Commun. Korean Math. Soc. **25** (2010), No. 3, pp. 335–342
DOI 10.4134/CKMS.2010.25.3.335

FUZZY TOPOLOGICAL SPACES BASED ON WFI-ALGEBRAS

Young Bae Jun

Abstract. The structure of fuzzy topological spaces based on WFI-algebras is considered. The notions of WFI-fuzzy topological spaces and WFI-fuzzy continuous mappings are introduced, and several properties are investigated. A relation between WFI-homomorphism and WFI-fuzzy continuous mapping is given.

1. Introduction

In 1990, Wu [10] introduced the notion of fuzzy implication algebras (FI-algebra, for short), and investigated several properties. In [7], Li and Zheng introduced the notion of distributive (resp. regular, commutative) FI-algebras, and investigated the relations between such FI-algebras and MV-algebras. In [3], Jun discussed several aspects of WFI-algebras. He introduced the notion of associative (resp. normal, medial) WFI-algebras, and investigated several properties. He gave conditions for a WFI-algebra to be associative/medial, and provided characterizations of associative/medial WFI-algebras, and showed that every associative WFI-algebra is a group in which every element is an involution. He also verified that the class of all medial WFI-algebras is a variety. Jun and Song [6] introduced the notions of simulative and/or mutant WFI-algebras and investigated some properties. They established characterizations of a simulative WFI-algebra, and gave a relation between an associative WFI-algebra and a simulative WFI-algebra. They also found some types for a simulative WFI-algebra to be mutant. Jun et al. [5] introduced the concept of ideals of WFI-algebras, and gave relations between a filter and an ideal. Moreover, they provided characterizations of an ideal, and established an extension property for an ideal. After Zadeh introduced the concept of fuzzy sets in [11], Chang [2] used the concept of fuzzy sets to introduce fuzzy topological spaces, and then several authors continued the investigation of such spaces. Caldas et al. [1] introduced and characterized fuzzy weakly θ-closed functions between fuzzy

Received May 24, 2009.
2000 *Mathematics Subject Classification.* 03G25, 03C05, 08A05.
Key words and phrases. WFI-algebra, fuzzy subalgebra, WFI-homomorphism, WFI-fuzzy topological space, WFI-fuzzy continuous mapping.

topological spaces. Lupiáñez [8] introduced the concept of C-scattered fuzzy topological spaces and obtained some properties. Lupiáñez [9] also introduced the notion of fuzzy partition of unity, and obtained some results about this concept.

In this paper, we consider the structure of fuzzy topological spaces based on WFI-algebras. We introduce the notion of WFI-fuzzy topological spaces and WFI-fuzzy continuous mappings, and investigates several properties. We give a relation between WFI-homomorphism and WFI-fuzzy continuous mapping.

2. Preliminaries

Let $K(\tau)$ be the class of all algebras of type $\tau = (2, 0)$. By a *WFI-algebra* we mean a system $\mathfrak{X} := (X, \ominus, 1) \in K(\tau)$ in which the following axioms hold:

(a1) $(x \in X)\ (x \ominus x = 1)$,
(a2) $(x, y \in X)\ (x \ominus y = y \ominus x = 1 \Rightarrow x = y)$,
(a3) $(x, y, z \in X)\ (x \ominus (y \ominus z) = y \ominus (x \ominus z))$,
(a4) $(x, y, z \in X)\ ((x \ominus y) \ominus \big((y \ominus z) \ominus (x \ominus z)\big) = 1)$.

We call the special element 1 the *unit*. For the convenience of notation, we shall write $[x, y_1, y_2, \ldots, y_n]$ for

$$(\cdots((x \ominus y_1) \ominus y_2) \ominus \cdots) \ominus y_n.$$

We define $[x, y]^0 = x$, and for $n > 0$, $[x, y]^n = [x, y, y, \ldots, y]$, where y occurs n-times. We use the notation $x^n \ominus y$ instead of $x \ominus (\cdots (x \ominus (x \ominus y)) \cdots)$ in which x occurs n-times.

