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SENSITIVITY ANALYSIS FOR A NEW SYSTEM OF VARIATIONAL INEQUALITIES

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Abstract. In this paper, we study the behavior and sensitivity analysis of the solution set for a new system of generalized parametric multi-valued variational inclusions with (A, η)-accretive mappings in q-uniformly smooth Banach spaces. The present results improve and extend many known results in the literature.

1. Introduction

Sensitivity analysis of solutions of variational inequalities with single-valued mappings have been studied by many authors via quite different techniques.

By using the projection method, Dafermos [2], Yen [12], Mukherjee and Verma [7], Noor [9] and Pan [10] studied the sensitivity analysis of solutions of some variational inequalities with single-valued mappings in finite-dimensional spaces or Hilbert spaces.

By using the resolvent operator technique, Agarwal et al. [1], Jeong [3] studied a new system of parametric generalized nonlinear mixed quasi-variational inclusions in a Hilbert space and in $L_p(p \geq 2)$ spaces, respectively.

In 2008, using the concept and technique of resolvent operators, Lan [4] introduced and studied the behavior and sensitivity analysis of the solution set for a system of generalized parametric (A, η)-accretive variational inclusions in Banach spaces.

Motivated and inspired by the research work going on this field, in this paper, we study the behavior and sensitivity analysis of the solution set for a new system of generalized parametric multi-valued variational inclusions with (A, η)-accretive mappings in q-uniformly smooth Banach spaces. The present results improve and extend many known results in the literature.

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2. Preliminaries

Let E be a real Banach space with dual space E^* and $\langle\cdot,\cdot\rangle$ be the dual pair between E and E^*, $CB(E)$ denote the family of all nonempty closed bounded subsets of E and 2^E denote the family of all the nonempty subsets of E. The generalized duality mapping $J_q : E \to 2^{E^*}$ is defined by

$$J_q(x) = \{f^* \in E^* : \langle x, f^*\rangle = \|x\|^q \quad \text{and} \quad \|f^*\| = \|x\|^{q-1}\}, \forall x \in E,$$

where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-2}J_2(x)$ for all $x \neq 0$ and J_q is single-valued if E^* is strictly convex. If $E = H$ is a Hilbert space, then J_2 becomes the identity mapping of H.

The modulus of smoothness of E is the function $\rho_E : [0,\infty) \to [0,\infty)$ defined by

$$\rho_E(t) = \sup\left\{\frac{1}{2}(\|x+y\| + \|x-y\|) - 1 : \|x\| \leq 1, \|y\| \leq t\right\}.$$

A Banach space E is called uniformly smooth if $\lim_{t\to 0}\frac{\rho_E(t)}{t} = 0$. E is called q-uniformly smooth if there exists a constant $c > 0$ such that $\rho_E(t) \leq ct^q$, $q > 1$. Note that J_q is single-valued if E is uniformly smooth.

We consider now a system of generalized parametric multi-valued variational inclusions with (A,η)-accretive mappings in q-uniformly smooth Banach spaces. To this end, let Ω and Λ be two nonempty open subsets of E in which the parameters ω and λ take values, $U : E \times E \times \Omega \to E$, $V : E \times E \times \Lambda \to E$, $f : E\times\Omega \to E$, $g : E\times\Lambda \to E$ are single-valued mappings and $S : E\times\Omega \to 2^E$, $T : E \times \Lambda \to 2^E$ are multi-valued mappings. Suppose that $M : E \times E \times \Omega \to 2^E$ and $N : E \times E \times \Lambda \to 2^E$ are any nonlinear mappings such that for all $(z,\omega) \in E \times \Omega$, $M(\cdot, z,\omega) : E \to 2^E$ is an (A,η)-accretive mapping with $f(E,\omega) \cap \text{dom}(M(\cdot, z,\omega)) \neq \phi$ and for all $(t,\lambda) \in E \times \Lambda$, $N(\cdot, t,\lambda) : E \to 2^E$ is an (A,η)-accretive mapping with $g(E,\lambda) \cap \text{dom}(N(\cdot, t,\lambda)) \neq \phi$. For each fixed $(\omega,\lambda) \in \Omega \times \Lambda$, the system of generalized parametric multi-valued variational inclusions with (A,η)-accretive mappings in q-uniformly smooth Banach spaces consist of finding $(x,y) \in E \times E$ such that $u \in S(x,\omega)$, $v \in T(y,\lambda)$ and

$$\begin{aligned} &0 \in A(f(x,\omega)) - x + \rho U(x,v,\omega) + \rho M(f(x,\omega),x,\omega), \\ &0 \in A(g(y,\lambda)) - y + \gamma V(u,y,\lambda) + \gamma N(g(y,\lambda),y,\lambda), \end{aligned} \tag{2.1}$$

where $\rho > 0$ and $\gamma > 0$ are two constants.

We now discuss some special cases.

Case I. Let $S : E \times \Omega \to E$ and $T : E \times \Lambda \to E$ be single-valued mappings. Then for each fixed $(\omega,\lambda) \in \Omega\times\Lambda$, the problem (2.1) reduces to finding $(x,y) \in E \times E$ such that

$$\begin{aligned} &0 \in A(f(x,\omega)) - x + \rho U(x,T(y,\lambda),\omega) + \rho M(f(x,\omega),x,\omega), \\ &0 \in A(g(y,\lambda)) - y + \gamma V(S(x,\omega),y,\lambda) + \gamma N(g(y,\lambda),y,\lambda). \end{aligned} \tag{2.2}$$

Case II. Let $A(f(x,\omega)) = x$ for all $(x,\omega) \in E \times \Omega$, $A(g(y,\lambda)) = y$ for all $(y,\lambda) \in E \times \Lambda$ and $\rho = \gamma = 1$. Then problem (2.1) reduces to the problem of finding $(x,y) \in E \times E$ such that

$$\begin{aligned} 0 &\in U(x,v,w) + M(f(x,\omega),x,\omega), \\ 0 &\in V(u,y,\lambda) + N(g(y,\lambda),y,\lambda), \end{aligned}$$

which has been studied by Lan [4].

Case III. Let $A = I$, the identity mapping, $f(x,\omega) = 2x$, $M(x,y,\omega) = M(\frac{1}{2}x,\omega)$ for all $(x,y,\omega) \in E \times E \times \Omega$ and $g(y,\lambda) = 2y$, $N(x,y,\lambda) = N(\frac{1}{2}x,\lambda)$ for all $(x,y,\lambda) \in E \times E \times \Lambda$. Let $U(x,T(y,\lambda),\omega) = G_1(y,\omega) + V_1(y,\omega) - \frac{1}{\rho}y$ and $V(S(x,\omega),y,\lambda) = G_2(x,\lambda) + V_2(x,\lambda) - \frac{1}{\gamma}x$ for all $(x,y,\omega,\lambda) \in E \times E \times \Omega \times \Lambda$, where $G_1, V_1 : E \times \Omega \to E$, $G_2, V_2 : E \times \Lambda \to E$ are nonlinear mappings. Then the problem (2.2) is equivalent to finding $(x,y) \in E \times E$ such that

$$\begin{aligned} 0 &\in x - y + \rho(G_1(y,\omega) + V_1(y,\omega)) + \rho M(x,\omega), \\ 0 &\in y - x + \gamma(G_2(x,\lambda) + V_2(x,\lambda)) + \gamma N(y,\lambda), \end{aligned} \tag{2.3}$$

which was studied by Jeong [3] for m-accretive mappings M, N in (2.3). Further, the problem (2.3) was introduced and studied by Agawal et al. [1] for a Hilbert space $E = H$, two maximal monotone mappings M, N in (2.3).

