Utkin 정리의 다입력 불확실 선형 시스템에 대한 증명

논 문
59-9-26

A Poof of Utkin's Theorem for a MI Uncertain Linear Case

이 정 훈*
(Jung-Hoon Lee)

Abstract – In this note, a proof of Utkin's theorem is presented for a MI(Multi Input) uncertain linear case. The invariance theorem with respect to the two transformation methods so called the two diagonalization methods are proved clearly and comparatively for MI uncertain linear systems. With respect to the sliding surface transformation and the control input transformation, the equation of the sliding mode i.e., the sliding surface is invariant. Both control inputs have the same gains. By means of the two transformation methods the same results can be obtained. Through an illustrative example and simulation study, the usefulness of the main results is verified.

Key Words : Variable structure system, Sliding mode control, Proof of Ukin's Theorem, Diagonalization methods

1. Introduction

The sliding mode control(SMC) can provide the effective means to the problem of controlling uncertain dynamical systems under parameter variations and external disturbances[1][2][3]. One of its essential advantages is the robustness of the controlled system to parameter uncertainties and external disturbances in the sliding mode on the predetermined sliding surface, *s=0*[4]. The proper design of the sliding surface can determine the almost output dynamics and its performances[5]. Many design algorithms including the linear(optimal control[6][9], geometric approach[7], pole assignment[8], eigenstructure assignment[9], differential geometric approach[12], output feedback[15], Lyapunov approach[16][17], integral augmentation[5][13][21], Ackermann's formula[18], dynamic sliding surface[19]), and nonlinear[14][20] techniques are reported. However, in these sliding design methods so for, the existence condition of the sliding mode on the predetermined sliding surface is not proved under uncertainties and disturbances.

To take the advantages of the sliding mode on the predetermined sliding surface, the existence condition of the sliding mode, $s \cdot \dot{s} < 0$ for the SI linear case and $s_i \cdot \dot{s}_i < 0,\ i = 1,2,...m$ for the MI(Multi Input) linear case are satisfied. Therefore the existence condition of the sliding mode must be proved. For the uncertain linear SI case and the uncertain integral SI systems, those proofs were reported in [22] and [23], respectively. For the linear MI case, a few control design method was studied, those are hierarchical control methodology[1][3], diagonalization methods[1][2][15][16], simplex algorithm[10], Lyapunov approach[16][17], and so on. Until now in MIMO VSSs, The proof is not presented and it is difficult to prove the precise existence condition of the sliding mode on the predetermined sliding surface theoretically, but in [16][16][21], only the results of the derivative of the Lyapunov function is negative, i,e. $\dot{V} < 0$ is obtained when $V = 1/2 s^T s$. Utkin presented the two methodologies to prove the existence condition of the sliding mode on the sliding surface for MI uncertain systems[1]. It is so called the invariance theorem, i.e the equation of the sliding mode is invariant with respect to the two nonlinear transformations. Those are the control input transformation and sliding surface transformation. so called the diagonalization methods. The essential feature of these methods is conversion of a multi-input design problem into m single-input design problems[2]. Those were reviewed in [2]. DeCarlo, Zak, and Matthews tried to prove Utkin's invariance theorem. But, the proof is not clear. In [15], Zak and Hui used the control input transformation without uncertainties and disturbance and suggested the sufficient condition for the existence and reachability of the sliding mode as in [1] and [2]. But, they did not explicitly prove for the control input transformation under uncertainties and disturbance. In

* 정 회 원 : 공학연구원, 경상대학교 공대 제어계측공학과 교수・공박
E-mail : jhleew@gnu.ac.kr

접수일자 : 2010년 5월 12일
최종완료 : 2010년 7월 13일

[17], Su, Drakunov, and Ozguner mentioned the sliding surface transformation, which would diagonalize the control coefficient matrix to the dynamics for s. But they did not prove the existence condition of the sliding mode on the predetermined sliding surface. Until now, only for SI uncertain linear (integral) systems, a rigorous proof of Utkin's theorem is reported in [22] and [23].

