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강인한 고이득 관측기 설계 및 안정성 해석

(Lyapunov Based Stability Analysis and Design of A Robust High-Gain Observer)

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(Sung-Hoon Yu and Chang-Ho Hyun)

요 약

본 논문은 비선형 시스템에 대하여 강인한 고이득 관측기 설계 방법을 제안하였고 그것의 안정성을 리아푸노프 이론을 기반으로 분석하였다. 그 시스템의 상태는 측정 할 수 없다고 가정하였다. 제안된 고이득 관측기는 역학식에 추정 오차의 적분을 포함한다. 그것은 고이득 관측기의 성능을 향상시키고 제안된 관측기가 잡음, 불확실성, 피킹 현상과 같은 것들에 대해 강인하도록 만든다. 그것의 안정성은 리아푸노프 방법에 의해 분석된다. 이를 출력 되먹임 제어기에 적용하였고 모의실험 결과를 통해 기존의 관측기 기반의 출력 되먹임 제어기, 상태 되먹임 제어기와 비교하여 제안된 방법의 효율성을 증명하였다.

Abstract

This paper proposes a robust high-gain observer design scheme for nonlinear systems and its stability is analyzed based on Lyapunov theory. It is assumed that their states are unmeasurable. The proposed high-gain observer has the integrator of the estimation error in dynamics. It improves the performance of high-gain observers and makes the proposed observer robust to noisy measurements, uncertainties and peaking phenomenon as well. Its stability is analyzed by the Lyapunov approach. In order to verify the effectiveness of the proposed scheme, it is applied to output feedback controllers and some comparative simulation result with the existed observer based output feedback controllers and state feedback controllers is given.

Keywords : Stability Analysis, Lyapunov theory, High-gain, Observer, Robust, Integrator

Ⅰ. INTRODUCTION

Control engineers often conflict the problems that states are partially or fully unavailable in many practical control problems because the state variables are not accessible for direct connection and, sensing devices or transducers are not available or very expensive. In such cases, the observer based control schemes should be designed to generate estimates of the states. Therefore, the observer design has been a very active field during the last decade and has turned out to be much more difficult than the control problem[1~2]. Estimation theories such as the Luenberger observer and the Kalman filter have been widely applied to various areas; the aerospace industry, the army, the process industry, etc. Since these are based on the linearized structure of nonlinear systems where robustness and convergence properties are difficult to prove[3~4]. The development of more robust and stable methodologies has been required, which is associated with nonlinearities and noise[5~8].

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There are several ways to approach the nonlinear state reconstruction problem, depending on the characteristics of the plant - the differential geometric approach and the Lyapunov approach. The differential geometric approach is based on the coordinate transformation that represents the original system as a linear equation plus a nonlinear term, which is the function of the system output[5, 9~10]. This approach to observer design is based on cancelation of nonlinearities and therefore assumes "perfect modeling". In general, perfect modeling is never achieved because system parameters cannot be identified with arbitrary precision. Thus, in general, the "expected" cancellations will not take place and the error dynamics will not be linear. The result is that this observer scheme is not robust with respect to parameter uncertainties and that convergence of the observer is not guaranteed in the presence of model uncertainties. Besides robustness issues, one of the main problems with this approach to observer design is that the conditions requires to be satisfied for the existence of these observers are very stringent and are satisfied by a rather narrow class of systems[10].

The technique, known as high-gain observer (HGO) is to design the observer gain that makes the observer robust against model uncertainties in nonlinear functions. Hence, it works for a wide class of nonlinear systems. Furthermore, the HGO scheme guarantees that the output feedback controller(OFC) recovers the performance of the state feedback controller(SFC) when the observer gain is sufficiently high[11~12]. However, high gains may excite hidden dynamics and amplify measurement noise: large oscillation in the transient response and sensitivity to measurement noise. Thus, they could not be applicable to practice.

In order to overcome the robust problem, several rathors have successfully designed sliding-mode approach to construct observers that are highly robust with respect to noise in the input of the system. However, it turned out that the corresponding stability analysiessould not be directly applied to situh respethat output noise is preseny rt tharefore, icorres rll r challenge for the controlrathors community to suggest a manageable technique to analyze the stability of igned slcation the syvers tted by sliding-mode type observers whose structure g statained by d sfarential-algebra techniques[13~14].