Proposition 2.1 ([3]). *In a WFI-algebra* $\mathfrak{X}$, *the following are true*:

(b1) $x \ominus [x, y]^2 = 1$,
(b2) $1 \ominus x = 1 \Rightarrow x = 1$,
(b3) $1 \ominus x = x$,
(b4) $x \ominus y = 1 \Rightarrow [y, z, x \ominus z] = 1 \,\&\, [z, x, z \ominus y] = 1$,
(b5) $[x, y, 1] = [x, 1, y \ominus 1]$,
(b6) $[x, y]^3 = x \ominus y$.

Denote by **WFI** the set of all WFI-algebras. Let $\mathfrak{X} \in$ **WFI**. A nonempty subset S of $\mathfrak{X}$ is called a *subalgebra* of $\mathfrak{X}$ if $x \ominus y \in S$ whenever $x, y \in S$.

A *fuzzy set* in a set X is a function $\bar{A}$ from X to the unit interval $[0, 1]$. For fuzzy sets $\bar{A}$ and $\bar{B}$ in a set X, we define:

(c1) $\bar{A} = \bar{B} \iff (\forall x \in X)(\bar{A}(x) = \bar{B}(x))$.
(c2) $\bar{A} \subseteq \bar{B} \iff (\forall x \in X)(\bar{A}(x) \leq \bar{B}(x))$.

The union $\bar{A} \cup \bar{B}$ of two fuzzy sets $\bar{A}$ and $\bar{B}$ in a set X is defined by

$$(\forall x \in X)\,((\bar{A} \cup \bar{B})(x) = \max\{\bar{A}(x), \bar{B}(x)\}).$$

The intersection $\bar{A} \cap \bar{B}$ of two fuzzy sets $\bar{A}$ and $\bar{B}$ in a set X is defined by

$$(\forall x \in X)\,((\bar{A} \cap \bar{B})(x) = \min\{\bar{A}(x), \bar{B}(x)\}).$$

More generally, for a family $\{\bar{A}_i \mid i \in \Lambda\}$ of fuzzy sets in a set X, the union $\bigcup_{i\in\Lambda} \bar{A}_i$ and the intersection $\bigcap_{i\in\Lambda} \bar{A}_i$ are defined by

$$\Big(\bigcup_{i\in\Lambda} \bar{A}_i\Big)(x) = \sup_{i\in\Lambda} \bar{A}_i(x), \quad \Big(\bigcap_{i\in\Lambda} \bar{A}_i\Big)(x) = \inf_{i\in\Lambda} \bar{A}_i(x),$$

respectively, for all $x \in X$. Let f be a mapping from a set X into a set Y. Let $\bar{B}$ be a fuzzy set in Y. Then $f^{-1}(\bar{B})$ is a fuzzy set in X defined by

$$(\forall x \in X)(f^{-1}(\bar{B})(x) = \bar{B}(f(x))).$$

A *fuzzy topology* on a set X is a family τ of fuzzy sets in X satisfying the following assertions:

(i) $\bar{0}, \bar{1} \in \tau$.
(ii) $(\forall \bar{A}, \bar{B} \in \tau)\ (\bar{A} \cap \bar{B} \in \tau)$.
(iii) $(\forall \{\bar{A}_i \mid i \in \Lambda\} \subseteq \tau)\ \Big(\bigcup_{i\in\Lambda} \bar{A}_i \in \tau\Big)$, where Λ is any index set.

3. WFI-fuzzy topological spaces

Definition 3.1. Let $\mathfrak{X} \in$ **WFI**. A fuzzy topology τ on $\mathfrak{X}$ is called a *WFI-fuzzy topology.* The pair $(\mathfrak{X}, \tau)$ is called a *fuzzy topological space* based on a WFI-algebra (briefly, *WFI-fuzzy topological space*).

Example 3.2. Let $X = \{1, a\}$ be a set with the following Cayley table:

$\ominus$	1	a
1	1	a
a	a	1

Then $\mathfrak{X} := (X, \ominus, 1) \in$ **WFI**.