In this paper, a proof of Utkin's theorem is presented for a MI uncertain linear case. The main results are extended from the SI uncertain case in [22] to the MI uncertain case. The invariance theorem with respect to the two transformation methods so called the two diagonalization methods are proved clearly and comparatively for MI uncertain linear systems. A design example and simulation study shows usefulness of the main results.

2. Main Results of Proof Utkin's Theorem for MI Uncertain Linear System

The invariant theorem of Utkin's for MI systems is as follows[1][2]:

Theorem 1: The equation of the sliding mode is invariant with respect to the two nonlinear transformations, i.e. the control input transformation and sliding surface transformation:

$$s^*(x) = H_s(x,t) \cdot s(x) \qquad (1)$$
$$u^*(x) = H_u(x,t) \cdot u(x)$$

for $\det H_s \neq 0$ and $\det H_u \neq 0$

For a MI uncertain linear system:

$$\dot{x} = (A_0 + \Delta A)x + (B_0 + \Delta B)u + \Delta D(t) \qquad (2)$$

where $x \in R^n$ is the state, $u \in R^m$ is the control input, $A_0 \in R^{n \times n}$ is the nominal system matrix, $B_0 \in R^{n \times m}$ is the nominal input matrix, $\triangle A$ and $\triangle B$ are the system matrix uncertainty and input matrix uncertainty, those are bounded, and $\Delta D(t)$ is bounded external disturbance, respectively.

The conventional typical sliding surface $s \in R^m$ is the linear combination of the full state variable as

$$s = C \cdot x \qquad (3)$$

Assumption 1:

CB_0 has the full rank and its inverse for a coefficient matrix of the sliding surface C.

Assumption 2:

$(CB_0)^{-1} C \triangle B = \triangle I$. $\triangle I$ is diagonal and $|\triangle I_i| \leq \rho_i < 1, \quad i = 1,2,\ldots,m$

Assumption 3:

$C \triangle B (CB_0)^{-1} = \triangle I'$. $\triangle I'$ is diagonal and $|\triangle I_i'| \leq \rho'_i < 1, \quad i = 1,2,\ldots,m$

The corresponding stabilizing VSS control input is as follows:

$$u = -K \cdot x - \Delta K \cdot x - G \cdot sign(s) \qquad (4)$$

where K is a constant gain, ΔK is a state dependent switching gain, G is a switching gain.

1) sliding surface transformation[1][2]

$$s^* = (CB_0)^{-1} \cdot s, \quad H_s(x,t) = (CB_0)^{-1} \qquad (5)$$

The transformation matrix is selected as $H_s(x,t) = (CB_0)^{-1}$. In [2], the proof of the sliding surface transformation theorem is not sufficient. In [16], without uncertainty and disturbance, it is mentioned that the sliding surface transformation would diagonalize the control coefficient matrix to the dynamics for s and the $\dot{V}(x) < 0$ is proved when $V(x) = x^T P x > 0$.

Now, the suggested VSS control input for the new transformed sliding surface is taken as follows:

$$u_1 = -K \cdot x - \Delta K \cdot x - G \cdot sign(s^*) \qquad (6)$$

where one takes the constant gain as

$$K = (CB_0)^{-1} C A_0 \qquad (7)$$

and takes the switching gains as follows:

$$\triangle k_{ij} = \begin{cases} \geq \dfrac{\max\{(CB_0)^{-1} C \triangle A - \triangle I' (CB_0)^{-1} C A_0\}_{ij}}{\min\{I + \triangle I'\}_{ii}} & sign(s_i^* x_j) > 0 \\ \leq \dfrac{\min\{(CB_0)^{-1} C \triangle A - \triangle I' (CB_0)^{-1} C A_0\}_{ij}}{\min\{I + \triangle I'\}_{ii}} & sign(s_i^* x_j) < 0 \end{cases}$$
$$i = 1,2,\ldots,m \quad j = 1,2,\ldots,n \qquad (8)$$

$$G_i = \begin{cases} \geq \dfrac{\max\{(CB_0)^{-1} C \Delta D(t)\}_i}{\min\{I + \triangle I\}_{ii}} & sign(s_i^*) > 0 \\ \leq \dfrac{\min\{(CB_0)^{-1} C \Delta D(t)\}_i}{\min\{I + \triangle I\}_{ii}} & sign(s_i^*) < 0 \end{cases}$$
$$i = 1,2,\ldots,m \qquad (9)$$