Remark 2.1. For appropriate and suitable choices of $U, V, M, N, S, T, A, f, g$ and E, it is easy to see that the problem (2.1) includes a number of quasi-variational inclusions, quasi-variational inequalities studied by many authors as special cases (see [1, 2, 3, 4, 7, 9, 10]).

Definition 2.1. Let $A : E \to E$, $\eta : E \times E \to E$ be single-valued mappings. The mapping A is said to be

(i) accretive if

$$\langle A(x) - A(y), J_q(x-y)\rangle \geq 0, \quad \forall x,y \in E;$$

(ii) γ-strongly accretive if

$$\langle A(x) - A(y), J_q(x-y)\rangle \geq \gamma\|x-y\|^q, \quad \forall x,y \in E;$$

(iii) r-strongly η-accretive if

$$\langle A(x) - A(y), J_q(\eta(x,y)) \geq r\|x-y\|^q, \quad \forall x,y \in E.$$

Definition 2.2. Let $A : E \to E$ and $\eta : E \times E \to E$ be single-valued mappings. Then a set-valued mapping $M : E \to 2^E$ is said to be

(i) m-relaxed η-accretive if there exists a constant $m > 0$ such that

$$\langle u - v, J_q(\eta(x,y))\rangle \geq -m\|x-y\|^q, \forall x,y \in E, u \in M(x), v \in M(y);$$

(ii) (A,η)-accretive if

(1) M is m-relaxed η-accretive,

(2) $(A + \rho M)(E) = E$ for every $\rho > 0$.

Definition 2.3. Let $S : E \times \Omega \to 2^E$ be a multi-valued mapping. Then S is called k-H-Lipschitz continuous in the first argument if there exists a constant $k > 0$ such that

$$H(S(x,\omega), S(y,\omega)) \leq k\|x - y\|, \forall x, y \in E, \omega \in \Omega,$$

where $H : 2^E \times 2^E \to (-\infty, \infty) \cup \{+\infty\}$ is the Hausdorff metric, i.e.,

$$H(A,B) = \max\{\sup_{x\in A} \inf_{y\in B} \|x - y\|, \sup_{x\in B} \inf_{y\in A} \|x - y\|\}, \forall A, B \subset 2^E.$$

In a similar way, we can define H-Lipschitz continuity of the mapping $S(\cdot,\cdot)$ in the second argument.

Definition 2.4. A mapping $f : E \times \Omega \to E$ is said to be

(i) δ-strongly accretive with respect to the first argument, $\delta \in (0,1)$, if

$$\langle f(x,\omega) - f(y,\omega), J_q(x - y)\rangle \geq \delta\|x - y\|^q, \forall x, y \in E;$$

(ii) σ-Lipschitz continuous with respect to the first argument if there exists a constant $\sigma > 0$ such that

$$\|f(x,\omega) - f(y,\omega)\| \leq \sigma\|x - y\|, \forall (x, y, \omega) \in E \times E \times \Omega.$$

Definition 2.5. A single-valued mapping $\eta : E \times E \to E$ is said to be τ-Lipschitz continuous if there exists a constant $\tau > 0$ such that

$$\|\eta(x,y)\| \leq \tau\|x - y\|, \quad \forall x, y \in E.$$

If $A : E \to E$ is a strictly η-accretive mapping and $M : E \to 2^E$ is an (A,η)-accretive mapping, then for a constant $\rho > 0$, the resolvent operator associated with A and M is defined by

$$R^{A,\eta}_{M,\rho}(u) = (A + \rho M)^{-1}(u), \quad \forall u \in E.$$

It is well known that $R^{A,\eta}_{M,\rho}$ is a single-valued mapping [5].

Remark 2.2. Since M is an (A,η)-accretive mapping with respect to the first argument, for any fixed $(z,\omega) \in E \times \Omega$, we define

$$R^{A,\eta}_{M(\cdot,z,\omega),\rho}(u) = (A + \rho M(\cdot, z, \omega))^{-1}(u), \quad \forall u \in D(M),$$

which is called the parametric resolvent operator associated with A and M $(\cdot, z, \omega)$.

Now we need some lemmas which will be used in the proofs for the main results in the next section.

Lemma 2.1 ([11]). *Let E be a real uniformly smooth Banach space. Then E is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that for all $x, y \in E$*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, J_q(x)\rangle + c_q\|y\|^q.$$

Lemma 2.2 ([5]). *Let E be a q-uniformly smooth Banach space, $\eta : E\times E \to E$ be a single-valued τ-Lipschitz continuous mapping, $A : E \to E$ be a r-strongly η-accretive mapping and $M : E \to 2^E$ be an (A,η)-accretive mapping. Then the resolvent operator $R^{A,\eta}_{M,\gamma} : E \to E$ is $\frac{\tau^{q-1}}{r-\gamma m}$-Lipschitz continuous, i.e.,*

$$\|R^{A,\eta}_{M,\gamma}(x) - R^{A,\eta}_{M,\gamma}(y)\| \le \frac{\tau^{q-1}}{r-\gamma m}\|x-y\|, \quad \forall x,y \in E.$$

Lemma 2.3 ([6]). *Let (X,d) be a complete metric space and $T_1,T_2 : X \to CB(X)$ be two set-valued contractive mappings with the same constant $\theta \in (0,1)$, i.e.,*

$$H(T_i(x),T_i(y)) \le \theta d(x,y), \quad \forall x,y \in X, i=1,2.$$

Then

$$H(F(T_1),F(T_2)) \le \frac{1}{1-\theta}\sup_{x\in X} H(T_1(x),T_2(x)),$$

where $F(T_1)$ and $F(T_2)$ are fixed point sets of T_1,T_2, respectively.

3. Sensitivity analysis of solution set

Throughout the rest of this paper, we always assume that E is a real q-uniformly smooth Banach space.

