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SOME APPLICATIONS OF SOFT SET TO BF-ALGEBRAS

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Abstract. In this paper we apply the notion of soft sets introduced by Molodtsov to the theory of BF-algebras. Soft BF-subalgebras and homomorphisms in soft BF-algebras are discussed.

1. Introduction

To solve complicated problem in economics, engineering, and environment, we can't successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which were pointed out in ([8]). Maji et al. ([8]) and Molodtsov ([9]) suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov ([9]) introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are

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progressing rapidly. Maji et al. ([8]) described the application of soft set theory to a decision making problem. Maji et al. ([7]) also studied several operations on the theory of soft sets. Chen et al. ([1]) presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors.

A. Walendziak introduce the notion of BF-algebras, which is a generalization of B-algebras, and investigated some properties of (normal) ideals in BF-algebras. Recently, H. S. Kim and N. R. Kye([12]) studied some properties of (normal, closed) ideals in BF-algebras, especially they showed that any ideal of BF-algebra can be decomposed into the union of some sets, and obtained the greatest closed ideal I^0 of an ideal I of a BF-algebra X contained in I. Moreover, they ([13]) introduced the notion of a quadratic BF-algebra, and obtained that quadratic BF-algebras, quadratic Q-algebras, BG-algebras and B-algebras are equivalent notions on a field X with $\mid X \mid \geq 3$, and hence every quadratic BF-algebra is a BCI-algebra.

In this paper we apply the notion of soft sets by Molodtsov to the theory of BF-algebras. Soft BF-subalgebras and homomorphisms in soft BF-algebras are discussed.

2. Preliminaries

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BF-algebra*([11]) if it satisfies the following conditions:

(I) $(\forall x \in X)\ (x * x = 0)$,

(II) $(\forall x \in X)\ (x * 0 = x)$,

(III) $(\forall x, y \in X)\ (0 * (x * y) = y * x)$.

A BF-algebra X is said to be *commutative* if $x \wedge y = y \wedge x$ for all $x, y \in X$ where $x \wedge y = y * (y * x)$. A commutative BF-algebra will be written by *cBF-algebra* for short. A non-empty subset S of a BF-algebra X is called a *BF-subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A mapping $f : X \to Y$ of BF-algebras is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$. For a homomorphism $f : X \to Y$ of BF-algebras, the *kernel* of f, denoted by $\ker(f)$, is defined to be the set

$$ker(f) := \{x \in X \mid f(x) = 0\}.$$

Let X be a BF-algebra. A fuzzy set $\mu : X \to [0,1]$ is called a *fuzzy subalgebra* of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

Molodtsov ([9]) defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denote the power set of U and $A \subset E$.

Definition 2.1. ([9]) A pair (F, A) is called a *soft set* over U, where F is a mapping given by

$$F : A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in ([9]).

Definition 2.2. ([7]) Let (F, A) and (G, B) be two soft sets over a common universe U. The *intersection* of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions:

(i) $C = A \cap B$,

(ii) $(\forall e \in C)$ $(H(e) = F(e)$ or $G(e)$, (as both are same set)).

In this case, we write $(F, A)\widetilde{\cap}(G, B) = (H, C)$.

Definition 2.3. ([7]) Let (F, A) and (G, B) be two soft sets over a common universe U. The *union* of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions:

(i) $C = A \cup B$,

(ii) for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

In this case, we write $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 2.4. ([7]) If (F, A) and (G, B) are two soft sets over a common universe U, then "(F, A) AND (G, B)" denoted by $(F, A)\widetilde{\wedge}(G, B)$ is defined by $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 2.5. ([7]) For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a *soft subset* of (G, B), denoted by $(F, A)\widetilde{\subset}\,(G, B)$, if it satisfies:

(i) $A \subset B$,

(ii) For every $\varepsilon \in A$, $F(\varepsilon)$ and $G(\varepsilon)$ are identical approximations.