(1) Let $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$ be fuzzy sets in $\mathfrak{X}$ defined by

$$\bar{A}(1) = 0.4,\ \bar{A}(a) = 0.2,\ \bar{B}(1) = 0.3,\ \bar{B}(a) = 0.7,$$
$$\bar{C}(1) = 0.3,\ \bar{C}(a) = 0.2,\ \bar{D}(1) = 0.4,\ \bar{D}(a) = 0.7,$$

respectively. Then $\tau := \{\bar{0}, \bar{1}, \bar{A}, \bar{B}, \bar{C}, \bar{D}\}$ is a WFI-fuzzy topology on $\mathfrak{X}$, and so $(\mathfrak{X}, \tau)$ is a WFI-fuzzy topological space.

(2) For every $n \in \mathbb{N}$, let $\bar{A}_n$ be a fuzzy set in $\mathfrak{X}$ given by $\bar{A}_n(1) = \frac{n}{n+1}$ and $\bar{A}_n(a) = \frac{n+1}{n+2}$. Then $(\mathfrak{X}, \sigma)$ is a WFI-fuzzy topological space, where $\sigma := \{\bar{0}, \bar{1}\} \cup \{\bar{A}_n \mid n \in \mathbb{N}\}$.

Example 3.3. Let $X = \{1, a, b, c\}$ be a set with the following Cayley table:

$\ominus_X$	1	a	b	c
1	1	a	b	c
a	a	1	c	b
b	b	c	1	a
c	c	b	a	1

Then $\mathfrak{X} := (X, \ominus_X, 1) \in \mathbf{WFI}$. Let $\bar{A}$ and $\bar{B}$ be fuzzy sets in $\mathfrak{X}$ defined by

$$\bar{A}(x) := \begin{cases} 1 & \text{if } x = 1, \\ 0 & \text{otherwise,} \end{cases} \qquad \bar{B}(x) := \begin{cases} 0 & \text{if } x = 1, \\ 1 & \text{otherwise,} \end{cases}$$

respectively. Then $\tau := \{\bar{0}, \bar{1}, \bar{A}, \bar{B}\}$ is a WFI-fuzzy topology on $\mathfrak{X}$.

Example 3.4. Let $Y = \{e, u, v, w\}$ be a set with the following Cayley table:

$\ominus_Y$	e	u	v	w
e	e	u	v	w
u	u	e	w	v
v	w	v	e	u
w	v	w	u	e

Then $\mathfrak{Y} := (Y, \ominus_Y, e) \in \mathbf{WFI}$. Let $\bar{C}$ and $\bar{D}$ be fuzzy sets in $\mathfrak{Y}$ defined by

$$\bar{C}(y) := \begin{cases} 1 & \text{if } y \in \{e, u\}, \\ 0 & \text{if } y \in \{v, w\}, \end{cases} \qquad \bar{D}(y) := \begin{cases} 0 & \text{if } y \in \{e, u\}, \\ 1 & \text{if } y \in \{v, w\}, \end{cases}$$

respectively. Then $\sigma_1 := \{\bar{0}, \bar{1}, \bar{C}\}$ and $\sigma_2 := \{\bar{0}, \bar{1}, \bar{C}, \bar{D}\}$ are WFI-fuzzy topologies on $\mathfrak{Y}$.

Proposition 3.5. *Let $\{\tau_i \mid i \in \Lambda\}$ be a family of WFI-fuzzy topologies on $\mathfrak{X} \in \mathbf{WFI}$, where Λ is any index set. Then $\bigcap_{i\in\Lambda} \tau_i$ is a WFI-fuzzy topology on $\mathfrak{X}$. Moreover, $\bigcap_{i\in\Lambda} \tau_i$ is the coarsest WFI-fuzzy topology on $\mathfrak{X}$ containing τ_i for all $i \in \Lambda$.*

Proof. Straightforward. □

Definition 3.6. Let $\mathfrak{X} \in \mathbf{WFI}$. A fuzzy set $\bar{A}$ in $\mathfrak{X}$ is called a *fuzzy subalgebra* of $\mathfrak{X}$ if it satisfies:

$$(\forall x, y \in X)\,(\bar{A}(x \ominus y) \geq \min\{\bar{A}(x), \bar{A}(y)\}). \tag{3.1}$$

Using the Transfer Principle (see [4]), we have a characterization of a fuzzy subalgebra.