Lemma 3.1. *Let $U : E\times E\times\Omega \to E$, $V : E\times E\times\Lambda \to E$, $f : E\times\Omega \to E$ and $g : E\times\Lambda \to E$ be single-valued mappings. Let $S : E\times\Omega \to 2^E$, $T : E\times\Lambda \to 2^E$ be multi-valued mappings. Suppose that $M : E\times E\times\Omega \to 2^E$ and $N : E\times E\times\Lambda \to 2^E$ are any nonlinear mappings such that for all $(z,\omega) \in E\times\Omega$, $M(\cdot,z,\omega) : E \to 2^E$ is an (A,η)-accretive mapping with $f(E,\omega)\cap dom(M(\cdot,z,\omega)) \neq \phi$ and for all $(t,\lambda) \in E\times\Lambda$, $N(\cdot,t,\lambda) : E \to 2^E$ is an (A,η)-accretive mapping with $g(E,\lambda)\cap dom(N(\cdot,t,\lambda)) \neq \phi$. Then for each fixed $(\omega,\lambda) \in \Omega\times\Lambda$, (x,y) is a solution of the system of generalized parametric multi-valued variational inclusions with (A,η)-accretive mappings in q-uniformly smooth Banach spaces (2.1) if and only if there are $x,y \in E$, $u \in S(x,\omega)$, $v \in T(y,\lambda)$ such that*

$$\begin{aligned} f(x,\omega) &= R^{A,\eta}_{M(\cdot,x,\omega),\rho}[x-\rho U(x,v,\omega)], \\ g(y,\lambda) &= R^{A,\eta}_{N(\cdot,y,\lambda),\gamma}[y-\gamma V(u,y,\lambda)], \end{aligned} \tag{3.1}$$

where $R^{A,\eta}_{M,\rho} = (A+\rho M)^{-1}$, $R^{A,\eta}_{N,\gamma} = (A+\gamma N)^{-1}$ and $\rho,\gamma > 0$ are constants.

Proof. The proof directly follows from definition of resolvent operator and some arguments. □

Theorem 3.1. *Let $A : E \to E$, $\eta : E\times E \to E$, $f : E\times\Omega \to E$, $g : E\times\Lambda \to E$ be mappings and $U : E\times E\times\Omega \to 2^E$, $V : E\times E\times\Lambda \to 2^E$, $M : E\times E\times\Omega \to 2^E$, $N : E\times E\times\Lambda \to 2^E$, $S : E\times\Omega \to CB(E)$, $T : E\times\Lambda \to CB(E)$ be set-valued mappings satisfying the following conditions*:

(1) *A is r-strongly η-accretive,*

(2) *η is τ-Lipschitz continuous,*

(3) *f is δ_1-strongly accretive and σ_1-Lipschitz continuous with respect to the first argument,*

(4) *g is δ_2-strongly accretive and σ_2-Lipschitz continuous with respect to the first argument,*

(5) *U is γ_1-strongly accretive, μ_1-Lipschitz continuous with respect to the first argument and μ_2-Lipschitz continuous with respect to the second argument,*

(6) *V is β_1-Lipschitz continuous with respect to the first argument and γ_2-strongly accretive, β_2-Lipschitz continuous with respect to the second argument,*

(7) *M and N are (A,η)-accretive with respect to the first argument,*

(8) *S is k_1-H-Lipschitz continuous with respect to the first argument,*

(9) *T is k_2-H-Lipschitz continuous with respect to the first argument.*

Suppose that

$$\begin{aligned} &\|R^{A,\eta}_{M(\cdot,x,\omega),\rho}(z) - R^{A,\eta}_{M(\cdot,y,\omega),\rho}(z)\| \le \nu_1\|x-y\|,\\ &\|R^{A,\eta}_{N(\cdot,x,\lambda),\gamma}(z) - R^{A,\eta}_{N(\cdot,y,\lambda),\gamma}(z)\| \le \nu_2\|x-y\| \end{aligned} \tag{3.2}$$

for all $(x,y,z,\omega,\lambda) \in E\times E\times E\times\Omega\times\Lambda$ and there exist $\rho>0$ and $\gamma>0$ such that

$$\begin{aligned} &h_1 = \sqrt[q]{1-q\delta_1+c_q\sigma_1^q}+\nu_1,\\ &h_2 = \sqrt[q]{1-q\delta_2+c_q\sigma_2^q}+\nu_2,\\ &\sqrt[q]{1-q\rho\gamma_1+c_q\rho^q\mu_1^q} < \tau^{1-q}(r-\rho m)\Big(1-h_1-\frac{\gamma\tau^{q-1}\beta_1 k_1}{r-\gamma m}\Big),\\ &\sqrt[q]{1-q\gamma\gamma_2+c_q\gamma^q\beta_2^q} < \tau^{1-q}(r-\gamma m)\Big(1-h_2-\frac{\rho\tau^{q-1}\mu_2 k_2}{r-\rho m}\Big), \end{aligned} \tag{3.3}$$

where c_q is the constant as in Lemma 2.1.
Then

(1) *for each $(\omega,\lambda)\in\Omega\times\Lambda$, the system of generalized parametric multi-valued variational inclusions with (A,η)-accretive mapping in q-uniformly smooth Banach space (2.1) has a nonempty solution set $K(\omega,\lambda)$.*

(2) *$K(\omega,\lambda)$ is a closed subset of $E\times E$.*

Proof. From (3.1) we first define mappings $F_1 : E\times E\times E\times\Omega\to E$, $F_2 : E\times E\times E\times\Lambda\to E$ as follows:

$$\begin{aligned} &F_1(x,y,v,\omega) = x - f(x,\omega) + R^{A,\eta}_{M(\cdot,x,\omega),\rho}[x-\rho U(x,v,w)],\\ &F_2(x,y,u,\lambda) = y - g(y,\lambda) + R^{A,\eta}_{N(\cdot,y,\lambda),\gamma}[y-\gamma V(u,y,\lambda)] \end{aligned} \tag{3.4}$$

for all $(x,y,\omega,\lambda)\in E\times E\times\Omega\times\Lambda$.

Now define $\|\cdot\|_1$ on $E\times E$ by

$$\|(x,y)\|_1 = \|x\|+\|y\|, \quad \forall (x,y)\in E\times E.$$

It is well known that $(E \times E, \|\cdot\|_1)$ is a Banach space. For any given $\rho > 0$ and $\gamma > 0$ we can define $F : E \times E \times \Omega \times \Lambda \to 2^E \times 2^E$ by

$$F(x, y, \omega, \lambda) = \{(F_1(x, y, v, \omega), F_2(x, y, u, \lambda)) : u \in S(x, \omega), v \in T(y, \lambda)\}$$

for every $(x, y, \omega, \lambda) \in E \times E \times \Omega \times \Lambda$. Since $S(x, \omega) \in CB(E)$, $T(y, \lambda) \in CB(E)$ and $f, g, U, V, R^{A,\eta}_{M,\rho}, R^{A,\eta}_{N,\gamma}$ are continuous, we have $F(x, y, \omega, \lambda) \in CB(E \times E)$ for every $(x, y, \omega, \lambda) \in E \times E \times \Omega \times \Lambda$.

Now for each fixed $(\omega, \lambda) \in \Omega \times \Lambda$, we prove that $F(x, y, \omega, \lambda)$ is a multi-valued contractive mapping.