3. Soft BF-algebras

In what follows let X and A be a BF-algebra and a non-empty set, respectively. We refer R to an arbitrary binary relation between an element of A and an element of X, that is, R is a subset of $A \times X$ unless otherwise specified. A set-valued function $F : A \to \mathscr{P}(X)$ can be defined as $F(x) = \{y \in X \mid xRy\}$ for all $x \in A$. The pair (F, A) is then a soft set over X. For any element x of a BF-algebra X, we define the order of x, denoted by $o(x)$, as

$$o(x) := \min\{n \in \mathbb{N} \mid 0 * x^n = 0\},$$

where $0 * x^n = (\cdots((0 * x) * x) * \cdots) * x$ in which x appears n-times.

Definition 3.1. Let (F, A) be a soft set over X. Then (F, A) is called a *soft BF-algebra* over X if $F(x)$ is a subalgebra of X for all $x \in A$.

Let us illustrate this definition using the following examples.

Example 3.2. Let $X := \{0, 1, 2, 3, 4\}$ be a BF-algebra with the following Cayley table:

$*$	0	1	2	3	4
0	0	1	2	3	4
1	1	0	1	0	1
2	2	1	0	1	0
3	3	0	1	0	1
4	4	1	0	1	0

Let (F, A) be a soft set over X, where $A = X$ and $F : A \to \mathscr{P}(X)$ is a set-valued function defined by

$$F(x) := \{0\} \cup \{y \in X \mid xRy \iff x \wedge y \in I\}$$

for all $x \in A$ where $I = \{0, 1\}$ and $x^{-1}I = \{y \in X \mid x \wedge y \in I\}$. Then $F(0) = X$, $F(1) = F(3) = \{0, 1, 2, 4\}$, and $F(2) = F(4) = \{0, 1, 3\}$ are subalgebras of X. Therefore (F, A) is a soft BF-algebra over X.

Example 3.3. Consider a BF-algebra $X := \{0, a, b, c, d\}$ with the following Cayley table:

$*$	0	a	b	c	d
0	0	d	c	b	a
a	a	0	d	c	b
b	b	a	0	d	c
c	c	b	a	0	d
d	d	c	b	a	0

Let $A = X$ and let $F : A \to \mathscr{P}(X)$ be a set-valued function defined as follows:

$$F(x) := \{y \in X \mid xRy \Longleftrightarrow y = x^n, n \in \mathbb{N}\}$$

for all $x \in A$ where $x^n = (\cdots((x * x) * x) * \cdots) * x$ in which x appears n-times. Then $F(0) = \{0\}$, $F(a) = F(b) = F(c) = F(d) = \{0, a, b, c, d\}$ which are subalgebras of X. Hence (F, A) is a soft BF-algebra. If we define a set-valued function $G : A \to \mathscr{P}(X)$ by

$$G(x) := \{y \in X \mid xRy \Longleftrightarrow y = 0 * x^n, n \in \mathbb{N}\}$$

for all $x \in A$, where $0 * x^n = (\cdots((0 * x) * x) * \cdots) * x$ in which x appears n-times. Then $G(0) = \{0\}$, $G(a) = G(b) = G(c) = G(d) = \{0, a, b, c, d\}$ which are subalgebras of X. Hence (G, A) is a soft BF-algebra. If we define a set-valued function $H : A \to \mathscr{P}(X)$ by

$$H(x) := \{y \in X \mid xRy \Longleftrightarrow o(x) = o(y)\}$$

for all $x \in A$, then $H(0) = \{0\}$ is a subalgebra of X, but $H(a) = H(b) = H(c) = H(d) = \{a, b, c, d\}$ is not a subalgebra of X. This shows that there exists a set-valued function $H : A \to \mathscr{P}(X)$ such that the soft set (H, A) is not a soft BF-algebra over X.