Then, the real dynamics of the sliding surface, i.e. the time derivative of s^* becomes

$$\begin{aligned} \dot{s}^* &= (CB_0)^{-1} \dot{s} = (CB_0)^{-1} C \dot{x} \\ &= (CB_0)^{-1} C (A_0 + \Delta A) x + (CB_0)^{-1} C (B_0 + \Delta B) u_1 \\ &\quad + (CB_0)^{-1} C \Delta D(t) \\ &= (CB_0)^{-1} C (A_0 + \Delta A) x + (I + \Delta I) u_1 + (CB_0)^{-1} C \Delta D(t) \\ &= (CB_0)^{-1} C (A_0 + \Delta A) x + (I + \Delta I)(-Kx - \Delta K x - G sign(s^*)) \\ &\quad + (CB_0)^{-1} C \Delta D(t) \\ &= (CB_0)^{-1} C A_0 x - Kx + (CB_0)^{-1} C \Delta A x - \Delta I K x - (I + \Delta I) \Delta K x \\ &\quad + (CB_0)^{-1} C \Delta D(t) - (I + \Delta I) G sign(s^*) \end{aligned} \qquad (10)$$

From (7), the real dynamics of s^* becomes

$$\begin{aligned} \dot{s}^* &= [(CB_0)^{-1} C \Delta A - \Delta I K] x - (I + \Delta I) \Delta K x \\ &\quad + (CB_0)^{-1} C \Delta D(t) - (I + \Delta I) G sign(s^*) \end{aligned} \qquad (11)$$

From inequalities of the switching gains (8)-(9) and (11), one can obtain the following equation then

$$s_i^* \cdot \dot{s}_i^* < 0, \quad i = 1,2,\ldots,m. \qquad (12)$$

If the sliding mode equation $s^* = 0$, then $s = 0$ since $CB_0 \neq 0$. The inverse augment also holds, therefore the both sliding surfaces are equal i.e. $s = s^* = 0$, which completes the proof of Theorem 1.

2) control input transformation[1][2][15]

$$u^{*}=(CB_0)^{-1}u, \quad H_u=(CB_0)^{-1} \qquad (13)$$
$$=(CB_0)^{-1}[-Kx-\Delta Kx-Gsign(s)]$$

where by letting the constant gain

$$K=CA_0 \qquad (14)$$

and if one take the switching gain as design parameters

$$\Delta k_{ij}=\begin{cases}\geq \dfrac{\max\{C\Delta A-\Delta I' CA_0\}_{ij}}{\min\{I+\Delta I'\}_{ii}} & sign(s_i x_j)>0\\ \leq \dfrac{\min\{C\Delta A-\Delta I' CA_0\}_{ij}}{\min\{I+\Delta I'\}_{ii}} & sign(s_i x_j)<0\end{cases}$$
$$i=1,2,\ldots,m \quad j=1,2,\ldots,n \qquad (15)$$

$$G_i=\begin{cases}\geq \dfrac{\max\{C\Delta D(t)\}_i}{\min\{I+\Delta I'\}_{ii}} & sign(s_i)>0\\ \leq \dfrac{\min\{C\Delta D(t)\}_i}{\min\{I+\Delta I'\}_{ii}} & sign(s_i)<0\end{cases} \quad i=1,2,\ldots,m \qquad (16)$$

for the MI uncertain linear case, the control input transformation matrix is selected as $H_u=(CB_0)^{-1}$. In [1] [2], and [15], the proofs for the nonlinear control input transformation are not clear under uncertainties and disturbances.