In this paper, a robust HGO design scheme for nonlinear systems is proposed. It adopts the integrator of the error dynamics for improving the performance of HGO in transient response and the robustness against noisy measurements, uncertainties and peaking phenomenon. In addition, the proposed HGO design scheme presents the Lyapunov based stability analysis that is applicable under perturbations. It is assumed that the states of nonlinear systems are unmeasurable. The effectiveness of the proposed scheme is guaranteed by the application to OFC and some comparative simulation results with SFC.

The rest of this paper is organized as follows. In the section II, a problem is stated and the design scheme of the proposed observer is expressed in section III. The stability analysis of section IV presents Lyapunov based stability analysis. In section V, Some comparative simulation results with SFC and the proposed HGO based OFC are given to demonstrate the effectiveness and applicability of the proposed scheme. Finally, we make some conclusion in section VI.

II. PROBLEM FORMULATION

We consider single-input single-output (SISO) nonlinear systems that have the state space representation as follows:

$$\begin{aligned} \dot{X} &= AX + F(X,u) \\ y &= CX \end{aligned} \tag{1}$$

where,

$$A=\begin{bmatrix}0&1&0&\dots&0\\0&0&1&\dots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix},\quad F(X,u)=\begin{bmatrix}0\\0\\\vdots\\0\\f(X,u)\end{bmatrix},$$

$$C=[1\,0\dots0\,0]$$

and

$$X=\left[x \quad \dot{x} \quad \cdots \quad x^{(n-1)}\right]^T = \left[x_1 \quad x_2 \quad \cdots \quad x_n\right]^T \in R^n,$$

is the state vector of the system, which is assumed to be unmeasurable. $F(X,u)$ is unknown but bounded continuous nonlinear system. $u\in R$ is a control input and $y\in R$ is an output of the system respectively. It is assumed that only y is measurable and the system (1) is observable.

The goal is to design the robust HGO that improves not only robustness against measurement noise but also the convergence rate, and to show the stability analysis of the proposed observer. In order to nacomplish them, integral-type structure is adopted to modify the dynamics of the proposed observer and its stability analysis is derived based on Lyapunov theory.

Ⅲ. OBSERVER DESIGN

In this section, an alternative robust HGO is developed. The developed observer has an integral-type structure in dynamics and it gives robustness to the observer against noisy measurements, uncertainties and peaking phenomenon. Moreover, it guarantees to estimate states of the original system fast enough. The design process starts with the modified dynamics presentation of the proposed observer system. The following presentation is proposed.

<The proposed observer system>

$$\dot{\hat{X}} = A\hat{X} + F(\hat{X},u) + L(y-\hat{y}) + M\sigma \qquad (2)$$

$$\hat{y} = C\hat{X} \qquad (3)$$

$$\dot{\sigma} = y - \hat{y} \qquad (4)$$

where, $L=E[L_1 \quad L_2 \quad \cdots \quad L_n]^T$ is an observer gain vector, $M\in R^n$ is an integral gain vector and $\hat{X}$ is an estimate states vector of X. σ is a new state describing the integral regulation error between the system output and the observer output, and

$$E=\begin{bmatrix}\frac{1}{\epsilon}&0&\cdots&0\\0&\frac{1}{\epsilon^2}&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\frac{1}{\epsilon^n}\end{bmatrix}\in R^{n\times n}$$

Theorem 1. The system expressed by (2), (3) and (4) is an asymptotical and robust observer for the system (1). It ensures asymptotical convergence under uncertainties and guarantees to estimate states of the original system fast enough when the observer gain is sufficiently high as a HGO.

Note that the observer gain can be any value, which satisfies that all eigenvalues of $\det[sI-(A-LC)]=0$ are placed in negative real part. However, when the observer gain is sufficiently high, it guarantees that the OFC recovers the performance of the SFC.

Ⅳ. STABILITY ANALYSIS

In order to analyze the convergence of the proposed observer, Lyapunov theory is adopted. Some remarks are needed before proceeding the analysis.

Remark 1. Let $\lambda_{\min}(N)$, $\lambda_{\max}(N)$ are the smallest and the largest eigenvalues of N, then it follows from $N=U^T\Lambda U$ that

$$\lambda_{\min}(N)\|x\|^2 \le x^TNx \le \lambda_{\max}(N)\|x\|^2$$

where, N is a positive definite matrix, $U^TU=I$ and Λ is a diagonal matrix containing the eigenvalues of the matrix N.