In fact, for any $(x_1, y_1, \omega, \lambda)$, $(x_2, y_2, \omega, \lambda) \in E \times E \times \Omega \times \Lambda$ and any $(a_1, a_2) \in F(x_1, y_1, \omega, \lambda)$, there exist $u_1 \in S(x_1, \omega)$, $v_1 \in T(y_1, \lambda)$ such that

$$\begin{aligned} a_1 &= x_1 - f(x_1, \omega) + R^{A,\eta}_{M(\cdot,x_1,\omega),\rho}[x_1 - \rho U(x_1, v_1, \omega)], \\ a_2 &= y_1 - g(y_1, \lambda) + R^{A,\eta}_{N(\cdot,y_1,\lambda),\gamma}[y_1 - \gamma V(u_1, y_1, \lambda)]. \end{aligned}$$

It follows from Nader's theorem [8] that there exist $u_2 \in S(x_2, \omega)$ and $v_2 \in T(y_2, \lambda)$ such that

$$\begin{aligned} \|u_1 - u_2\| &\le H(S(x_1, \omega), S(x_2, \omega)), \\ \|v_1 - v_2\| &\le H(T(y_1, \lambda), T(y_2, \lambda)). \end{aligned} \tag{3.5}$$

Let

$$\begin{aligned} b_1 &= x_2 - f(x_2, \omega) + R^{A,\eta}_{M(\cdot,x_2,\omega),\rho}[x_2 - \rho U(x_2, v_2, \omega)], \\ b_2 &= y_2 - g(y_2, \lambda) + R^{A,\eta}_{N(\cdot,y_2,\lambda),\gamma}[y_2 - \gamma V(u_2, y_2, \lambda)]. \end{aligned}$$

Then we have $(b_1, b_2) \in F(x_2, y_2, \omega, \lambda)$. By (3.2) and Lemma 2.2, we have

$$\begin{aligned} &\|a_1 - b_1\| \\ &\le \|x_1 - x_2 - (f(x_1, \omega) - f(x_2, \omega))\| \\ &\quad + \|R^{A,\eta}_{M(\cdot,x_1,\omega),\rho}[x_1 - \rho U(x_1, v_1, \omega)] - R^{A,\eta}_{M(\cdot,x_2,\omega),\rho}[x_1 - \rho U(x_1, v_1, \omega)]\| \\ &\quad + \|R^{A,\eta}_{M(\cdot,x_2,\omega),\rho}[x_1 - \rho U(x_1, v_1, \omega)] - R^{A,\eta}_{M(\cdot,x_2,\omega),\rho}[x_2 - \rho U(x_2 v_2, \omega)]\| \\ &\le \|x_1 - x_2 - (f(x_1, \omega) - f(x_2, \omega))\| + \nu_1\|x_1 - x_2\| \\ &\quad + \frac{\tau^{q-1}}{r - \rho m}\|x_1 - x_2 - \rho(U(x_1, v_1, \omega) - U(x_2, v_1, \omega))\| \\ &\quad + \frac{\rho\tau^{q-1}}{r - \rho m}\|U(x_2, v_1, \omega) - U(x_2, v_2, \omega)\|, \end{aligned} \tag{3.6}$$

$$\begin{aligned} &\|a_2 - b_2\| \\ &\le \|y_1 - y_2 - (g(y_1, \lambda) - g(y_2, \lambda))\| \\ &\quad + \|R^{A,\eta}_{N(\cdot,y_1,\lambda),\gamma}[y_1 - \gamma V(u_1, y_1, \lambda)] - R^{A,\eta}_{N(\cdot,y_2,\lambda),\gamma}[y_1 - \gamma V(u_1, y_1, \lambda)]\| \\ &\quad + \|R^{A,\eta}_{N(\cdot,y_2,\lambda),\gamma}[y_1 - \gamma V(u_1, y_1, \lambda)] - R^{A,\eta}_{N(\cdot,y_2,\lambda),\gamma}[y_2 - \gamma V(u_2, y_2, \lambda)]\| \end{aligned} \tag{3.7}$$

$$\begin{aligned}&\le \|y_1-y_2-(g(y_1,\lambda)-g(y_2,\lambda))\|+\nu_2\|y_1-y_2\|\\&\quad+\frac{\tau^{q-1}}{r-\gamma m}\|y_1-y_2-\gamma(V(u_1,y_1,\lambda)-V(u_1,y_2,\lambda))\|\\&\quad+\frac{\gamma\tau^{q-1}}{r-\gamma m}\|V(u_1,y_2,\lambda)-V(u_2,y_2,\lambda)\|.\end{aligned}$$

From Lemma 2.1, the δ_1-strongly accretivity and σ_1-Lipschitz continuity of f, and the δ_2-strongly accretivity and σ_2-Lipschitz continuity of g with respect to the first argument we have

$$\begin{aligned}&\|x_1-x_2-(f(x_1,\omega)-f(x_2,\omega))\|^q\\&\le \|x_1-x_2\|^q-q\langle f(x_1,\omega)-f(x_2,\omega),J_q(x_1-x_2)\rangle+c_q\|f(x_1,\omega)-f(x_2,\omega)\|^q\\&\le (1-q\delta_1+c_q\sigma_1^q)\|x_1-x_2\|^q,\end{aligned}\tag{3.8}$$

$$\|y_1-y_2-(g(y_1,\lambda)-g(y_2,\lambda))\|^q\le(1-q\delta_2+c_q\sigma_2^q)\|y_1-y_2\|^q.\tag{3.9}$$

Since U is γ_1-strongly accretive, μ_1-Lipschitz continuous with respect to the first argument and V is γ_2-strongly accretive, β_2-Lipschitz continuous with respect to the second argument,

$$\begin{aligned}&\|x_1-x_2-\rho(U(x_1,v_1,\omega)-U(x_2,v_1,\omega))\|^q\\&\le \|x_1-x_2\|^q-q\rho\langle U(x_1,v_1,\omega)-U(x_2,v_1,\omega),J_q(x_1-x_2)\rangle\\&\quad+c_q\rho^q\|U(x_1,v_1,\omega)-U(x_2,v_1,\omega)\|^q\\&\le(1-q\rho\gamma_1+c_q\rho^q\mu_1^q)\|x_1-x_2\|^q,\end{aligned}\tag{3.10}$$

$$\|y_1-y_2-\gamma(V(u_1,y_1,\lambda)-V(u_1,y_2,\lambda))\|^q\le(1-q\gamma\gamma_2+c_q\gamma^q\beta_2^q)\|y_1-y_2\|^q.\tag{3.11}$$

Since U is μ_2-Lipschitz continuous with respect to the second argument, V is β_1-Lipschitz continuous with respect to the first argument, T is k_2-H-Lipschitz continuous with respect to the first argument and S is k_1-H-Lipschitz continuous with respect to the first argument, we obtain

$$\begin{aligned}\|U(x_2,v_1,\omega)-U(x_2,v_2,\omega)\|&\le\mu_2\|v_1-v_2\|\\&\le\mu_2H(T(y_1,\lambda),T(y_2,\lambda))\\&\le\mu_2k_2\|y_1-y_2\|,\end{aligned}\tag{3.12}$$

$$\|V(u_1,y_2,\lambda)-V(u_2,y_2,\lambda)\|\le\beta_1k_1\|x_1-x_2\|.\tag{3.13}$$

It follows from (3.6)-(3.13) that

$$\begin{aligned}&\|a_1-b_1\|\\&\le\left[\sqrt[q]{1-q\delta_1+c_q\sigma_1^q}+\nu_1+\frac{\tau^{q-1}}{r-\rho m}\sqrt[q]{1-q\rho\gamma_1+c_q\rho^q\mu_1^q}\right]\|x_1-x_2\|\\&\quad+\frac{\rho\tau^{q-1}\mu_2k_2}{r-\rho m}\|y_1-y_2\|\end{aligned}\tag{3.14}$$