The real dynamics of s, i.e. the time derivative of s is as follows:

$$\dot{s}=C\dot{x} \qquad (17)$$
$$=C(A_0+\Delta A)x+C(B_0+\Delta B)u^{*}+C\Delta D(t)$$
$$=C(A_0+\Delta A)x+(I+\Delta I')u+C\Delta D(t)$$
$$=C(A_0+\Delta A)x+(I+\Delta I')(-Kx-\Delta Kx-Gsign(s))$$
$$+C\Delta D(t)$$
$$=CA_0x-Kx+C\Delta Ax-\Delta I' Kx-(I+\Delta I')\Delta Kx$$
$$+C\Delta D(t)-(I+\Delta I')Gsign(s)$$

From (14), then the real dynamics of s becomes

$$\dot{s}=[C\Delta A-\Delta I' K]x-(I+\Delta I')\Delta Kx \qquad (18)$$
$$+C\Delta D(t)-(I+\Delta I')Gsign(s)$$

From (15), (16), and (18), one can obtain the following equation

$$s_i\cdot\dot{s}_i<0, \quad i=1,2,\ldots,m \qquad (19)$$

The existence condition of the sliding mode by the transformed control input is proved for the MI uncertain linear system. The equation of the sliding mode, i.e. the sliding surface is invariant to the control input transformation. The sliding mode equation i.e. the sliding surface $s=0$ is the same as that of $s^{*}=0$. To compare the control inputs, u_1 and u^{*}, the form and the gain is the same. The both methods equivalently diagonalize the system, so those are called the diagonalization methods.

3. Illustrative Example

Consider a fifth-order system described by the state equation which is slightly modified from that in [24]

$$\dot{x}=\begin{bmatrix}0.0 & 1.0 & 0.0 & 3.20 & 1.98\\ 0.0 & 0.0 & 1.0 & -14.72 & 0.49\\ -8.86 & 8.0 & 9.36 & -7.92 & 36.01\\ 1.69 & 1.26 & 0.08 & 0.0 & 1.0\\ -7.52 & -5.23 & -0.45 & 32.32 & -1.36\end{bmatrix}x$$
$$+\begin{bmatrix}0.0 & 0.0\\ 0.0 & 0.0\\ 2.0\pm0.1 & 0.0\\ 0.0 & 0.0\\ 0.0 & 2.0\pm0.2\end{bmatrix}u+\begin{bmatrix}0.0 & .0.0\\ 0.0 & 0.0\\ \pm3.0 & 0.0\\ 0.0 & 0.0\\ 0.0 & \pm5.0\end{bmatrix} \qquad (20)$$

where the nominal parameter A_0 and B_0, matched uncertainties $\triangle A$ and $\triangle B$, and disturbance $\triangle D(t)$ are

$$A_0=\begin{bmatrix}0.0 & 1.0 & 0.0 & 3.20 & 1.98\\ 0.0 & 0.0 & 1.0 & -14.72 & 0.49\\ -8.86 & 8.0 & 9.36 & -7.92 & 36.01\\ 1.69 & 1.26 & 0.08 & 0.0 & 1.0\\ -7.52 & -5.23 & -0.45 & 32.32 & -1.36\end{bmatrix}, B_0=\begin{bmatrix}0.0 & 0.0\\ 0.0 & 0.0\\ 2.0 & 0.0\\ 0.0 & 0.0\\ 0.0 & 2.0\end{bmatrix},$$

$$\triangle A=0, \triangle B=\begin{bmatrix}0.0 & 0.0\\ 0.0 & 0.0\\ \pm0.1 & 0.0\\ 0.0 & 0.0\\ 0.0 & \pm0.2\end{bmatrix}, \& \triangle D(t)=\begin{bmatrix}0.0 & .0.0\\ 0.0 & 0.0\\ \pm3.0 & 0.0\\ 0.0 & 0.0\\ 0.0 & \pm5.0\end{bmatrix} \qquad (21)$$

The stable coefficient matrix of the sliding surface is determined as

$$C=\begin{bmatrix}-0.436 & 1.802 & 1.0 & -14.568 & 0.0\\ 1.010 & 0.505 & 0.0 & 1.616 & 0.5\end{bmatrix} \qquad (22)$$

1) sliding surface transformation

$$s^{*}=(CB_0)^{-1}\cdot s, \; H_s(x,t)=(CB_0)^{-1}=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}^{-1}=\begin{bmatrix}0.5 & 0\\ 0 & 1\end{bmatrix} \qquad (23)$$