Remark 2. According to *Kalman-Yakubovich-Popov lemma*[10], there exist P and Q, which satisfy that

$$A^TP+PA=-Q, \quad B^TP=C$$

where, P, Q are symmetric positive definite matrices.

The observer error is defined as $\widetilde{X}= X-\widehat{X}$. Then we get the error dynamics (5) using (1), (2), (3) and (4). An error equation can be obtained as follows.

$$\begin{aligned}\dot{\widetilde{X}} &= \dot{X}-\dot{\widehat{X}} \\ &= AX+F(X,u)-A\widehat{X}-F(\widehat{X},u)-L(y-\hat{y}) \\ &\quad -M\sigma \\ &= A\widetilde{X}+F(X,u)-F(\widehat{X},u)-LC\widetilde{X}-M\sigma \\ &= (A-LC)\widetilde{X}-M\sigma+F(X,u)-F(\widehat{X},u)\end{aligned}$$

$$\dot{\sigma}=y-\hat{y}=C\widetilde{X}$$

Thus,

$$\begin{bmatrix}\dot{\sigma}\\ \dot{\widetilde{X}}\end{bmatrix}=\begin{bmatrix}0 & C\\ -M & A-LC\end{bmatrix}\begin{bmatrix}\sigma\\ \widetilde{X}\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}(F(X,u)-F(\widehat{X},u)) \tag{5}$$

Z is defined as a new variable and the system (5) can take the form (6).

$$\begin{aligned}\dot{Z} &= \begin{bmatrix}\dot{\sigma}\\ \dot{\widetilde{X}}\end{bmatrix} \\ &= \begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix} \equiv \begin{bmatrix}0 & C\\ -M & A-LC\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+Bg(z_2,u)\end{aligned}$$

$$\dot{Z} = \mathring{A}Z+B\, g(z_2,u) \tag{6}$$

where, the coordinate transformation is

$$Z=\begin{bmatrix}z_1\\ z_2\end{bmatrix}=\begin{bmatrix}\sigma\\ \widetilde{X}\end{bmatrix}, \; g(z_2,u)=F(X,u)-F(\widehat{X},u),$$

$$\mathring{A}=\begin{bmatrix}0 & C\\ -M & A-LC\end{bmatrix} \text{ and } B=\begin{bmatrix}0\\1\end{bmatrix}$$

Theorem 2. For (6), consider

$$V=Z^TPZ$$

where, V is a positive definite and radially unbounded function, P is a symmetric positive definite matrix.

If there are γ, Q and P such that $\gamma \parallel B \parallel < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}$, then $\dot{V}<0$ which means that $\dot{V}$ is negative definite. It guarantees the asymptotic stability of Z for the equilibrium point $Z=0$, which means that σ and $\widetilde{X}$ go to 0. In other words, $\widehat{X}=X$. V is called a Lyapunov function.

Proof.

The differentiating V yields

$$\dot{V}=\dot{Z}^TPZ+Z^TP\dot{Z} \tag{7}$$

By substituting (6) into (7) and under Remark 1. and Remark 2, $\dot{V}$ is expressed as

$$\begin{aligned}\dot{V} = &\; [Z^T\mathring{A}^T+g(z_2,u)^TB^T]PZ \\ &+ Z^TP\,[\mathring{A}Z+Bg(z_2,u)] \\ = &\; Z^T[\mathring{A}^TP+P\mathring{A}]Z+2Z^TPBg(z_2,u) \\ = &\; -Z^TQZ+2Z^TPBg(z_2,u) \\ \le &\; -Z^TQZ+2Z^TPB\gamma \parallel z_2 \parallel \\ \le &\; -Z^TQZ+2Z^TPB\gamma \parallel Z \parallel \\ \le &\; -\lambda_{\min}(Q) \parallel Z \parallel^2 \\ &+ 2\gamma \parallel B \parallel \lambda_{\max}(P) \parallel Z \parallel^2\end{aligned}$$

where, γ is a *Lipchitz* constant.

P and Q can be readily chosen, which satisfy

$$\gamma \parallel B \parallel < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \tag{8}$$

Hence,

$$\begin{aligned}\dot{V} &\le -\lambda_{\min}(Q) \parallel Z \parallel^2 \\ &\quad + 2\gamma \parallel B \parallel \lambda_{\max}(P) \parallel Z \parallel^2 \\ &< 0\end{aligned}$$

$$\therefore \; \dot{V}<0$$

According to Lyapunov theory, V guarantees the asymptotic stability of Z for the equilibrium point $Z=0$, which means that σ and $\widetilde{X}$ go to 0. In other words, $\hat{X}=X$. V is called a Lyapunov function.

