Convergence Characteristics of the Normalized Blind Equalization Algorithm

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Abstract— **We derived Stop-and-go normalized DD, dual-mode normalized Sato, dual-mode NCMA blind equalization algorithm for complex data in this research. And then, the convergence characteristics of the proposed SG-NDD, dual-mode NSato blind equalization algorithms are compared with those of SG-DD, dual-mode Sato algorithms. In general, the normalized blind equalization algorithms have better convergence characteristics than the conventional algorithms.**

Index Terms— **Convergence characteristics of blind equalization algorithm, Stop-and-go normalized DD, dual-mode NCMA blind equalization algorithm, Sato algorithm.**

I. INTRODUCTION

The blind equalizer, to avoid initial training period has Sato Algorithm and Godard Algorithm as basic algorithms, and calls Constant Modulus Algorithm (CMA) especially in case the coefficients of Godard Algorithm has the second order[1-3].

But, this Sato Algorithm and Constant Modulus Algorithm continue to improve its poor convergence characteristics through the many studies.

The algorithm proposed by Picchi and Prati is to improve blind convergence characteristics containing simple of Decision-Directed (DD) algorithm, is to stop adaptive procedure if the reliability of output error decided by the above conditions is not enough. And it is the decision algorithm general DD algorithm is to the present decision is whether the output error use efficiently or not with a binary flag.

The stop-and-go DD (SG-DD) algorithm, stop-and-go scheme based code of the Sato error is proposed.[5] on the one hand, Ready is proposed toward radius method shaped of combined CMA with DD algorithm and Weerackody is proposed a new adaptive dual mode blind equalization algorithm to has a different operation according to each decision area.

Recently, it is proposed NDD algorithm as a normalized NCMA algorithm and DD algorithm with a application of the improved Least Mean Square (LMS) algorithm, Normalized Least Mean Square algorithm (NLMS)[7].

In this research, we describe DD and NCMA algorithm as a normalized algorithm of an improved convergence characteristics of DD and CMA. Also, we propose the blind equalization algorithm, applied to the SG-DD and dual mode algorithm. And, we compared convergence characteristics of the proposed algorithm with that's of the existing algorithms.

II. NORMALIZED CMA BLIND EQUALIZATION ALGORITHM

The CMA to adaptive the initial adaptive filter coefficients minimize modulus error J. where, A is the target modulus and the output of equalizer represented equation (2).

$$J = E[||y(n)|^2 - A^2)|^2] \tag{1}$$

$$y(n) = \sum_{k=0}^{N-1} w_n(k)x(n-k) = \underline{W}_n^t \underline{X}_n \tag{2}$$

In equation (2), $x(n)$ is the equalizer input, $w_n(k)$ is the adaptive filter tap, $\underline{X}_n = [x(n), x(n-1), \cdots, x(n-N+1)]^t$ and $\underline{W}_n = [W_n(0), W_n(1), \cdots, W_n(N-1)]^t$. The renewal of the filter coefficients are represented as follow.

$$w_k(n+1) = w_k(n) - 4\mu(|y(n)|^2 - A^2)y(n)x^*(n-k)$$
$$= w_k(n) - \mu Kx^*(n-k) \tag{3}$$

$$K = 4(|y(n)|^2 - A^2)y(n) \tag{4}$$

Or

$$\underline{W}_{n+1} = \underline{W}_n - 4\mu(|y(n)|^2 - A^2)y(n)\underline{X}_n^*$$
$$= \underline{W}_n - \mu K \underline{X}_n^* \tag{5}$$

Where, μ is step size, and is obtained with the constant as experimental experience. Also, the optimum convergence rates applied with a normalized least mean square analysis method is obtained in case posterior error ε_n is made to zero.

$$\varepsilon_n = |\underline{W}_{n+1}^t \underline{X}_n|^2 - A^2 \tag{6}$$

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Equation (7) is determined by equation (5) and equation (6), and equation (8) is obtained as solving to μ.

$$|K|^2|\underline{X}_n|^2 - \mu^2 - 2y^*(n)K\|\underline{X}_n|^2\mu + |y(n)|^2 - A^2 = 0 \quad (7)$$

$$\mu_{\pm} = \frac{y^*(n)K \pm A|K|}{|K|^2|\underline{X}_n|^2} \quad (8)$$

The smallest positive solution that K in equation (4) applied to equation (8) is as follow, and it is step size of NCMA.

$$\mu_{+\min} = \frac{|y(n)|^2 - A|y(n)|}{4|y(n)|^2(|y(n)|^2 - A^2\|\underline{X}_n|^2} \quad (9)$$

III. NORMALIZED STOP-AND-GO ALGORITHM

We defined weighting vector are represented as follow, to apply into the Decision-Directed equalization with the normalized scheme presented in section Ⅱ.