Example 3.4. Let $X := \{0, a, b, c\}$. Consider the following Cayley table:

$*$	0	a	b	c
0	0	a	b	c
a	a	0	b	c
b	b	b	0	a
c	c	c	a	0

Then $(X; *, 0)$ is a BF-algebra. Let (F, A) be a soft set over X, where $A = X$ and $F : A \to \mathscr{P}(X)$ is a set-valued function defined as follows:

$$F(x) := \{y \in X \mid xRy \Longleftrightarrow o(x) = o(y)\}$$

for all $x \in A$. Then $F(0) = \{0\}$ is a subalgebra of X, but $F(a) = F(b) = F(c) = \{a, b, c\}$ is not a subalgebras of X. Hence (F, A) is not a soft BF-algebra over X. If we take $B = \{0\} \subset X$ and if we define a set-valued function $G : B \to \mathscr{P}(X)$ by

$$G(x) := \{y \in X \mid xRy \Longleftrightarrow o(x) = o(y)\}$$

for all $x \in B$, then (G, B) is a soft BF-algebra over X since $G(0) = \{0\}$ is a subalgebra of X.

Let A be a subalgebra of X with membership function μ_A. Let us consider the family of α -level sets for the function μ_A given by

$$F(\alpha) := \{x \in X \mid \mu_A(x) \geq \alpha\}, \alpha \in [0, 1].$$

Then $F(\alpha)$ is a subalgebra of X. If we know the family F, we can find the function $\mu_A(x)$ by means of the following formula:

$$\mu_A(x) := \sup\{\alpha \in [0, 1] \mid x \in F(\alpha)\}$$

Thus, every subalgebra A may be considered as the soft BF-algebra $(F, [0, 1])$.

Theorem 3.5. *Let (F, A) be a soft BF-algebra over X. If B is a subset of A, then $(F|_B, B)$ is a soft BF-algebra over X, where $F|_B$ is the restriction of B.*

Proof. Straightforward. □

The following example shows that there exists a soft set (F, A) over X such that

(i) (F, A) is not a soft BF-algebra over X.

(ii) there exists a subset B of A such that $(F|_B, B)$ is a soft BF-algebra over X.

Example 3.6. Let (F, A) be a soft set over X given in Example 3.4. Note that (F, A) is not a soft BF-algebra over X. But if we take $B = \{0\} \subset A$, then $(F|_B, B)$ is a soft BF-algebra over X.

Theorem 3.7. *Let (F, A) and (G, B) be two soft BF-algebras over X. If $A \cap B \neq \emptyset$, then the intersection $(F, A)\widetilde{\cap}(G, B)$ is a soft BF-algebra over X.*

Proof. Using Definition 2.2, we can write $(F, A)\widetilde{\cap}(G, B) = (H, C)$, where $C = A \cap B$ and $H(x) = F(x)$ or $G(x)$ for all $x \in C$. Note that $H : C \to \mathscr{P}(X)$ is a mapping, and therefore (H, C) is a soft set over X. Since (F, A) and (G, B) are soft BF-algebras over X, it follows that $H(x) = F(x)$ is a subalgebra of X, or $H(x) = G(x)$ is a subalgebra of X for all $x \in C$. Hence $(H, C) = (F, A)\widetilde{\cap}(G, B)$ is a soft BF-algebra over X. □

Corollary 3.8. *Let (F, A) and (G, A) be two soft BF-algebras over X. Then their intersection $(F, A)\widetilde{\cap}(G, A)$ is a soft BF-algebra over X.*

Proof. Straightforward. □

Theorem 3.9. *Let (F, A) and (G, B) be two soft BF-algebras over X. If A and B are disjoint, then the union $(F, A)\widetilde{\cup}(G, B)$ is a soft BF-algebra over X.*

Proof. Using Definition 2.3, we can write $(F, A)\widetilde{\cup}(G, B) = (H, C)$, where $C = A \cup B$ and for every $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B, \\ G(e) & \text{if } e \in B \setminus A, \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

Since $A \cap B = \emptyset$, either $x \in A \setminus B$ or $x \in B \setminus A$ for all $x \in C$. If $x \in A \setminus B$, then $H(x) = F(x)$ is a subalgebra of X, since (F, A) is a soft BF-algebra over X. If $x \in B \setminus A$, then $H(x) = G(x)$ is a subalgebra of X, since (G, B) is a soft BF-algebra over X. Hence $(H, C) = (F, A)\widetilde{\cup}(G, B)$ is a soft BF-algebra over X. □