$$= \theta_1\|x_1 - x_2\| + \theta_2\|y_1 - y_2\|,$$

$$\begin{aligned} &\|a_2 - b_2\| \\ &\leq \left[\sqrt[q]{1 - q\delta_2 + c_q\sigma_2^q} + \nu_2 + \frac{\tau^{q-1}}{r - \gamma m}\sqrt[q]{1 - q\gamma\gamma_2 + c_q\gamma^q\beta_2^q}\right]\|y_1 - y_2\| \\ &\quad + \frac{\gamma\tau^{q-1}\beta_1 k_1}{r - \gamma m}\|x_1 - x_2\| \\ &= \theta_3\|x_1 - x_2\| + \theta_4\|y_1 - y_2\|, \end{aligned} \tag{3.15}$$

where

$$\begin{aligned} \theta_1 &= \sqrt[q]{1 - q\delta_1 + c_q\sigma_1^q} + \nu_1 + \frac{\tau^{q-1}}{r - \rho m}\sqrt[q]{1 - q\rho\gamma_1 + c_q\rho^q\mu_1^q}, \\ \theta_2 &= \frac{\rho\tau^{q-1}\mu_2 k_2}{r - \rho m}, \\ \theta_3 &= \frac{\gamma\tau^{q-1}\beta_1 k_1}{r - \gamma m}, \\ \theta_4 &= \sqrt[q]{1 - q\delta_2 + c_q\sigma_2^q} + \nu_2 + \frac{\tau^{q-1}}{r - \gamma m}\sqrt[q]{1 - q\gamma\gamma_2 + c_q\gamma^q\beta_2^q}. \end{aligned}$$

By (3.14) and (3.15), we have

$$\|a_1 - b_1\| + \|a_2 - b_2\| \leq \theta(\|x_1 - x_2\| + \|y_1 - y_2\|), \tag{3.16}$$

where $\theta = \max\{\theta_1 + \theta_3, \theta_2 + \theta_4\}$. Hence we have

$$\begin{aligned} d((a_1, a_2), F(x_2, y_2, \omega, \lambda)) &= \inf_{(b_1,b_2)\in F(x_2,y_2,\omega,\lambda)}(\|a_1 - b_1\| + \|a_2 - b_2\|) \\ &\leq \theta(\|x_1 - x_2\| + \|y_1 - y_2\|) \\ &= \theta\|(x_1, y_1) - (x_2, y_2)\|_1 \end{aligned}$$

and

$$d((b_1, b_2), F(x_1, y_1, \omega, \lambda)) \leq \theta\|(x_1, y_1) - (x_2, y_2)\|_1.$$

By the definition of the Hausdorff metric H on $CB(E \times E)$, we have

$$\begin{aligned} &H(F(x_1, y_1, \omega, \lambda), F(x_2, y_2, \omega, \lambda)) \\ &= \max\{\sup_{(a_1,a_2)\in F(x_1,y_1,\omega,\lambda)} d((a_1, a_2), F(x_2, y_2, \omega, \lambda)), \\ &\qquad \sup_{(b_1,b_2)\in F(x_2,y_2,\omega,\lambda)} d((b_1, b_2), F(x_1, y_1, \omega, \lambda))\} \\ &\leq \theta\|(x_1, y_1) - (x_2, y_2)\|_1 \end{aligned} \tag{3.17}$$

for all $(x_1, x_2, y_1, y_2, \omega, \lambda) \in E \times E \times E \times E \times \Omega \times \Lambda$. It follows from condition (3.3) that $\theta < 1$. Thus, (3.17) implies that F is a contractive mapping which is uniform with respect to $(\omega, \lambda) \in \Omega \times \Lambda$. Since $F(x, y, \omega, \lambda)$ is a uniform θ-contractive mapping with respect to $(\omega, \lambda) \in \Omega \times \Lambda$, by the Nadler fixed point theorem [8], $F(x, y, \omega, \lambda)$ has a fixed point $(\bar{x}, \bar{y})$ for each $(\omega, \lambda) \in \Omega \times \Lambda$. From the definition of F there exist $\bar{u} \in S(\bar{x}, \omega)$ and $\bar{v} \in T(\bar{y}, \lambda)$ such that (3.1) holds. By Lemma 3.1, $K(\omega, \lambda) \neq \phi$.

(2) For each $(\omega, \lambda) \in \Omega \times \Lambda$, let $(x_n, y_n) \in K(\omega, \lambda)$ and $x_n \to x_0$, $y_n \to y_0$ as $n \to \infty$. Then we have

$$(x_n, y_n) \in F(x_n, y_n, \omega, \lambda), \quad n = 1, 2, \ldots.$$

By (1), we have

$$H(F(x_n, y_n, \omega, \lambda), F(x_0, y_0, \omega, \lambda)) \leq \theta \|(x_n, y_n) - (x_0, y_0)\|_1.$$

It follows that

$$\begin{aligned} & d((x_0, y_0), F(x_0, y_0, \omega, \lambda)) \\ \leq\ & \|(x_0, y_0) - (x_n, y_n)\|_1 + d((x_n, y_n), F(x_n, y_n, \omega, \lambda)) \\ & + H(F(x_n, y_n, \omega, \lambda), F(x_0, y_0, \omega, \lambda)) \\ \leq\ & (1 + \theta)\|(x_n, y_n) - (x_0, y_0)\|_1 \to 0 \quad \text{as} \quad n \to \infty. \end{aligned}$$

Hence we have $(x_0, y_0) \in F(x_0, y_0, \omega, \lambda)$. From Lemma 3.1 we have $(x_0, y_0) \in K(\omega, \lambda)$. Therefore, $K(\omega, \lambda)$ is a nonempty closed subset of $E \times E$. □

Theorem 3.2. *Under the hypotheses of Theorem 3.1, further assume that for any $x, y \in E$, the mappings $\omega \mapsto U(x, y, \omega)$, $\lambda \mapsto V(x, y, \lambda)$, $\omega \mapsto f(x, \omega)$ and $\lambda \mapsto g(y, \lambda)$ are Lipschitz continuous with constants l_U, l_V, l_f, l_g, respectively. Let $\omega \to S(x, \omega)$ be l_S-H-Lipschitz continuous and $\lambda \mapsto T(y, \lambda)$ be l_T-H-Lipschitz continuous for any $x, y \in E$. Suppose that for any $(t, \omega, \bar{\omega}) \in E \times \Omega \times \Omega$ and $(z, \lambda, \bar{\lambda}) \in E \times \Lambda \times \Lambda$*

$$\begin{aligned} \|R^{A,\eta}_{M(\cdot,x,\omega),\rho}(t) - R^{A,\eta}_{M(\cdot,x,\bar{\omega}),\rho}(t)\| &\leq \xi_1 \|\omega - \bar{\omega}\|, \\ \|R^{A,\eta}_{N(\cdot,y,\lambda),\gamma}(z) - R^{A,\eta}_{N(\cdot,y,\bar{\lambda}),\gamma}(z)\| &\leq \xi_2 \|\lambda - \bar{\lambda}\|, \end{aligned} \tag{3.18}$$

where $\xi_1 > 0$ and $\xi_2 > 0$ are two constants.
Then the solution mapping $K(\omega, \lambda)$ for the system of generalized parametric multi-valued variational inclusions with (A, η)-accretive mapping in q-uniformly smooth Banach spaces (2.1) is Lipschitz continuous from $\Omega \times \Lambda$ to $E \times E$.