Now, the VSS control input is taken as follows:

$$u_1=-K\cdot x-\Delta K\cdot x-G\cdot sign(s^{*}) \qquad (24)$$

The real dynamics of the transformed sliding surface, I.e., the time derivative of s^{*} becomes

$$\dot{s}^{*}=(CB_0)^{-1}\dot{s}=(CB_0)^{-1}C\dot{x} \qquad (25)$$
$$=(CB_0)^{-1}CA_0x-Kx+(CB_0)^{-1}C\Delta Ax-\Delta IKx$$
$$-(I+\Delta I)\Delta Kx+(CB_0)^{-1}C\Delta D(t)-(I+\Delta I)Gsign(s^{*})$$

By letting the gain

$$K=(CB_0)^{-1}CA_0$$
$$=\begin{bmatrix}0.5 & 0.0\\ 0.0 & 1.0\end{bmatrix}\begin{bmatrix}-33.4799 & -10.7917 & 9.966 & -35.8406 & 21.4617\\ -1.0290 & 0.4312 & 0.4093 & 11.9584 & 3.1833\end{bmatrix}$$
$$=\begin{bmatrix}-16.7400 & -5.3958 & 4.9983 & -17.9203 & 10.7309\\ -1.0290 & 0.4312 & 0.4093 & 11.9584 & 3.1833\end{bmatrix}$$
$$(26)$$

then the real dynamics of s^{*} becomes

$$\dot{s}^{*}=(CB_0)^{-1}C\Delta Ax-\Delta IKx-(I+\Delta I)\Delta Kx \qquad (27)$$
$$+(CB_0)^{-1}C\Delta D(t)-(I+\Delta I)Gsign(s^{*})$$

and $\triangle I$ is

$$\triangle I=(CB_0)^{-1}C\triangle B=\begin{bmatrix}0.5 & 0.0\\ 0.0 & 1.0\end{bmatrix}\begin{bmatrix}-0.436 & 1.802 & 1.0 & -14.568 & 0.0\\ 1.010 & 0.505 & 0.0 & 1.616 & 0.5\end{bmatrix}$$
$$\begin{bmatrix}0.0 & 0.0\\ 0.0 & 0.0\\ \pm0.1 & 0.0\\ 0.0 & 0.0\\ 0.0 & \pm0.2\end{bmatrix}=\begin{bmatrix}\pm0.05 & 0.0\\ 0.0 & \pm0.1\end{bmatrix} \qquad (28)$$

If one take the switching gains as follows:

$$\Delta k_{11}=\begin{cases}1.2 & \text{if } s_1{*}x_1>0\\ -1.2 & \text{if } s_1{*}x_1<0\end{cases}, \quad \Delta k_{12}=\begin{cases}4.2 & \text{if } s_1{*}x_2>0\\ -4.2 & \text{if } s_1{*}x_2<0\end{cases},$$
$$\Delta k_{13}=\begin{cases}6.5 & \text{if } s_1{*}x_3>0\\ -6.5 & \text{if } s_1{*}x_3<0\end{cases}, \quad \Delta k_{14}=\begin{cases}6.5 & \text{if } s_1{*}x_4>0\\ -6.5 & \text{if } s_1{*}x_4<0\end{cases},$$
$$\Delta k_{15}=\begin{cases}6.5 & \text{if } s_1{*}x_5>0\\ -6.5 & \text{if } s_1{*}x_5<0\end{cases}, \quad G_{11}=\begin{cases}1.8 & \text{if } s_1>0\\ -1.8 & \text{if } s_1<0\end{cases} \qquad (29)$$