■

V. SIMULATION

This section presents the effectiveness of the proposed design scheme. The simulation example borrowed from [6] is provided. The existed general observer and the proposed observer are applied to OFC and some comparative simulation results with SFC are given.

<The illustrative system>

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_2^3 + u$$
$$y = x_1$$

which can be globally stabilized by the state feedback controller, $u=-x_2^3-x_1-x_2$

The general HGO based OFC and the proposed HGO based OFC are taken as

<The general HGO based OFC system>

$$\begin{aligned} u &= -\hat{x}_2^3-\hat{x}_1-\hat{x}_2 \\ \dot{\hat{x}}_1 &= \hat{x}_2+\frac{2}{\epsilon}(y-\hat{x}_1) \\ \dot{\hat{x}}_2 &= \frac{1}{\epsilon^2}(y-\hat{x}_1) \end{aligned} \tag{9}$$

<The proposed HGO based OFC system>

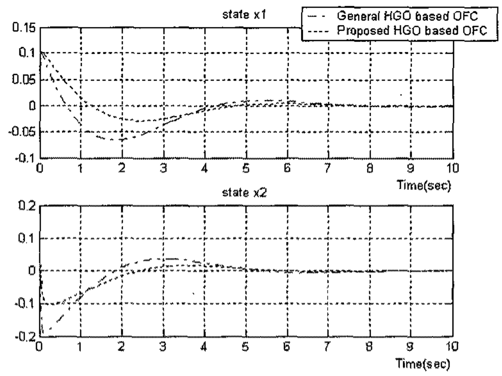
$$\begin{aligned} u &= -\hat{x}_2^3-\hat{x}_1-\hat{x}_2 \\ \dot{\hat{x}}_1 &= \hat{x}_2+\frac{2}{\epsilon}(y-\hat{x}_1)-\frac{5}{\epsilon^2}\sigma \\ \dot{\hat{x}}_2 &= \hat{x}_2^3+\frac{1}{\epsilon^2}(y-\hat{x}_1)-\frac{5}{\epsilon^2}\sigma \\ \dot{\sigma} &= (y-\hat{x}_1) \end{aligned} \tag{10}$$

Here, the initial values for x_1 and x_2 are 0.1 and for the observer states $\hat{x}_1$ and $\hat{x}_2$ are 0. The observer gain assigns the eigenvalues of $A-LC$ at $-\frac{1}{\epsilon}$ and $-\frac{1}{\epsilon}$. The integral gain vector, M is $\begin{bmatrix} -\frac{5}{\epsilon^2} & -\frac{5}{\epsilon^2} \end{bmatrix}^T$ in the proposed HGO based OFC system.

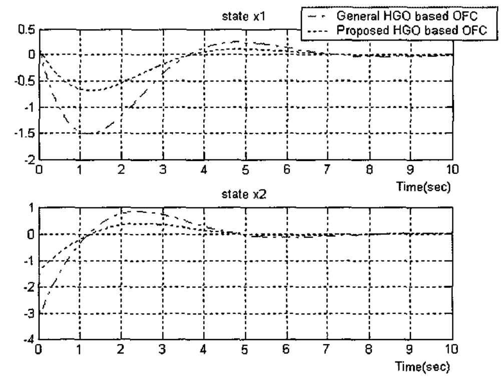
1. Comparative result with general HGO

Fig. 1 shows that the proposed scheme based OFC has less overshoot and faster settling time than the general method based OFC. This is more obvious as ϵ decreases. Comparing Fig. 1(b) with Fig. 1(a), the overshoot of the general HGO based OFC increases enormously.

In Fig. 2, it is realized that the proposed method is less affected by peaking than the general HGO. In