Proof. For each $(\omega, \lambda), (\bar{\omega}, \bar{\lambda}) \in \Omega \times \Lambda$, by Theorem 3.1, $K(\omega, \lambda)$ and $K(\bar{\omega}, \bar{\lambda})$ are both nonempty closed subsets. Also, $F(x, y, \omega, \lambda)$ and $F(x, y, \bar{\omega}, \bar{\lambda})$ are contractive mappings with same constant $\theta \in (0, 1)$ and have fixed points $(x(\omega, \lambda), y(\omega, \lambda))$ and $(x(\bar{\omega}, \bar{\lambda}), y(\bar{\omega}, \bar{\lambda}))$, respectively. For any fixed $(\omega, \lambda), (\bar{\omega}, \bar{\lambda}) \in \Omega \times \Lambda$, by Lemma 2.3, we have

$$\begin{aligned} & H(K(\omega, \lambda), K(\bar{\omega}, \bar{\lambda}) \\ \leq\ & \frac{1}{1 - \theta} \sup_{(x,y) \in E \times E} H(F(x(\omega, \lambda), y(\omega, \lambda), \omega, \lambda), F(x(\bar{\omega}, \bar{\lambda}), y(\bar{\omega}, \bar{\lambda}), \bar{\omega}, \bar{\lambda}). \end{aligned} \tag{3.19}$$

For any $(a_1, a_2) \in F(x(\omega,\lambda), y(\omega,\lambda), \omega, \lambda)$ there exist $u(\omega,\lambda) \in S(x(\omega,\lambda),\omega)$, $v(\omega,\lambda) \in T(y(\omega,\lambda),\lambda)$ such that

$$\begin{aligned} a_1 &= x(\omega,\lambda) - f(x(\omega,\lambda),\omega) \\ &\quad + R^{A,\eta}_{M(\cdot,x(\omega,\lambda),\omega),\rho}[x(\omega,\lambda) - \rho U(x(\omega,\lambda), v(\omega,\lambda),\omega)], \\ a_2 &= y(\omega,\lambda) - g(y(\omega,\lambda),\lambda) \\ &\quad + R^{A,\eta}_{N(\cdot,y(\omega,\lambda),\lambda),\gamma}[y(\omega,\lambda) - \gamma V(u(\omega,\lambda), y(\omega,\lambda),\lambda)]. \end{aligned} \tag{3.20}$$

By Nader's theorem [8], there exist

$$u(\bar\omega,\bar\lambda) \in S(x(\bar\omega,\bar\lambda),\bar\omega), v(\bar\omega,\bar\lambda) \in T(y(\bar\omega,\bar\lambda),\bar\lambda)$$

such that

$$\begin{aligned} \|u(\omega,\lambda) - u(\bar\omega,\bar\lambda)\| &\le H(S(x(\omega,\lambda),\omega), S(x(\bar\omega,\bar\lambda),\bar\omega)), \\ \|v(\omega,\lambda) - v(\bar\omega,\bar\lambda)\| &\le H(T(y(\omega,\lambda),\lambda), T(y(\bar\omega,\bar\lambda),\bar\lambda)). \end{aligned} \tag{3.21}$$

Let

$$\begin{aligned} b_1 &= x(\bar\omega,\bar\lambda) - f(x(\bar\omega,\bar\lambda),\bar\omega) \\ &\quad + R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\bar\omega),\rho}[x(\bar\omega,\bar\lambda) - \rho U(x(\bar\omega,\bar\lambda), v(\bar\omega,\bar\lambda),\bar\omega)], \\ b_2 &= y(\bar\omega,\bar\lambda) - g(y(\bar\omega,\bar\lambda),\bar\lambda) \\ &\quad + R^{A,\eta}_{N(\cdot,y(\bar\omega,\bar\lambda),\bar\lambda),\gamma}[y(\bar\omega,\bar\lambda) - \gamma V(u(\bar\omega,\bar\lambda), y(\bar\omega,\bar\lambda),\bar\lambda)]. \end{aligned} \tag{3.22}$$

Then we have

$$(b_1, b_2) \in F(x(\bar\omega,\bar\lambda), y(\bar\omega,\bar\lambda),\bar\omega,\bar\lambda).$$

By (3.18), (3.20), (3.22) and Lemma 2.2, we have

$$\begin{aligned} \|a_1 - b_1\| &\le \|x(\omega,\lambda) - x(\bar\omega,\bar\lambda) - (f(x(\omega,\lambda),\omega) - f(x(\bar\omega,\bar\lambda),\omega))\| \\ &\quad + \|f(x(\bar\omega,\bar\lambda),\omega) - f(x(\bar\omega,\bar\lambda),\bar\omega)\| \\ &\quad + \|R^{A,\eta}_{M(\cdot,x(\omega,\lambda),\omega),\rho}[x(\omega,\lambda) - \rho U(x(\omega,\lambda), v(\omega,\lambda),\omega)] \\ &\qquad - R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\omega),\rho}[x(\omega,\lambda) - \rho U(x(\omega,\lambda), v(\omega,\lambda),\omega)]\| \\ &\quad + \|R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\omega),\rho}[x(\omega,\lambda) - \rho U(x(\omega,\lambda), v(\omega,\lambda),\omega)] \\ &\qquad - R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\omega),\rho}[x(\bar\omega,\bar\lambda) - \rho U(x(\bar\omega,\bar\lambda), v(\bar\omega,\bar\lambda),\omega)]\| \\ &\quad + \|R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\omega),\rho}[x(\bar\omega,\bar\lambda) - \rho U(x(\bar\omega,\bar\lambda), v(\bar\omega,\bar\lambda),\omega)] \\ &\qquad - R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\bar\omega),\rho}[x(\bar\omega,\bar\lambda) - \rho U(x(\bar\omega,\bar\lambda), v(\bar\omega,\bar\lambda),\omega)]\| \\ &\quad + \|R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\bar\omega),\rho}[x(\bar\omega,\bar\lambda) - \rho U(x(\bar\omega,\bar\lambda), v(\bar\omega,\bar\lambda),\omega)] \\ &\qquad - R^{A,\eta}_{M(\cdot,x(\bar\omega,\bar\lambda),\bar\omega),\rho}[x(\bar\omega,\bar\lambda) - \rho U(x(\bar\omega,\bar\lambda), v(\bar\omega,\bar\lambda),\bar\omega)]\| \\ &\le \|x(\omega,\lambda) - x(\bar\omega,\bar\lambda) - (f(x(\omega,\lambda),\omega) - f(x(\bar\omega,\bar\lambda),\omega))\| \\ &\quad + l_f\|\omega - \bar\omega\| + \nu_1\|x(\omega,\lambda) - x(\bar\omega,\bar\lambda)\| \end{aligned} \tag{3.23}$$

$$
\begin{aligned}
&+ \frac{\tau^{q-1}}{r-\rho m}\|x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda}) - \rho(U(x(\omega,\lambda), v(\omega,\lambda),\omega) \\
&\qquad - U(x(\bar{\omega},\bar{\lambda}), v(\omega,\lambda),\omega)\| \\
&+ \frac{\tau^{q-1}\rho}{r-\rho m}\|U(x(\bar{\omega},\bar{\lambda}), v(\omega,\lambda),\omega) - U(x(\bar{\omega},\bar{\lambda}), v(\bar{\omega},\bar{\lambda}),\omega)\| \\
&+ \xi_1\|\omega-\bar{\omega}\| + \frac{\tau^{q-1}\rho}{r-\rho m} l_U\|\omega-\bar{\omega}\|,
\end{aligned}
$$