Theorem 3.7. *For a fuzzy set $\bar{A}$ in a WFI-algebra $\mathfrak{X}$, the following assertions are equivalent*:

(i) *$\bar{A}$ is a fuzzy subalgebra of $\mathfrak{X}$.*
(ii) $(\forall t \in [0,1])$ $(U(\bar{A}; t) \neq \emptyset \implies U(\bar{A}; t)$ is a subalgebra of $\mathfrak{X})$,

where $U(\bar{A}; t) = \{x \in X \mid \bar{A}(x) \geq t\}$.

Let $f : (\mathfrak{X}, \tau_1) \to (\mathfrak{Y}, \tau_2)$ be a mapping of WFI-fuzzy topological spaces. It is important to make questions of the following:

(i) If $\bar{A} \in \tau_2$, then is $f^{-1}(\bar{A}) \in \tau_1$?
(ii) If $\bar{B}$ is a fuzzy subalgebra of $\mathfrak{Y}$ which is contained in τ_2, then is $f^{-1}(\bar{B})$ a fuzzy subalgerba of $\mathfrak{X}$?

The answer is negative as given by the following example.

Example 3.8. Consider WFI-algebras $\mathfrak{X}$ and $\mathfrak{Y}$ which are described in Examples 3.3 and 3.4, respectively. Define a mapping $f : (\mathfrak{Y}, \sigma_1) \to (\mathfrak{X}, \tau)$ by

$$f(y) := \begin{cases} 1 & \text{if } y \in \{e, u\}, \\ b & \text{if } y = v, \\ c & \text{if } y = w. \end{cases}$$

Obviously, $f^{-1}(\bar{0}) \in \sigma_1$ and $f^{-1}(\bar{1}) \in \sigma_1$. But we know that

$$f^{-1}(\bar{B})(e) = \bar{B}(f(e)) = \bar{B}(1) = 0 = \bar{D}(e),$$

$$f^{-1}(\bar{B})(u) = \bar{B}(f(u)) = \bar{B}(1) = 0 = \bar{D}(u),$$

$$f^{-1}(\bar{B})(v) = \bar{B}(f(v)) = \bar{B}(b) = 1 = \bar{D}(v),$$

$$f^{-1}(\bar{B})(w) = \bar{B}(f(w)) = \bar{B}(c) = 1 = \bar{D}(w).$$

Hence $f^{-1}(\bar{B}) = \bar{D} \notin \sigma_1$ for $\bar{B} \in \tau$. But if we take $\sigma_2 := \{\bar{0}, \bar{1}, \bar{C}, \bar{D}\}$ instead of $\sigma_1 := \{\bar{0}, \bar{1}, \bar{C}\}$, then we can check that the answer to the question (i) above is positive. Now let $\bar{E}$ be a fuzzy set in $\mathfrak{X}$ defined by

$$\bar{E}(x) = \begin{cases} 0.2 & \text{if } x = 1, \\ 0.4 & \text{if } x \in \{a, b\}, \\ 0.5 & \text{if } x = c. \end{cases}$$

It is easily to check that $\bar{E}$ is a fuzzy subalgebra of $\mathfrak{X}$, and $\delta := \{\bar{0}, \bar{1}, \bar{E}\}$ is a WFI-fuzzy topology on $\mathfrak{X}$. Then $f^{-1}(\bar{E})$ is given by

$$f^{-1}(\bar{E})(y) = \begin{cases} 0.2 & \text{if } y \in \{e, u\}, \\ 0.4 & \text{if } y = v, \\ 0.5 & \text{if } y = w, \end{cases}$$

which is not a fuzzy subalgebra of $\mathfrak{Y}$ since

$$f^{-1}(\bar{E})(w \ominus_Y v) = f^{-1}(\bar{E})(u) = 0.2 < 0.4 = \min\{f^{-1}(\bar{E})(w), f^{-1}(\bar{E})(v)\}.$$

This shows that the answer to the second question above is negative.