Theorem 3.10. *If (F, A) and (G, B) are soft BF-algebras over X, then $(F, A)\widetilde{\wedge}(G, B)$ is a soft BF-algebra over X.*

Proof. By Definition 2.4, we know that

$$(F, A)\widetilde{\wedge}(G, B) = (H, A \times B),$$

where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$. Since $F(x)$ and $G(y)$ are subalgebras of X, the intersection $F(x) \cap G(y)$ is also a subalgebra of X. Hence $H(x, y)$ is a subalgebra of X for all $(x, y) \in A \times B$, and therefore $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$ is a soft BF-algebra over X. □

Definition 3.11. A soft BF-algebra (F, A) over X is said to be *trivial* (resp., *whole*) if $F(x) = \{0\}$ (resp., $F(x) = X$) for all $x \in A$.

Example 3.12. Consider the BF-algebra $X := \{0, a, b, c\}$ in Example 3.3. For $A := \{a, b, c\}$, let $F : A \to \mathscr{P}(X)$ be a set-valued function defined by

$$F(x) = \{0\} \cup \{y \in X \mid xRy \Leftrightarrow o(x) = o(y)\}$$

for all $x \in A$. Then $F(x) = X$ for all $x \in A$, and so (F, A) is a whole soft BF-algebra over X.

4. Soft BF-subalgebras

Definition 4.1. Let (F, A) and (G, B) be two soft BF-algebras over X. Then (F, A) is called a *soft BF-subalgebra* of (G, B), denoted by $(F, A)\widetilde{<}(G, B)$, if it satisfies:

(i) $A \subset B$,
(ii) $F(x)$ is a subalgebra of $G(x)$ for all $x \in A$.

Example 4.2. Let (F, A) be a soft BF-algebra over X which is given in Example 3.2. Let $B := \{1, 2, 3\}$ be a subset of A and let $G : B \to \mathscr{P}(X)$ be a set-valued function defined by

$$G(x) = \{0\} \cup \{y \in X \mid xRy \iff x \wedge y \in I\}$$

for all $x \in B$, where $I = \{0, 1\}$ and $x^{-1}I = \{y \in X \mid x \wedge y \in I\}$. Then $G(1) = G(3) = \{0, 1, 2, 4\}$ and $G(2) = \{0, 1, 3\}$ are subalgebras of $F(1) = F(3)$, and $F(2)$, respectively. Hence (G, B) is a soft BF-subalgebra of (F, A).

Theorem 4.3. *Let (F, A) and (G, A) be two soft BF-algebras over X.*

(i) *If* $F(x) \subset G(x)$ *for all* $x \in A$*, then* $(F,A)\widetilde{<}(G,A)$.
(ii) *If* $B = \{0\}$ *and* (F,B), (F,X) *are soft* BF*-algebras over* X*, then* $(F,B)\widetilde{<}(F,X)$.

Proof. Straightforward. □

Theorem 4.4. *Let* (F,A) *be a soft* BF*-algebra over* X *and let* (G_1,B_1) *and* (G_2,B_2) *be soft* BF*-subalgebras of* (F,A)*. Then*
(i) $(G_1,B_1)\widetilde{\cap}(G_2,B_2)\widetilde{<}(F,A)$.
(ii) $B_1 \cap B_2 = \emptyset \Longrightarrow (G_1,B_1)\widetilde{\cup}(G_2,B_2)\widetilde{<}(F,A)$.