(3.24)

$$
\begin{aligned}
\|a_2-b_2\| \le\ & \|y(\omega,\lambda) - y(\bar{\omega},\bar{\lambda}) - (g(y(\omega,\lambda),\lambda) - g(y(\bar{\omega},\bar{\lambda}),\lambda))\| \\
&+ \|g(y(\bar{\omega},\bar{\lambda}),\lambda) - g(y(\bar{\omega},\bar{\lambda}),\bar{\lambda})\| \\
&+ \|R^{A,\eta}_{N(\cdot,y(\omega,\lambda),\lambda),\gamma}[y(\omega,\lambda) - \gamma V(u(\omega,\lambda), y(\omega,\lambda),\lambda)] \\
&\quad - R^{A,\eta}_{N(\cdot,y(\bar{\omega},\bar{\lambda})\lambda),\gamma}[y(\omega,\lambda) - \gamma V(u(\omega,\lambda), y(\omega,\lambda),\lambda)]\| \\
&+ \|R^{A,\eta}_{N(\cdot,y(\bar{\omega},\bar{\lambda}),\lambda),\gamma}[y(\omega,\lambda) - \gamma V(u(\omega,\lambda), y(\omega,\lambda),\lambda)] \\
&\quad - R^{A,\eta}_{N(\cdot,y(\bar{\omega},\bar{\lambda}),\lambda),\gamma}[y(\bar{\omega},\bar{\lambda}) - \gamma V(u(\bar{\omega},\bar{\lambda}), y(\bar{\omega},\bar{\lambda}),\lambda)]\| \\
&+ \|R^{A,\eta}_{N(\cdot,y(\bar{\omega},\bar{\lambda}),\lambda),\gamma}[y(\bar{\omega},\bar{\lambda}) - \gamma V(u(\bar{\omega},\bar{\lambda}), y(\bar{\omega},\bar{\lambda}),\lambda)] \\
&\quad - R^{A,\eta}_{N(\cdot,y(\bar{\omega},\bar{\lambda}),\bar{\lambda}),\gamma}[y(\bar{\omega},\bar{\lambda}) - \gamma V(u(\bar{\omega},\bar{\lambda}), y(\bar{\omega},\bar{\lambda}),\lambda)]\| \\
&+ \|R^{A,\eta}_{N(\cdot,y(\bar{\omega},\bar{\lambda}),\bar{\lambda}),\gamma}[y(\bar{\omega},\bar{\lambda}) - \gamma V(u(\bar{\omega},\bar{\lambda}), y(\bar{\omega},\bar{\lambda}),\lambda)] \\
&\quad - R^{A,\eta}_{N(\cdot,y(\bar{\omega},\bar{\lambda}),\bar{\lambda}),\gamma}[y(\bar{\omega},\bar{\lambda}) - \gamma V(u(\bar{\omega},\bar{\lambda}), y(\bar{\omega},\bar{\lambda}),\bar{\lambda})]\| \\
\le\ & \|y(\omega,\lambda) - y(\bar{\omega},\bar{\lambda}) - (g(y(\omega,\lambda),\lambda) - g(y(\bar{\omega},\bar{\lambda}),\lambda))\| \\
&+ l_g\|\lambda-\bar{\lambda}\| + \nu_2\|y(\omega,\lambda) - y(\bar{\omega},\bar{\lambda})\| \\
&+ \frac{\tau^{q-1}}{r-\gamma m}\|y(\omega,\lambda) - y(\bar{\omega},\bar{\lambda}) - \gamma(V(u(\omega,\lambda), y(\omega,\lambda),\lambda) \\
&\quad - V(u(\omega,\lambda), y(\bar{\omega},\bar{\lambda}),\lambda))\| \\
&+ \frac{\tau^{q-1}\gamma}{r-\gamma m}\|V(u(\omega,\lambda), y(\bar{\omega},\bar{\lambda}),\lambda) - V(u(\bar{\omega},\bar{\lambda}), y(\bar{\omega},\bar{\lambda}),\lambda)\| \\
&+ \xi_2\|\lambda-\bar{\lambda}\| + \frac{\tau^{q-1}\gamma}{r-\gamma m} l_V\|\lambda-\bar{\lambda}\|,
\end{aligned}
$$

$$
\begin{aligned}
&\|x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda}) - (f(x(\omega,\lambda),\omega) - f(x(\bar{\omega},\bar{\lambda}),\omega))\|^q \\
&\le \|x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda})\|^q - q\langle f(x(\omega,\lambda),\omega) \\
&\quad - f(x(\bar{\omega},\bar{\lambda}),\omega), J_q(x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda}))\rangle \\
&\quad + c_q\|f(x(\omega,\lambda),\omega) - f(x(\bar{\omega},\bar{\lambda}),\omega)\|^q \\
&\le (1 - q\delta_1 + c_q\sigma_1^q)\|x(\omega,\lambda) - x(\bar{\omega},\bar{\lambda})\|^q,
\end{aligned} \tag{3.25}
$$

$$
\begin{aligned}
&\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)-\rho(U(x(\omega,\lambda),v(\omega,\lambda),\omega)-U(x(\bar\omega,\bar\lambda),v(\omega,\lambda),\omega))\|^q\\
\le{}& \|x(\omega,\lambda)-x(\bar\omega,\bar\lambda\|^q\\
&-q\rho\langle U(x(\omega,\lambda),v(\omega,\lambda),\omega)-U(x(\bar\omega,\bar\lambda),v(\omega,\lambda),\omega),J_q(x(\omega,\lambda)-x(\bar\omega,\bar\lambda))\rangle\\
&+c_q\rho^q\|U(x(\omega,\lambda),v(\omega,\lambda),\omega)-U(x(\bar\omega,\bar\lambda),v(\omega,\lambda),\omega)\|^q\\
\le{}&(1-q\rho\gamma_1+c_q\rho^q\mu_1^q)\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\|^q,
\end{aligned}
\tag{3.26}
$$

$$
\begin{aligned}
&\|U(x(\bar\omega,\bar\lambda),v(\omega,\lambda),\omega)-U(x(\bar\omega,\bar\lambda),v(\bar\omega,\bar\lambda),\omega)\|\\
\le{}&\mu_2\|v(\omega,\lambda)-v(\bar\omega,\bar\lambda)\|\\
\le{}&\mu_2H(T(y(\omega,\lambda),\lambda),T(y(\bar\omega,\bar\lambda),\bar\lambda))\\
\le{}&\mu_2[H(T(y(\omega,\lambda),\lambda),T(y(\bar\omega,\bar\lambda),\lambda))+H(T(y(\bar\omega,\bar\lambda),\lambda),T(y(\bar\omega,\bar\lambda),\bar\lambda))]\\
\le{}&\mu_2(k_2\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)\|+l_T\|\lambda-\bar\lambda\|),
\end{aligned}
\tag{3.27}
$$