Based on this result, we proceed on to define the notion of WFI-fuzzy continuous mappings.

Definition 3.9. Let $(\mathfrak{X}, \tau)$ and $(\mathfrak{Y}, \sigma)$ be WFI-fuzzy topological spaces. A mapping $f : \mathfrak{X} \to \mathfrak{Y}$ is said to be *WFI-fuzzy continuous* if it satisfies the following assertions:

(i) $(\forall \bar{A} \in \sigma)$ $(f^{-1}(\bar{A}) \in \tau)$,
(ii) If $\bar{B}$ is a fuzzy subalgebra of $\mathfrak{Y}$ which is contained in σ, then $f^{-1}(\bar{B})$ is a fuzzy subalgebra of $\mathfrak{X}$.

We provide examples of a WFI-fuzzy continuous mapping.

Example 3.10. Let $(\mathfrak{X}, \tau)$ and $(\mathfrak{Y}, \sigma_2)$ be WFI-fuzzy topological spaces which are given in Examples 3.3 and 3.4, respectively. Let $g : \mathfrak{Y} \to \mathfrak{X}$ be a function defined by

$$g(y) := \begin{cases} 1 & \text{if } y \in \{e, u\}, \\ b & \text{if } y = v, \\ c & \text{if } y = w. \end{cases}$$

Obviously $g^{-1}(\bar{0}) \in \tau$ and $g^{-1}(\bar{1}) \in \tau$. Now we have

$$g^{-1}(\bar{A})(e) = \bar{A}(g(e)) = \bar{A}(1) = 1 = \bar{C}(e),$$
$$g^{-1}(\bar{A})(u) = \bar{A}(g(u)) = \bar{A}(1) = 1 = \bar{C}(u),$$
$$g^{-1}(\bar{A})(v) = \bar{A}(g(v)) = \bar{A}(b) = 0 = \bar{C}(v),$$
$$g^{-1}(\bar{A})(w) = \bar{A}(g(w)) = \bar{A}(c) = 0 = \bar{C}(w)$$

and

$$g^{-1}(\bar{B})(e) = \bar{B}(g(e)) = \bar{B}(1) = 0 = \bar{D}(e),$$
$$g^{-1}(\bar{B})(u) = \bar{B}(g(u)) = \bar{B}(1) = 0 = \bar{D}(u),$$
$$g^{-1}(\bar{B})(v) = \bar{B}(g(v)) = \bar{B}(b) = 1 = \bar{D}(v),$$
$$g^{-1}(\bar{B})(w) = \bar{B}(g(w)) = \bar{B}(c) = 1 = \bar{D}(w).$$

It follows that $g^{-1}(\bar{A}) = \bar{C} \in \sigma_2$ and $g^{-1}(\bar{B}) = \bar{D} \in \sigma_2$. Note that $\bar{0}$, $\bar{1}$ and $\bar{A}$ are fuzzy subalgebras of $\mathfrak{X}$. But $\bar{B}$ is not a fuzzy subalgebra of $\mathfrak{X}$ since

$$\bar{B}(b \ominus_X b) = \bar{B}(1) = 0 < 1 = \min\{\bar{B}(b), \bar{B}(b)\}.$$

It is clear that $g^{-1}(\bar{0})$ and $g^{-1}(\bar{1})$ are fuzzy subalgebras of $\mathfrak{Y}$, and it is easy to verify that $g^{-1}(\bar{A})$ is a fuzzy subalgebra of $\mathfrak{Y}$. Hence g is a WFI-fuzzy continuous mapping.

