of the proposed MRL estimators given in section 3 and 4 through the Monte Carlo simulation study. Some remarks and conclusion are given in section 6.

2. Approximation of the MRL

When an object is given of time x, the remaining life after any time x is random. The expected value of this random residual life is called the mean residual life (MRL). Since the MRL is defined for each time x, it is spoken of the MRL function. Let F be a life distribution function with a finite mean μ, and let X be a continuous random variable with distribution F. Let $R_F = 1-F(x)$ denote the reliability function (or survival function). Then, the MRL function at time x is described as following;

$$e(x) = E[X-x|X>x] = \begin{cases} \dfrac{\int_x^\infty R_F(t)\,dt}{R_F(x)}, & R_F(x)>0, \\ 0, & R_F(x)=0. \end{cases}$$

It can be rewritten as;

$$e(x) = \frac{\mu - \int_x^\infty t dF(t)}{R_F(x)} - x$$

In above equation,

$$\int_0^x t dF(t)$$

is called as the first partial moment of a random variable X with the distribution F about the origin over (0, x). And, the partial moment approximation of $e(x)$ is given by

$$e_p(x) = \mu + (\frac{F(x)}{1-F(x)})^{\frac{1}{2}}\sigma - x.$$

There have been many works on estimating *e(x)*. Under the random censoring model, Kumazawa proposed estimators of MRL function based on Nelson-Aalen estimator and Kaplan-Meier estimator of reliability function, respectively [16]. Ghorai and Rejtwo presented the estimator of the MRL function based on the maximum likelihood estimator for reliability function and obtained the strong consistency and weak convergence of the estimator under the proportional hazard mode [6]. Moon et al. proposed an estimation procedure for MRL function with consistency and asymptotic normality on the left truncated and right censoring model [4].

The partial moment estimator can be induced as following;

$$\hat{e_p}(x) = \overline{X} + (\frac{\hat{F}(x)}{1-\hat{F}(x)})^{\frac{1}{2}} S - x$$

where $\overline{X}$ and S^2 are the sample mean and variance, respectively and

$$\hat{F}(x) = \frac{1}{n}\sum I(X_i \le x)$$

is the empirical estimator of *F*. It is shown that the partial moment estimator is expected to perform better than the empirical MRL estimator, when the sample size is small because the one fully utilizes the entire sample through the statistics $\overline{X}$ and S^2, whereas the other utilizes only the observations exceeding *x*.

In the reliability study, having censoring observations is commonly occurred.

Determining the reliability of manufactured items often requires performing a life test and analyzing observed times to failure. Such data is frequently censored, in that some items being tested may not have failed when the test is ended.

Let $X_1,...,X_n$ be observed survival time of an item with distribution function F and $C_1,...,C_n$ be the corresponding censoring time. And define the indicator function

$$\delta_i = \begin{cases} 1, \ X_i < C_i \ (uncensored), \\ 0, \ X_i \ge C_i \ (censored). \end{cases}$$

Then, $Z_i = \min(X_i, C_i) \ (i=1,...,n)$ and, the observed data is $(Z_1,\delta_1),...,(Z_n,\delta_n)$. This sampling scheme usually called as random censoring model.

3. Estimation of the MRL on Kaplan–Meier Estimator

The Kaplan-Meier estimator of a distribution is represented as following;

$$\hat{F}^{km}(x) = \begin{cases} 1 - \prod_{i;Z_{(i)} \le x} (\frac{n-1}{n-i+1})^{\delta^{(i)}}, & x < Z_{(n)}, \\ 1, & x \ge Z_{(n)}, \delta_{(n)} = 1. \end{cases}$$

where $Z_{(i)}$ is the *i*th order statistic based on $Z_{(1)},...,Z_{(n)}$ and $\delta_{(i)}$ is the corresponding indicator value. Let

$$d\widehat{F^{km}}(x_i) = (\prod_{Z(j) < x_i} (\frac{n-j}{n-j+1}))(\frac{1}{n-i+1})$$

be the size of the jump at an uncensored point x_i.

Let $\hat{\mu}^{km} = \int_0^\infty x d\hat{F}^{km}(x)$

and $(\hat{\sigma}^{km})^2 = \int_0^\infty x^2 d\hat{F}^{km}(x) - (\hat{\mu}^{km})^2$

be sample mean and sample variance based on the Kaplan-Meier distribution F^{km}, respectively.