Proof. (i) Using Definition 2.2, we can write

$$(G_1,B_1)\widetilde{\cap}(G_2,B_2) = (G,B),$$

where $B = B_1 \cap B_2$ and $G(x) = G_1(x)$ or $G_2(x)$ for all $x \in B$. Obviously, $B \subset A$. Let $x \in B$. Then $x \in B_1$ and $x \in B_2$. If $x \in B_1$, then $G(x) = G_1(x)$ is a subalgebra of $F(x)$ since $(G_1,B_1)\widetilde{<}(F,A)$. If $x \in B_2$, then $G(x) = G_2(x)$ is a subalgebra of $F(x)$ since $(G_2,B_2)\widetilde{<}(F,A)$. Hence $(G_1,B_1)\widetilde{\cap}(G_2,B_2) = (G,B)\widetilde{<}(F,A)$

(ii) Assume that $B_1 \cap B_2 = \emptyset$. We can write $(G_1,B_1)\widetilde{\cup}(G_2,B_2) = (G,B)$ where $B = B_1 \cup B_2$

$$G(x) = \begin{cases} G_1(x) & \text{if } x \in B_1 \setminus B_2, \\ G_2(x) & \text{if } x \in B_2 \setminus B_1, \\ G_1(x) \cup G_2(x) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

for all $x \in B$. Since $(G_i,B_i)\widetilde{<}(F,A)$ for $i = 1,2$, $B = B_1 \cup B_2 \subset A$ and $G_i(x)$ is a subalgebra of $F(x)$ for all $x \in B_i$, $i = 1,2$. Since $B_1 \cap B_2 = \emptyset$, $G(x)$ is a subalgebra of $F(x)$ for all $x \in B$. Therefore $(G_1,B_1)\widetilde{\cup}(G_2,B_2) = (G,B)\widetilde{<}(F,A)$. □

5. Homomorphisms in soft BF-algebras

Let $f : X \to Y$ be a mapping of BF-algebras. For a soft set (F,A) over X, $(f(F),A)$ is a soft set over Y where $f(F) : A \to \mathscr{P}(Y)$ is defined by $f(F)(x) := f(F(x))$ for all $x \in A$.

Lemma 5.1. *Let* $f : X \to Y$ *be a homomorphism of* BF*-algebras. If* (F,A) *is a soft* BF*-algebra over* X*, then* $(f(F),A)$ *is a soft* BF*-algebra over* Y*.*

Proof. For every $x \in A$, we have $f(F)(x) = f(F(x))$ is a subalgebra of Y since $F(x)$ is a subalgebra of X and its homomorphic image is also a subalgebra of Y. Hence $(f(F), A)$ is a soft BF-algebra over Y. □

Theorem 5.2. *Let $f : X \to Y$ be an homomorphism of BF-algebras and let (F, A) be a soft BF-algebra over X.*

(i) *If $F(x) = \ker(f)$ for all $x \in A$, then $(f(F), A)$ is the trivial soft BF-algebra over Y.*
(ii) *If f is onto and (F, A) is whole, then $(f(F), A)$ is the whole soft BF-algebra over Y.*

Proof. (i) Assume that $F(x) = \ker(f)$ for all $x \in A$. Then $f(F)(x) = f(F(x)) = \{0_Y\}$ for all $x \in A$. Hence $(f(F), A)$ is the trivial soft BF-algebra over Y by Lemma 5.1 and Definition 3.11.

(ii) Suppose that f is onto and (F, A) is whole. Then $F(x) = X$ for all $x \in A$, and so $f(F)(x) = f(F(x)) = f(X) = Y$ for all $x \in A$. It follows from Lemma 5.1 and Definition 3.11 that $(f(F), A)$ is the whole soft BF-algebra over Y. □

Theorem 5.3. *Let $f : X \to Y$ be a homomorphism of BF-algebras and let (F, A) and (G, B) be soft BF-algebras over X. Then*

$$(F, A)\widetilde{<}(G, B) \Rightarrow (f(F), A)\widetilde{<}(f(G), B).$$

Proof. Assume that $(F, A)\widetilde{<}(G, B)$. Let $x \in A$. Then $A \subset B$ and $F(x)$ is a subalgebra of $G(x)$. Since f is a homomorphism, $f(F)(x) = f(F(x))$ is a subalgebra of $f(G(x)) = f(G)(x)$, and hence $(f(F), A)\widetilde{<}$ $(f(G), B)$. □

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