$$
\begin{aligned}
&\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)-(g(y(\omega,\lambda),\lambda)-g(y(\bar\omega,\bar\lambda),\lambda))\|^q\\
\le{}&(1-q\delta_2+c_q\sigma_2^q)\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)\|^q,
\end{aligned}
\tag{3.28}
$$

$$
\begin{aligned}
&\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)-\gamma(V(u(\omega,\lambda),y(\omega,\lambda),\lambda)-V(u(\omega,\lambda),y(\bar\omega,\bar\lambda),\lambda))\|^q\\
\le{}&(1-q\gamma\gamma_2+c_q\gamma^q\beta_2^q)\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)\|^q,
\end{aligned}
\tag{3.29}
$$

$$
\begin{aligned}
&\|V(u(\omega,\lambda),y(\bar\omega,\bar\lambda),\lambda)-V(u(\bar\omega,\bar\lambda),y(\bar\omega,\bar\lambda),\lambda)\|\\
\le{}&\beta_1(k_1\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\|+l_S\|\omega-\bar\omega\|).
\end{aligned}
\tag{3.30}
$$

By (3.23)-(3.30), we have

$$
\begin{aligned}
&\|a_1-b_1\|+\|a_2-b_2\|\\
\le{}&\left[\sqrt[q]{1-q\delta_1+c_q\sigma_1^q}+\nu_1+\frac{\tau^{q-1}}{r-\rho m}\sqrt[q]{1-q\rho\gamma_1+c_q\rho^q\mu_1^q}+\frac{\tau^{q-1}\gamma\beta_1k_1}{r-\gamma m}\right]\\
&\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\|\\
&+\left[\frac{\tau^{q-1}\rho\mu_2k_2}{r-\rho m}+\sqrt[q]{1-q\delta_2+c_q\sigma_2^q}+\nu_2+\frac{\tau^{q-1}}{r-\gamma m}\sqrt[q]{1-q\gamma\gamma_2+c_q\gamma^q\beta_2^q}\right]\\
&\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)\|\\
&+\left[l_f+\xi_1+\frac{\tau^{q-1}\rho l_U}{r-\rho m}+\frac{\tau^{q-1}\gamma\beta_1l_S}{r-\gamma m}\right]\|\omega-\bar\omega\|\\
&+\left[\frac{\tau^{q-1}\rho\mu_2l_T}{r-\rho m}+l_g+\xi_2+\frac{\tau^{q-1}\gamma l_V}{r-\gamma m}\right]\|\lambda-\bar\lambda\|
\end{aligned}
\tag{3.31}
$$

$$
\begin{aligned}
&= \theta_1\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\| + \theta_2\|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)\| + n_1\|\omega-\bar\omega\| + n_2\|\lambda-\bar\lambda\| \\
&\le \theta[\|x(\omega,\lambda)-x(\bar\omega,\bar\lambda)\| + \|y(\omega,\lambda)-y(\bar\omega,\bar\lambda)\|] + n_1\|\omega-\bar\omega\| + n_2\|\lambda-\bar\lambda\| \\
&\le \theta[\|a_1-b_1\|+\|a_2-b_2\|] + n_1\|\omega-\bar\omega\| + n_2\|\lambda-\bar\lambda\|,
\end{aligned}
$$

where

$$\theta_1 = \sqrt[q]{1-q\delta_1+c_q\sigma_1^q} + \nu_1 + \frac{\tau^{q-1}}{r-\rho m}\sqrt[q]{1-q\rho\gamma_1+c_q\rho^q\mu_1^q} + \frac{\tau^{q-1}\gamma\beta_1 k_1}{r-\gamma m},$$

$$\theta_2 = \frac{\tau^{q-1}\rho\mu_2 k_2}{r-\rho m} + \sqrt[q]{1-q\delta_2+c_2\sigma_2^q} + \nu_2 + \frac{\tau^{q-1}}{r-\gamma m}\sqrt[q]{1-q\gamma\gamma_2+c_q\gamma^q\beta_2^q},$$

$$n_1 = l_f + \xi_1 + \frac{\tau^{q-1}\rho l_U}{r-\rho m} + \frac{\tau^{q-1}\gamma\beta_1 l_S}{r-\gamma m},$$

$$n_2 = \frac{\tau^{q-1}\rho\mu_2 l_T}{r-\rho m} + l_g + \xi_2 + \frac{\tau^{q-1}\gamma l_V}{r-\gamma m},$$

$$\theta = \max\{\theta_1,\theta_2\}.$$

It follows from (3.3) and (3.31) that

$$
\begin{aligned}
\|a_1-b_1\|+\|a_2-b_2\| &\le \frac{1}{1-\theta}[n_1\|\omega-\bar\omega\| + n_2\|\lambda-\bar\lambda\|] \\
&\le \frac{1}{1-\theta}\max\{n_1,n_2\}(\|\omega-\bar\omega\|+\|\lambda-\bar\lambda\|) \\
&= \vartheta(\|\omega-\bar\omega\|+\|\lambda-\bar\lambda\|),
\end{aligned}
$$

where $\vartheta = \frac{1}{1-\theta}\max\{n_1,n_2\}$. Then we obtain

$$
\begin{aligned}
&d((a_1,a_2),F(x(\bar\omega,\bar\lambda),y(\bar\omega,\bar\lambda),\bar\omega,\bar\lambda)) \\
&= \inf_{(b_1,b_2)\in F(x(\bar\omega,\bar\lambda),y(\bar\omega,\bar\lambda),\bar\omega,\bar\lambda)}(\|a_1-b_1\|+\|a_2-b_2\|) \\
&\le \vartheta(\|\omega-\bar\omega\|+\|\lambda-\bar\lambda\|) \\
&\le \vartheta\|(\omega,\lambda)-(\bar\omega,\bar\lambda)\|_1,
\end{aligned}
\tag{3.32}
$$

$$d((b_1,b_2),F(x(\omega,\lambda),y(\omega,\lambda),\omega,\lambda)) \le \vartheta\|(\omega,\lambda)-(\bar\omega,\bar\lambda)\|_1. \tag{3.33}$$

Hence, from (3.19), (3.32) and (3.33), we have

$$
\begin{aligned}
&H(K(\omega,\lambda),K(\bar\omega,\bar\lambda)) \\
&\le \frac{1}{1-\theta}\sup_{(x,y)\in E\times E} H(F(x(\omega,\lambda),y(\omega,\lambda),\omega,\lambda),F(x(\bar\omega,\bar\lambda),y(\bar\omega,\bar\lambda),\bar\omega,\bar\lambda)) \\
&\le \frac{\vartheta}{1-\theta}\|(\omega,\lambda)-(\bar\omega,\bar\lambda)\|_1.
\end{aligned}
$$

This proves that $K(\omega,\lambda)$ is Lipschitz continuous with respect to $(\omega,\lambda)\in\Omega\times\Lambda$. $\square$

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