Thus, the first nonparametric MRL estimator based on the Kaplan-Meier estimator can be obtained as following;

$$\hat{e}^{km}(x) = \begin{cases} \dfrac{\hat{\mu}^{km} - \int_0^x t d\hat{F}^{km}(t)}{1-\hat{F}^{km}(t)} - x, & 1-\hat{F}^{km}(x) > 0, \\ 0, & 1-\hat{F}^{km}(x) = 0. \end{cases}$$

The second estimator of MRL based on Kaplan-Meier estimator $\hat{F}^{km}$ using partial moment approximation is obtained as following;

$$\hat{e}_p^{km}(x) = \begin{cases} \hat{\mu}^{km} + (\dfrac{\hat{F}^{km}(x)}{1-\hat{F}^{km}(x)})^{\frac{1}{2}} \hat{\sigma}^{km} - x, & x < Z(n), \\ 0, & x \ge Z(n). \end{cases}$$

4. Estimation of the MRL Based on the B-spline Estimator

Consider the sorted uncensored life time $X_{(i)}(i=1,...,k)$ as knots, where k is the number of uncensored lifetime. When the largest value is censored, the convention is also to redefine it as uncensored.

The cumulative hazard rate estimator by approximating a B-spline function is

$$\hat{H}(x)=\sum_{i=1}^{k}\frac{\sum_{l\geq i-1}B_{l,3}(x)}{\sum_{j=1}^{n}\sum_{r\geq i-1}B_{r,3}(x_j)}.$$

With knots $X_{(i-1)}<X_{(i)}<X_{(i+1)}$,

$$\sum_{l\geq i-1}B_{l,3}(x)=$$

$$\begin{cases}0, & x\leq x_{(i-1)},\\ \dfrac{(x-x_{(i-1)})^2}{(x_{(i)}-x_{(i-1)})(x_{(i+1)}-x_{(i-1)})}, & x_{(i-1)}<x\leq x_{(i)},\\ 1-\dfrac{(x_{(i+1)}-x)^2}{(x_{(i+1)}-x_{(i)})(x_{(i+1)}-x_{(i-1)})}, & x_{(i)}<x\leq x_{(i+1)},\\ 1, & x>x_{(i+1)}.\end{cases}$$

where $x_{(0)}=2x_{(1)}-x_{(2)},\ x_{(k-1)}=2x_{(k)}-x_{(k+1)}.$

The estimator of the distribution function is obtained by

$$\hat{F}^{sp}(x)=1-\exp(-\hat{H}(x))$$

and the estimator $\widehat{F^{sp}}$ is shown to be consistent to F.

Let $\widehat{\mu^{sp}}=\int_0^{\infty}xd\hat{F}^{sp}(x)$

and $(\hat{\sigma}^{sp})^2=\int_0^{\infty}x^2d\hat{F}^{sp}(x)-\hat{\mu}^{sp2}$

be sample mean and sample variance based on the B-spline estimator $\hat{F}^{sp}$, respectively. Then the estimator $\hat{e}^{sp}(x)$ is obtained by substituting $\hat{F}^{sp}(x)$ and $\hat{\mu}^{sp}$, respectively, for $F(x)$ and μ.

$$\hat{e}^{sp}(x)=\begin{cases}\dfrac{\hat{\mu}^{sp}-\int_0^{x}td\hat{F}^{sp}(t)}{1-\hat{F}^{sp}(t)}-x, & 1-\hat{F}^{sp}(x)>0,\\ 0, & 1-\hat{F}^{sp}(x)=0.\end{cases}$$

At last, the MRL estimator using partial moment approximation is proposed as;

$$\hat{e}_p^{sp}(x)=\begin{cases}\hat{\mu}^{sp}+\left(\dfrac{\hat{F}^{sp}(x)}{1-\hat{F}^{sp}(x)}\right)^{\frac{1}{2}}\hat{\sigma}^{sp}-x, & x<Z(n),\\ 0, & x\geq Z(n).\end{cases}$$

5. Monte Carlo Simulation

A Monte Carlo simulation was conducted to compare the behavior of the proposed estimators by using RNEXP, RNWIB, and RNUN in IMSL. This paper intends to identify the most proper estimation method in various situations trough the comparison by simulation. The simulation structure adopted in this paper was designed with various combinations of censoring percentages (10%, 20%, and 30%), different sample sizes(n = 10, 20, 30), three life distribution, and two censoring distribution as Table 1.

The underlying lifetime distribution is assumed to follow a Weibull distribution with the scale parameter 1 and the shape parameter Θ, where Θ is used 0.5, 1, and 2.

When Θ is 0.5, 1, and 2, the life distribution has decreasing failure rate, constant failure rate, and increasing failure rate, respectively.

[Table 1] Simulation Structure

Life Distribution	Censoring Distribution		Censoring Percentage
WEI(1, 0.5)	UNIF(0, 17)	EXP(15)	10
(decreasing failure rate)	UNIF(0, 7.5)	EXP(5.3)	20
	UNIF(0, 4)	EXP(2.6)	30
WEI(1, 1)	UNIF(0, 9)	EXP(9)	10
(constant failure rate)	UNIF(0, 5)	EXP(4)	20
	UNIF(0, 3)	EXP(2.3)	30
WEI(1, 2)	UNIF(0, 8.3)	EXP(7.8)	10
(increasing failure rate)	UNIF(0, 4)	EXP(3.7)	20
	UNIF(0, 2.9)	EXP(2.3)	30

Weibull distribution is very widely used as life time distribution because it is very powerful and can be applied to describe various failure processes such as electronics, mechanical components, and material.