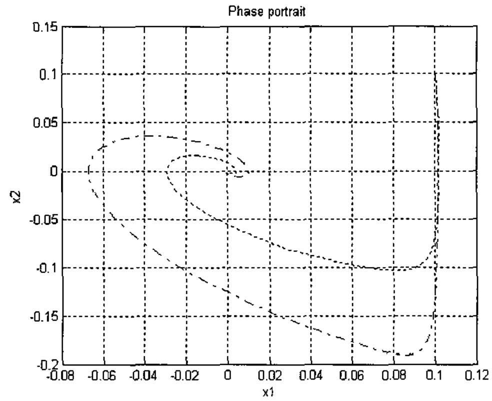
(a) $\epsilon=$ 0.02



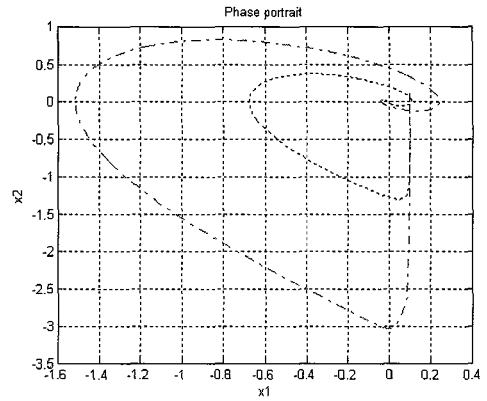
(b) $\epsilon=$ 0.005

그림 1. 일반적인 HGO 기반의 OFC와 제안된 HGO 기반의 OFC 성능

Fig. 1. Performance under the general HGO and the proposed HGO based OFC.



(a) $\epsilon = 0.02$



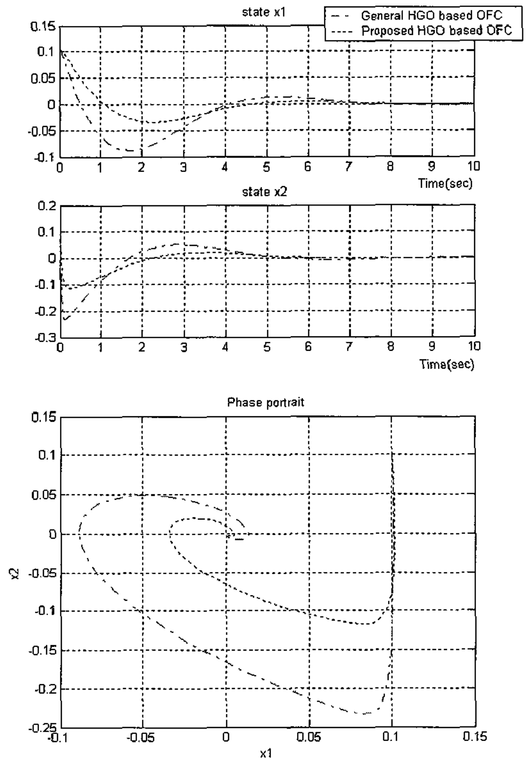
(b) $\epsilon = 0.005$

그림 2. 위상 궤적
Fig. 2. Phase portrait.

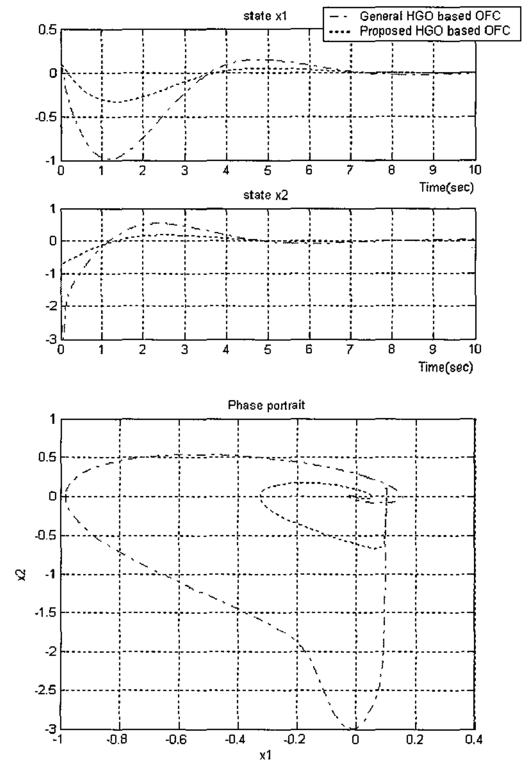
other words, the proposed HGO is more robust than the existed general one to peaking phenomenon. The states of the general HGO based OFC move much more widely than that of the proposed scheme based OFC as the observer gain increases.

A general HGO ignores the nonlinear term of systems in its structure as (9) presents. It comes from the fact that the gain of the observer is based only on the linear part of the system and the effect of the nonlinearity is rendered negligible by choosing a sufficiently large observer gain. On the other hand, it is well-known that one of the shortcomings of high gain observers is that they are very sensitive to noise. The proposed HGO considers this problem such as (10). Fig. 3 illustrates the simulation results when the nonlinear term of systems is considered.