$$w_n(k+1) = w_n(n) - \mu\varepsilon_{DD}(n)x^*(n-k) \quad (10)$$

Or

$$\underline{W}_{n+1} = \underline{W}_n - \mu\varepsilon_{DD}\underline{X}_n^* \quad (11)$$

Where, $\breve{p} = \arg\min|y(n) - \alpha_p|$, $\varepsilon_{DD}(n) = y(n) - \alpha_{\breve{p}}$. It is obtained equation (12) if posterior error ε_n is made to zero. And it is step size of NDD in equation (13), NDD algorithm presented by equation (14).

$$w_k(n+1)X^T(n) = \alpha_p, \quad p = 0,1,\cdots,p-1 \quad (12)$$

$$\mu = \frac{1}{\|\underline{X}_n\|^2} \quad (13)$$

$$w_k(n+1) = w_k(n) - \frac{1}{\|\underline{X}_n\|^2}\varepsilon_{DD}(n)x^*(n-k) \quad (14)$$

The Sato algorithm presented by equation (15) and equation (16) in QAM signal.

$$w_k(n+1) = w_k(n) - \mu_{sato}\varepsilon_{sato}(n)x^*(n-k) \quad (15)$$

$$\varepsilon_{sato}(n) = y(n) - \gamma c\,\mathrm{sgn}[y(n)] \quad (16)$$

Equation (14) divide by real part and imaginary part, and if it arrange as stop-and-go algorithm with flag constants, it is obtained equation (17) and equation (18). Where, if equation (13) applied to equation (17) and equation (18), we can obtain proposed SG-NDD formula in this research.

$$\underline{w}_R(n+1) = \underline{w}_n(n) - \mu[f_R(n)\underline{y}_R(n)e_R^D(n) + f_I(n)\underline{y}_I(n)e_I^D(n)] \quad (17)$$

$$\underline{w}_I(n+1) = \underline{w}_I(n) - \mu[f_R(n)\underline{y}_I(n)e_R^D(n) - f_I(n)\underline{y}_R(n)e_I^D(n)] \quad (18)$$

$$f_J(i) = \begin{cases} 1, if\ \mathrm{sgn}[e_J^D(n)] = \mathrm{sgn}[e_J^S(n)] \\ 0, otherwise \end{cases}$$

IV. NORMALIZED DUAL MODE SHAPED ALGORITHM

A. Normalized dual mode Sato algorithm

Similar to stop-and-go algorithm, dual mode shaped Sato algorithm obey to a good convergence decision directed method within some range. It is method to improve convergence characteristics as obeying Sato algorithm in other cases.

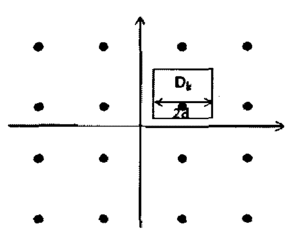

Fig. 1 Decision area of dual mode Sato's algorithm for 16-QAM

As shown figure 1, the error signal is equation (19) if the output *y(n)* exist within decision range. If not, it is equation (20) respectively.

$$\varepsilon = \{y_R(n) - d_R(n)\} + j\{y_I(n) - d_I(n)\} \quad (19)$$

$$\varepsilon = \{y_R(n) - \gamma\mathrm{sgn}(y_R(n))\} + j\{y_I(n) - \gamma\mathrm{sgn}d_I(n))\} \quad (20)$$

Therefore, the renewal of the weighting value for 16-QAM is as follow.

$$\underline{w}(n+1) = \underline{w}(n) - \mu[(y_R(n) - d_R(n)) + j(y_I(n) - d_I(n))]\underline{X}^n(n), \quad y(n) \in D_k \quad (21)$$

$$\underline{w}(n+1) = \underline{w}(n) - \mu[(y_R(n) - \gamma\,\mathrm{sgn}\,y_R(n)) + j(y_I(n) - \gamma\mathrm{sgn}(y_I(n))]\underline{X}^*(n), \quad y(n) \notin D_k \quad (22)$$

And, it change into the step size, $\mu = 1/\|\underline{X}_n\|^2$ if the above formula applying into the normalization scheme.

Where, α is the initial condition to converge with a speed in season. if this step size applying into the equation (20) and equation (21), it comes to the proposed normalized dual mode Sato algorithm(Dual-NSato)

B. Normalized dual mode CMA algorithm

As well as Sato algorithm in the prior section, the CMA algorithm can apply into dual mode.