The parameters of censoring distribution are obtained to fit the censoring percentage.

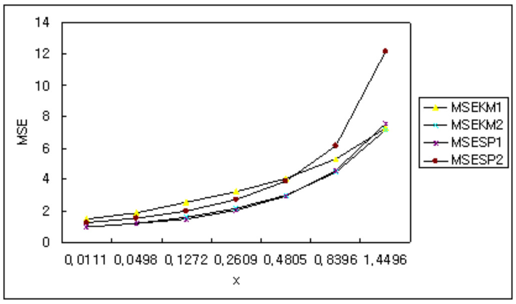
The true value of *e(x)* in Table 2 is compared as following;

$$e(x) = \begin{cases} 2\sqrt{x}+2, & X \sim WEI(1,0.5) \\ 1, & X \sim WEI(1,1) \\ \dfrac{0.5\sqrt{\pi}-\Gamma(\frac{3}{2},x^2)}{e^{-x^2}} - x, & X \sim WEI(1,2) \end{cases}$$

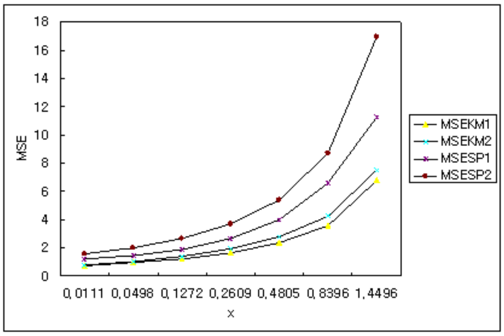
And the values of x given as conditionals are obtained by the inverse of R_F , i.e., $R_F^{-1}(0.9), R_F^{-1}(0.8), \ldots, R_F^{-1}(0.3)$. To compare the proposed estimators, bias and mean squared error(MSE) are used. The total numbers of table are 18 and the total numbers of figure are 54. The other tables and pictures are omitted for want of space. In the figures, MSEKM1= $\hat{e}_p^{km}$, MSEKM2= $\hat{e}^{km}$, MSESP1= $\hat{e}_p^{sp}$, and MSESP2= $\hat{e}^{km}$.

[Table 2] Bias and MSE of $\hat{e}_p^{km}$, $\hat{e}^{km}$, $\hat{e}_p^{sp}$, and $\hat{e}^{km}$ *for n=10, F(x) = WEI(1, 0.5), and G(x)=UNIF(0, b)*(Bias and MSE(the value in parenthesis) are multiplied by 100)

r(%)		True value of at time x (conditional points)						
		2.21072 (0.0111)	2.44429 (0.04979)	2.71335 (0.12722)	3.02165 (0.26094)	3.38629 (0.48045)	3.83258 (0.83959)	4.40795 (1.44955)
10	$\hat{e}_p^{km}$	0.01769 (1.47237)	0.13884 (1.8884)	0.20268 (2.56525)	0.109 (3.24345)	-0.06645 (4.03888)	-0.4003 (5.28268)	-0.97485 (7.28854)
	$\hat{e}^{km}$	-0.38116 (0.98084)	-0.4363 (1.1884)	-0.4775 (1.56059)	-0.59152 (2.13998)	-0.72477 (2.99451)	-0.96918 (4.45159)	-1.40992 (7.15202)
	$\hat{e}_p^{sp}$	-0.61412 (0.98397)	-0.62205 (1.16383)	-0.69967 (1.47869)	-0.87764 (2.02773)	-1.18129 (2.91341)	-1.66724 (4.57454)	-2.38376 (7.55582)
	$\hat{e}^{km}$	-0.91682 (1.21766)	-1.04459 (1.52489)	-1.20472 (1.97169)	-1.42271 (2.68572)	-1.74606 (3.90187)	-2.24441 (6.14939)	-3.18481 (12.17471)
20	$\hat{e}_p^{km}$	-0.37037 (0.75746)	-0.34984 (0.97982)	-0.44584 (1.21517)	-0.61997 (1.62029)	-0.9381 (2.35662)	-1.42176 (3.56974)	-2.21533 (6.79803)
	$\hat{e}^{km}$	-0.63996 (0.8012)	-0.72748 (1.05917)	-0.8569 (1.39407)	-1.01398 (1.90985)	-1.27042 (2.79401)	-1.67341 (4.21463)	-2.38841 (7.50361)
	$\hat{e}_p^{sp}$	-0.95587 (1.1832)	-1.04901 (1.43585)	-1.21348 (1.87808)	-1.47445 (2.64111)	-1.87254 (4.02263)	-2.45868 (6.58015)	-3.28899 (11.29244)
	$\hat{e}^{km}$	-1.16931 (1.55675)	-1.34041 (2.01269)	-1.55333 (2.6707)	-1.84026 (3.68479)	-2.25118 (5.40738)	-2.88075 (8.72578)	-3.9967 (16.95378)
30	$\hat{e}_p^{km}$	-0.72111 (0.76582)	-0.79289 (0.93662)	-0.95891 (1.29078)	-1.22122 (1.91942)	-1.65211 (3.18924)	-2.24509 (5.53286)	-3.17835 (10.58597)
	$\hat{e}^{km}$	-0.8933 (0.95738)	-1.0225 (1.25716)	-1.1881 (1.70216)	-1.41374 (2.37841)	-1.80169 (3.66872)	-2.33912 (5.95321)	-3.25537 (11.08692)
	$\hat{e}_p^{sp}$	-1.28532 (1.74968)	-1.43084 (2.15539)	-1.65431 (2.85951)	-1.98127 (4.05795)	-2.44589 (6.12463)	-3.09676 (9.73472)	-3.97583 (15.9552)
	$\hat{e}^{km}$	-1.43511 (2.13462)	-1.63532 (2.75332)	-1.89113 (3.66364)	-2.23995 (5.11579)	-2.72793 (7.55788)	-3.44815 (12.06762)	-4.72106 (22.91229)