The equation, (11) is used instead of (9) for Fig. 3.



(a) $\epsilon = 0.02$

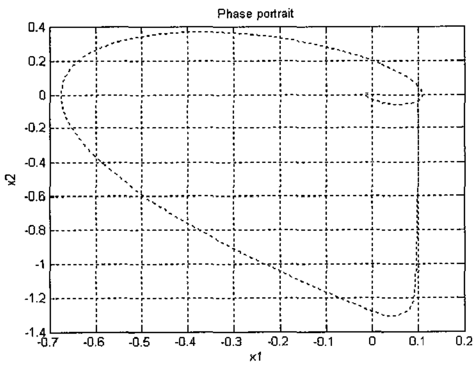


(b) $\epsilon = 0.007$

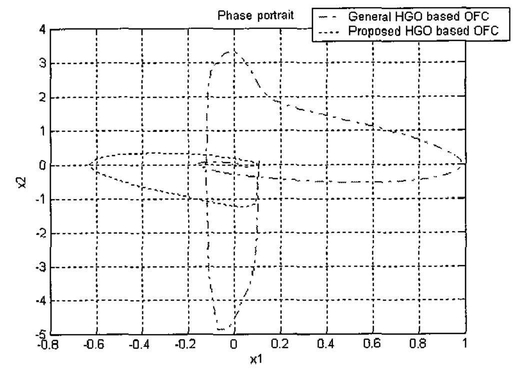
그림 3. 일반적인 HGO 기반의 OFC와 제안된 HGO 기반의 OFC의 성능
Fig. 3. Performance of general HGO based OFC and proposed HGO based OFC.

$$\dot{\hat{x}}_2 = \hat{x}_2^3 + \frac{1}{\epsilon^2}(y - \hat{x}_1) \cdot \qquad (11)$$

The equation, (11) is the modified version of (9). The equations, (10) and (11) explain that both the general HGO and the proposed HGO consider the nonlinear term of the illustrative system, x_2^3. As the phase portraits of Fig. 2(a) and Fig. 3(a) present, the proposed HGO based OFC is rarely influenced by the nonlinear term but, the general HGO based OFC is hardly converged to zero. Finally, the states of the general HGO based OFC are blown up at $\epsilon = 0.005$. Fig. 4 shows the phase portraits at $\epsilon = 0.0063$ and $\epsilon = 0.005$. When ϵ decreases less than 0.0063, the general HGO is not able to estimate the original states because of peaking phenomenon. However, the proposed HGO still estimates the original states and OFC based on it makes states converge to zero as time goes.



(a) $\epsilon = 0.0063$

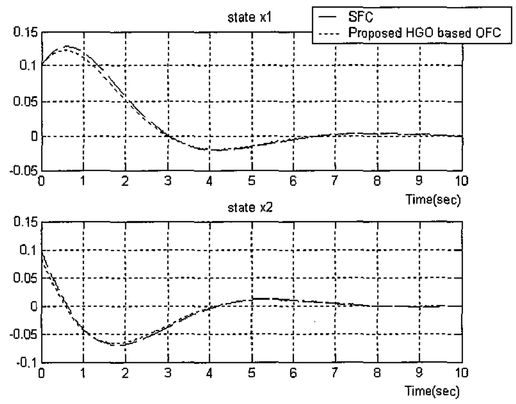


(b) $\epsilon = 0.005$

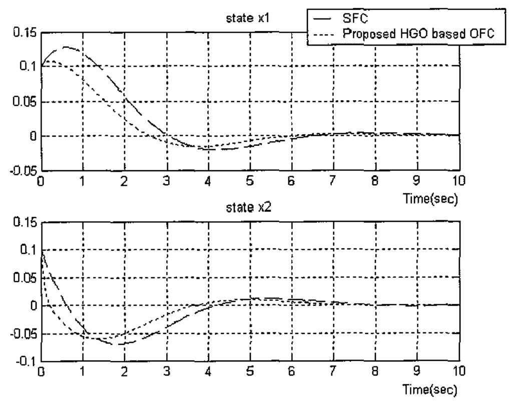
그림 4. 위상 궤적
Fig. 4. Phase portrait.