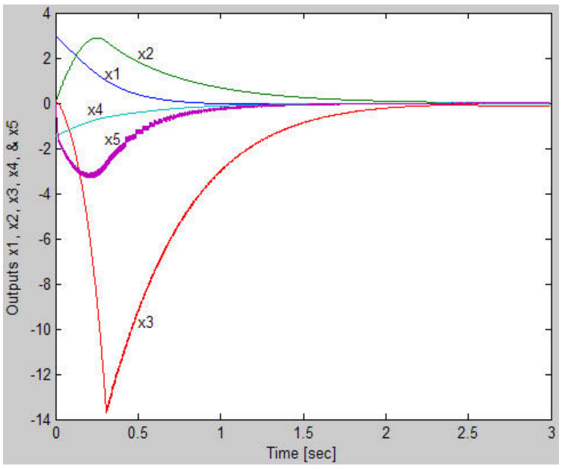

Fig. 1 Five state output responses, x_1, x_2, x_3, x_4, and x_5

$$\Delta k_{21}=\begin{cases}5.2 & \text{if } s_2*x_1>0\\ -5.2 & \text{if } s_2*x_1<0\end{cases},\quad \Delta k_{22}=\begin{cases}4.2 & \text{if } s_2*x_2>0\\ -4.2 & \text{if } s_2*x_2<0\end{cases},$$
$$\Delta k_{23}=\begin{cases}6.5 & \text{if } s_2*x_3>0\\ -6.5 & \text{if } s_2*x_3<0\end{cases},\quad \Delta k_{24}=\begin{cases}6.5 & \text{if } s_2*x_4>0\\ -6.5 & \text{if } s_2*x_4<0\end{cases},$$
$$\Delta k_{25}=\begin{cases}6.5 & \text{if } s_2*x_5>0\\ -6.5 & \text{if } s_2*x_5<0\end{cases},\quad G_{22}=\begin{cases}2.8 & \text{if } s_2>0\\ -2.8 & \text{if } s_2<0\end{cases} \tag{30}$$

then

$$s_i^*\cdot \dot{s}_i^*<0, \quad i=1,2 \tag{31}$$

If $s_i^*=0$, then $s_i=0$. The inverse augment also holds. The simulation is carried out under $1[msec]$ sampling time and with $x(0)=[3\ 0\ 0\ -1.5\ 0]^T$ initial condition.

Fig. 1 shows the five state output responses, x_1, x_2, x_3, x_4, and x_5

2) control input transformation

$$u^*=(CB_0)^{-1}u, \quad H_u=(CB_0)^{-1}=\begin{bmatrix}0.5 & 0.0\\ 0.0 & 1.0\end{bmatrix}$$
$$=\begin{bmatrix}0.5 & 0.0\\ 0.0 & 1.0\end{bmatrix}[-Kx-\Delta Kx-Gsign(s)] \tag{32}$$

Then, the real dynamics of s, I.e., the time derivative of s is as follows:

$$\dot{s}=C\dot{x}$$
$$=CA_0x-Kx+C\Delta Ax-\Delta I'Kx-(I+\Delta I')\Delta Kx$$
$$+C\Delta D(t)-(I+\Delta I')Gsign(s) \tag{33}$$

where

$$\Delta I'=C\Delta B(CB_0)^{-1}=\begin{bmatrix}-0.436 & 1.802 & 1.0 & -14.568 & 0.0\\ 1.010 & 0.505 & 0.0 & 1.616 & 0.5\end{bmatrix}$$
$$\begin{bmatrix}0.0 & 0.0\\ 0.0 & 0.0\\ \pm 0.1 & 0.0\\ 0.0 & 0.0\\ 0.0 & \pm 0.2\end{bmatrix}\begin{bmatrix}0.5 & 0\\ 0 & 1.0\end{bmatrix}=\begin{bmatrix}\pm 0.05 & 0.0\\ 0.0 & \pm 0.1\end{bmatrix} \tag{34}$$

By letting the constant gain as

$$K=CA_0$$
$$=\begin{bmatrix}-33.4799 & -10.7917 & 9.9966 & -35.8406 & 21.4617\\ -4.7890 & -2.0938 & 0.1843 & 28.4064 & 2.6814\end{bmatrix} \tag{35}$$