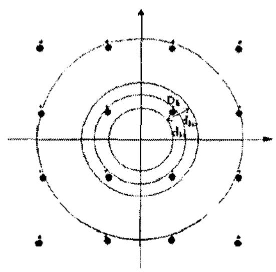

Fig. 2 Decision area of dual mode CMA algorithm for 16-QAM

As shown figure 1, that is, it goes with the specific radius if the output exists within decided radius range. If not, it goes with the original radius. Therefore, the renewal of the weighting value is as follow.

$$\underline{w}(n+1)=\underline{w}(n)-\mu[|y(n)|^2-R_k]y(n)\underline{X}^*(n),\quad y(n)\in D_k \qquad (23)$$

$$\underline{w}(n+1)=\underline{w}(n)-\mu[|y(n)|^2-R]y(n)\underline{X}^*(n),\quad y(n)\notin D_k \qquad (24)$$

Where, R_k has chosen $R_1=0.2$, $R_2=1$ and $R_3=1.8$ in case of 16-QAM and R comes to 1.32. And, d is $d_{ki}=d_{ko}=d$ in $k=1, 2, 3$ if d is lower than 0.2, d have to comes to $d_{ki}=d_{ko}=d$ and $d_{1o}=d$, $d_{1i}=0.2$ in $k=2, 3$ if d is higher than 0.2. Step size change to step size of NCMA if the dual mode algorithm apply into normalization scheme.

$$\mu = \frac{|y(n)|-A}{|y(n)|(|y(n)|^2-A^2)\|\underline{X}_n\|^2} \qquad (25)$$

V. SIMULATIONS AND CONSIDERATIONS

We adopted to verify performance of proposed algorithm that input is the complex QAM signal with uniform distribution, dispersion is 1 and the additional noise is the Gaussian random white noise with mean, dispersion is 1 and SNR is 30[dB] respectively. Channel is linear FIR filter with the minimum phase characteristics. it is as follow.

$$h = h_R + jh_I \qquad (26)$$

$\{h_R\}$ = {-0.005, 0.009, -0.024, 0.854, -0.218, 0.049,-0.016}

$\{h_I\}$ = {-0.004, 0.030, -0.104, 0.520, -0.273, 0.074,-0.020}

The equalizer, $w(n)$ use FIR filter with 8 taps, the step size choose as experimental in each simulation. The convergence characteristics of the algorithm considered using Inter Symbol Interference (ISI) as defining the following formula.

$$ISI(\underline{s}) = \frac{\|\underline{s}\|_2^2 - \|\underline{s}\|_\infty^2}{\|\underline{s}\|_\infty^2} \qquad (27)$$

Where, $\underline{s}$ is vector from doing convolve with channel coefficient vector, $\underline{h}$ and equalizer coefficient vector. $\underline{w}$, is $\|\bullet\|_2$ is l_2-norm, $\|\bullet\|_\infty$ is infinity-norm. In case the perfect equalization, $\underline{s}$ is impulse form. Where, $ISI(\underline{s})$ is zero.

Therefore, it means the equalizer characteristics shows that the higher performance the lower ISI.

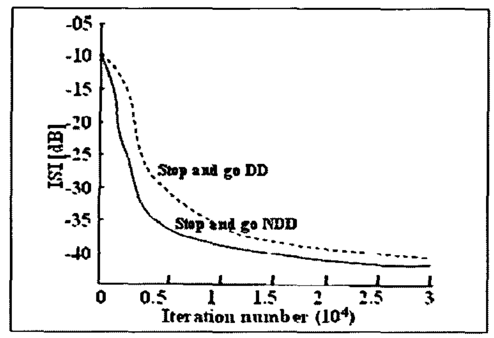

Fig. 3 Convergence characteristics of SG-DD and SG-NDD

Figure 3 shows convergence characteristics of SG-DD and SG-NDD. The step size of SG-DD is 0.00078, the initial value of the step size of SG-DD is 0.01. The iteration is $29,000$, mean of ensemble. We can acknowledged that the characteristic of the normalized has the higher performance than not, as shown Figure 3.

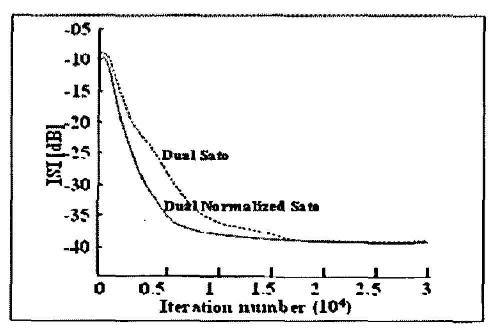

Fig. 4 Convergence characteristics of Dual-Sato and Dual-NSato

The characteristics of the normalized in figure 4 shows that has the higher performance in applying into dual mode under same condition than not. And we applied into dual mode and normalized dual mode to CMA. As results, we can recognize that normalized algorithm has the similar performance than that of dual mode.

VI. CONCLUSIONS

In this research, we described DD and NCMA algorithm as a normalized algorithm of the improved convergence characteristics of DD and CMA. Also, we proposed the blind equalization algorithm, applied into the SG-DD and dual mode algorithm. And, we compared convergence characteristics of the proposed algorithm with that's of the existing algorithm. In this research, normalized scheme apply into stop-and-go algorithm and dual mode blind algorithm.

As results, we verified that normalized algorithm has the higher performance than that of the existing algorithm. But, we can recognize that normalized algorithm has the similar performance than that of dual mode. Therefore, the results of this research will be included in further works.

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