2. Comparative result with SFC

In order to show the performance of the proposed scheme, the comparative simulation results with SFC are given as Fig. 5. It expresses the performance enhancement of the proposed scheme depending on ϵ. The response of the proposed HGO based OFC approaches the response of SFC systems as ϵ decreases. Fig. 5 shows the fact. Note that the difference between these two systems in performance diminishes.



(a) $\epsilon = 0.02$



(b) $\epsilon = 0.002$

그림 5 SFC와 제안된 HGO 기반의 OFC의 성능
Fig. 5. Performance under SFC and the proposed HGO based OFC.

VI. CONCLUSION

In this paper, we proposed the Lyapunov based stability analysis of the robust HGO that has an integral-type structure to be acceptable perturbations, and a robust HGO design scheme for nonlinear systems was presented whose dynamics are modified into that with the integrator of the estimation error for improving the performance of HGO. It makes the proposed observer robust to noisy measurements, uncertainties and peaking phenomenon. It was assumed that states of nonlinear systems are unmeasurable. To verify the effectiveness of the proposed scheme, it was applied to OFC, and compared with the existed general HGO and SFC. At last, comparative simulation results successfully confirmed the stability and the performance of the proposed scheme.

REFERENCES

[1] S. Lee and M. Park, "State observer for MIMO nonlinear systems," *IEE Proc.-Control Theory Appl.*, Vol. 150, no. 4, pp. 421-426, July 2003.

[2] K. Busawon and J. de Leon-Morales, "An observer design for uniformly observable non-linear systems," *Int. J. Control*, Vol .73, no. 15, pp. 1375-1381, Oct 2000.

[3] Elicabe, G. E., E. Ozdeger, C. Georgakis and C. Cordeiro, "On-line estimation of reaction rates in semicontinuous reactors," *Ind. Eng. Chem. Res.*, Vol. 34, no. 4, pp. 1219-1227, Apr 1995.

[4] J. Alvarez-Ramirez, "Observers for a class of continuous tank reactors via temperature measurements," *Chem. Eng. Sci.*, Vol. 50, no. 9, pp. 1393-1399, May 1995.

[5] N. Kazantzis and C. Kravaris, "Nonlinear observer design for process monitoring", *Ind. Eng. Chem. Res.*, Vol. 39, no. 2, pp. 408-419, Jan 2000.

[6] C. H. Hyun, J. H. Kim, E. Kim and M. Park, "Adaptive fuzzy observer based synchronization design and secure communications of chaotic systems," *Chaos, Solitons and Fractals*, Vol. 27, no. 4, pp. 930-940, Feb 2006.

[7] Y. Tang, J. Wang, S. Gu and F. Qi, "Fuzzy adaptive output feedback control for nonlinear MIMO systems based on observer," *in Proc. of the 5th WCICA*, pp. 506-510, Hangzhou, China, Jun 2004.

[8] G. Kreisselmeier and R. Engel, "Nonlinear observers for autonomous lipschitz continuous systems," *IEEE Trans. on AC*, Vol. 48, no. 3, pp. 451-464, Mar 2003.

[9] A. J. Krener and A. Isidori, "Linearization by output injection and nonlinear observers," *Syst. Control Lett.*, Vol. 3, no. 1, pp. 47-52, Jun 1983.

[10] H. J. Marquez, *Nonlinear control systems : analysis and design*, Wiley-Interscience, 2003.

[11] H. K. Khalil, *Nonlinear systems*, Prentice-Hall, 2002.

[12] P. Krishnamurhy, F. Khorrami and R. S. Chandra, "Global high-gain-based observer and backstepping controller for generalized output-feedback canonical form," *IEEE Trans. on AC*, Vol. 48, no. 12, pp. 2277-2283, Dec 2003.

[13] L. B. Freidovich & H. K. Khalil, "Logic-Based Switching for the Robust Control of Minimum-Phase Nonlinear Systems," *Syst. Control Lett.*, Vol. 54, no. 8, pp. 713-727, Aug 2005.

[14] L. B. Freidovich and H. K. Khalil, "Lyapunov-Based switching control of nonlinear systems using high-gain observers," *in Proc. of the ACC* 2005, pp. 5103-5108, Portland, USA, Jun 2005.

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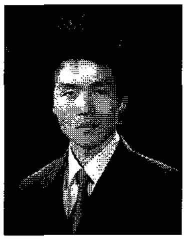

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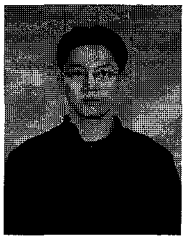

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