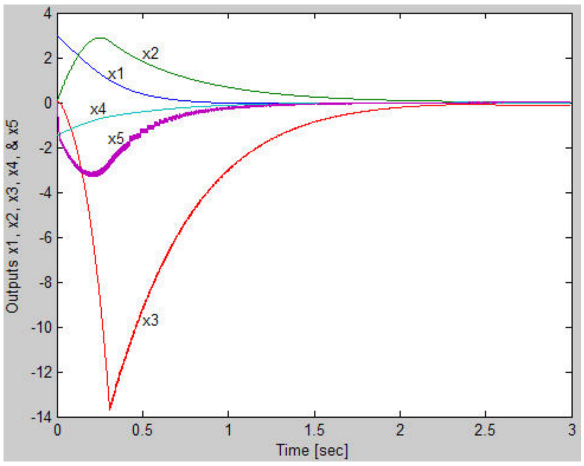

Fig. 2 Five state output responses, x_1, x_2, x_3, x_4, and x_5

then, the real sliding dynamics becomes

$$\dot{s}=C\Delta Ax-\Delta I'Kx-(I+\Delta I')\Delta Kx$$
$$+C\Delta D(t)-(I+\Delta I')Gsign(s) \tag{36}$$

If one take the switching gain as the design parameters as the same in (29)-(30)

then one can obtain the following equation

$$s_i^*\cdot \dot{s}_i^*<0, \quad i=1,2 \tag{37}$$

The existence condition of the sliding mode is proved. The equation of the sliding mode that is the sliding surface is invariant to the control input transformation

The simulation is carried out under $1[msec]$ sampling time and with $x(0)=[3\ 0\ 0\ -1.5\ 0]^T$ initial condition. Fig. 2 shows the five state output responses, x_1, x_2, x_3, x_4, and x_5 by u^* with s. Those Fig. 1 and Fig. 2 are almost identical because the sliding surface $s=0=s^*$ is equal and the continuous gains and switching gains of the both control u_1 and u^* are equal.

4. Conclusions

In this note, the invariant theorem of Utkin is rigorously proved for MI uncertain linear systems. The invariance theorem of the two diagonal methods i.e., the control input transformation and sliding surface transformation is proved clearly and comparatively. Therefore, the equation of the sliding mode, i.e., the sliding surface is invariant with respect to the two diagonalization methods. These two methods diagonalize the input system of the real sliding dynamics of the sliding surface s or s^* so that the existence condition of the sliding mode on the predetermined sliding surface is easily proved. During the proof of Utkin's theorem for MI uncertain systems, the design rules of both control inputs are proposed. Through an illustrative example and simulation study, the effectiveness of the proposed main results is verified. The same results of the outputs by the two diagonalization methods are obtained. The equation of the sliding mode, i.e., the sliding surface is invariant with respect to the two diagonalization methods for MI uncertain systems.

참 고 문 헌

[1] V.I. Utkin,, Sliding Modes and Their Application in Variable Structure Systems. Moscow, 1978.

[2] Decarlo, R.A., Zak, S.H., and Mattews, G.P., "Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial," Proc. IEEE, 1988, 76, pp.212-232.

[3] Young, K.D., Utkin, V.I., Ozguner, U, "A Control Engineer's Guide to Sliding Mode Control," 1996 IEEE Workshop on Variable Structure Systems, pp.1-14

[4] Drazenovic, B.:The invariance conditions in variable structure systems, Automatica, 1969, (5), pp.287-295.

[5] J. H. Lee and M. J. Youn, "An Integral-Augmented Optimal Variable Structure control for Uncertain dynamical SISO System, KIEE(The Korean Institute of Electrical Engineers), vol.43, no.8, pp.1333-1351, 1994.

[6] V. I. Utkin and K. D. Yang, "Methods for Constructing Discontinuity Planes in Multidimentional Variable Structure Systems," Automat. Remote Control, vol. 39 no. 10 pp.1466-1470, 1978.

[7] D. M. E. El-Ghezawi, A. S. I. and S. A. Bilings, "Analysis and Design of Variable Structure Systems Using a Geometric Approach," Int. J. Control, vol,38, no.3 pp.657-671,1 1983.

[8] A. Y. Sivaramakrishnan, M. Y. Harikaran, and M. C. Srisailam, " Design of Variable Structure Load Frequency cntroller Using Pole Assignment Technique," Int. J. Control, vol,40 no.3, pp. 487-498, 1984.