[Fig. 1] Comparison of MRL with respect to MSE *(n=10, F(x) = WEI(1, 0.5), and G(x)=UNIF(0, 17))*



[Fig. 2] Comparison of MRL with respect to MSE *n=10, F(x) = WEI(1, 0.5), and G(x)=UNIF(0, 7.5)*

6. Conclusion

Totally, the overall results are summarized in Table 3 and the best estimator in each case is tabulated. In table, we can know that it is good to use the partial moment approximation even in large sample under random censoring model and in the lifetime distribution having the decreasing failure rate. In addition, it is good to use the estimator based on the B-spline function.

In a real practice, this paper can be applied to predict the residual life time of components having a similar failure or survival pattern(distribution) with the one proposed in this paper. Since components life can be measured in terms of the time to failure, MRL estimation is very crucial and it allows the producer to make assurances to customers about the quality, safety, and useful "life" range of the component. So, the overall system reliability is then obtained by simulating system operation and empirically calculating the reliability values for a series of time values.

The results of this study shows that 1) B-spline estimator using partial moment approximation is better if a component has a decreasing failure rate and censoring percentage is low, 2) Kaplan-Meier estimator using partial moment approximation is better if a component has a decreasing failure rate and censoring percentage is high, 3) Kaplan-Meier estimator using partial moment approximation is better if a component has a constant failure rate and censoring distribution follows Uniform, 4) B-spline estimator using partial moment approximation is better if a component has a constant failure rate and censoring distribution follows Exponential, 5) B-spline estimator using partial moment approximation is better if a component has a increasing failure rate and regardless of censoring distribution.

[Table 3] The overall results of simulation

Life dist.				WEI(1, 0.5)		WEI(1, 1)		(WEI(1, 2)	
Censoring dist.				UNIF	EXP	UNIF	EXP	UNIF	EXP
n	10	r(%)	10	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$
			20	$\hat{e}_p^{km}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$
			30	$\hat{e}_p^{km}$	$\hat{e}_p^{km}$	$\hat{e}_p^{km}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$
	20		10	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}^{km}$	$\hat{e}^{km}$	$\hat{e}^{km}$	$\hat{e}_p^{sp}$
			20	$\hat{e}_p^{km}$	$\hat{e}_p^{sp}$	$\hat{e}^{km}$	$\hat{e}^{km}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$
			30	$\hat{e}_p^{km}$	$\hat{e}_p^{km}$	$\hat{e}^{km}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$
	30		10	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}^{km}$	$\hat{e}^{km}$	$\hat{e}^{km}$	$\hat{e}^{km}$
			20	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}^{km}$	$\hat{e}^{km}$	$\hat{e}^{km}$	$\hat{e}^{km}$
			30	$\hat{e}_p^{km}$	$\hat{e}_p^{km}$	$\hat{e}_p^{km}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$	$\hat{e}_p^{sp}$

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Mi-Sook Lee [Regular member]

- Feb. 1998 : Kyungpook National University, MS
- Feb. 2008 : Kyungpook National University, PhD
- Mar. 2009 ~ current : Kyungpook National Univ., School of Business Admin., Research Professor

<Research Interests>
Technology Innovation, RFID, SCM, Scheduling