[9] C. M. Dorling and A. S. I. Zinober, "Two approaches to Hyperplane Design in Muntivariable Variable Structure Control systems," Int. J. Control, vol,44 no.1, pp. 65-82 1986.

[10] S. V. Baida and D. B. Izosimov, "Vector Method of Design of Sliding Motion and Simplex Algorithms." Automat. Remote Control, vol. 46 pp.830-837, 1985. Variable Structure Control systems," Int. J. Control, vol,44 no.1, pp. 65-82 1986.

[11] B. Fernandez and J. K. Hedrick, "Control of Multivariable Nonlinear System by the Sliding Mode Method," I. J. Control, vol.46, no.3 pp.1019-1040, 1987.

[12] S. R. Hebertt, "Differential Geometric Methods in Variable Structure Control," Int. J. Control, vol,48 no.4, pp.1359-1390, 1988.

[13] T. L. Chern and Y. C. Wu, "An Optimal Variable Structure Control with Integral Compensation for Electrohydraulic postion Servo Control Systems," IEEE T. Industrial Electronics, vol.39, no.5 pp460-463, 1992.

[14] K. Yeung, C. Cjiang, and C. Man Kwan, "A Unifying Design of Sliding Mode and Classical Controller' IEEE T. Automatic Control, vol.38, no.9 pp1422-1427, 1993.

[15] S. H. Zak and S. Hui, "Output Feedback Variable Structure Controllers and State Estimators for Uncertain/Nonlinear Dynamic Systems," IEE Proc. vol. 140, no.1 pp.41-50, 1993

[16] R. DeCarlo and S. Drakunov, "Sliding Mode Control Design via Lyapunov Approach," Proc. 33rd IEEE Conference on CDC, pp.1925-1930, 1994.

[17] W. C. Su S. V. Drakunov, and U. Ozguner, "Constructing Discontinuity Surfaces for Variable Structure Systems: A Lyapunov Approach," Automatica, vol.32 no.6 pp.925-928,1996

[18] J. Ackermann and V. I. Utkin, " Sliding Mode Control Design based on Ackermann's Formula," IEEE T. Automatic Control, vol.43, no.9 pp.234-237, 1998.

[19] T. Acarman and U. Ozguner, "Relation of Dynamic Sliding surface Design and High Order Sliding Mode Controllers," Proc. 40th IEEE Conference on CDC, pp.934-939, 2001.

[20] W. J. Cao and J. X. Xu, "Nonlinear Integral-Type Sliding Surface for Both Matched and Unmatched Uncertain Systems," IEEE T. Automatic Control, vol.49, no83 pp1355-1360,2004.

[21] H. H. Choi, " LMI-Based Sliding Surface Design for Integral Sliding Mode Control of Mismatched Uncertain Systems, IEEE T. Automatic Control, vol.52, no.2 pp736-742, 2007.

[22] J. H. Lee, 'A Poof of Utkin's Theorem for a SI Uncertain Linear Case," KIEE (to be appear)

[23] J. H. Lee, 'A Poof of Utkin's Theorem for a SI Uncertain Integral System," KIEE (to be appear)

[24] M. Zohdy, M. S. Fadali, and J. Liu, "Variable Structure Control Using Decomposition," IEEE T. Automatic Control, vol.37, no.10 pp1514-1517, 1992.

저 자 소 개

이 정 훈 (李 政 勳)

1966년 2월 1일생. 1988년 경북대학교 전자공학과 졸업(공학사), 1990년 한국과학기술원 전기 및 전자공학과 졸업(석사). 1995년 한국과학기술원 전기 및 전자공학과 졸업(공박). 현재 2010년 현재 경상대학교 전기전자공학부 제어계측공학과 교수. 1997-1999 경상대학교 제어계측공학과 학과장. 마르퀘스사의 Who's Who in the world 2000년 판에 등재. American Biograhpical Institute(ABI)의 500 Leaders of Influence에 선정.

Tel : +82-591-751-5368

Fax : +82-591-757-3974

E-mail : jhleew@gnu.ac.kr