Example 3.11. Let $\mathfrak{X}$ be a WFI-algebra which is given in Example 3.2. Let $Y = \{e, x, y\}$ be a set with the following Cayley table:

$\ominus_Y$	e	x	y
e	e	x	y
x	y	e	x
y	x	y	e

Then $\mathfrak{Y} := (Y; \ominus_Y, e)$ is a WFI-algebra. Let τ_X and τ_Y be discrete WFI-fuzzy topologies on $\mathfrak{X}$ and $\mathfrak{Y}$, respectively. Define a mapping $f : (\mathfrak{X}, \tau_X) \to (\mathfrak{Y}, \tau_Y)$ by $f(1) = e$ and $f(a) = x$. Clearly, $f^{-1}(\bar{A}) \in \tau_X$ for all $\bar{A} \in \tau_Y$. Let $\bar{B}$ be a fuzzy subalgebra of $\mathfrak{Y}$ which is contained in τ_Y. If $\bar{B} = \bar{0}$ or $\bar{B} = \bar{1}$, then clearly $f^{-1}(\bar{B})$ is a fuzzy subalgebra of $\mathfrak{X}$. Assume that $\bar{B} \neq \bar{0}$ and $\bar{B} \neq \bar{1}$. Since $\mathfrak{Y}$ has only two subalgebras $\{e\}$ and Y itself, using Theorem 3.7 implies that $\bar{B}$ has the following form:

$$\bar{B}(u) = \begin{cases} t_1 & \text{if } u = e, \\ t_2 & \text{if } u \in \{x, y\}, \end{cases}$$

where $t_1, t_2 \in [0,1]$ with $t_1 > t_2$. For every $x \in X$, we have

$$f^{-1}(\bar{B})(x) = \bar{B}(f(x)) = \begin{cases} t_1 & \text{if } x = 1, \\ t_2 & \text{if } x = a. \end{cases}$$

Hence $f^{-1}(\bar{B})$ is a fuzzy subalgebra of $\mathfrak{X}$. Therefore f is a WFI-fuzzy continuous mapping.

Theorem 3.12. *Let $(\mathfrak{X}, \tau_X)$ and $(\mathfrak{Y}, \tau_Y)$ be WFI-fuzzy topological spaces and let $f : \mathfrak{X} \to \mathfrak{Y}$ be a mapping. If τ_Y is indiscrete, then f is WFI-fuzzy continuous.*

Proof. Straightforward. □

Remark 3.13. Although τ_X is discrete, a mapping $f : (\mathfrak{X}, \tau_X) \to (\mathfrak{Y}, \tau_Y)$ may not be a WFI-fuzzy continuous mapping (see Example 3.8).

Definition 3.14. A mapping $f : \mathfrak{X} \to \mathfrak{Y}$ of WFI-algebras $\mathfrak{X}$ and $\mathfrak{Y}$ is called a *WFI-homomorphism* if it satisfies:

$$(\forall x, y \in X)(f(x \ominus y) = f(x) \ominus f(y)).$$

We now discuss the relation between WFI-homomorphisms and WFI-fuzzy continuous mappings. It is natural to give the following question.

Question 3.15. *Is any WFI-homomorphism a WFI-fuzzy continuous mapping?*

The following example provide a negative answer to the above question.

Example 3.16. Let $X = \{e, a, b, c\}$ be a set with the following Cayley table:

$\ominus_X$	e	a	b	c
e	e	a	b	c
a	b	e	c	a
b	a	c	e	b
c	c	b	a	e

Then $\mathfrak{X} := (X, \ominus_X, e) \in$ **WFI**. Let $\bar{A}$ and $\bar{B}$ be fuzzy sets in G defined by

$$\bar{A}(x) = \begin{cases} 1 & \text{if } x = e, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \bar{B}(x) = \begin{cases} 0 & \text{if } x = e, \\ 1 & \text{otherwise,} \end{cases}$$

respectively. Then $\tau_1 = \{\bar{0}, \bar{1}, \bar{A}\}$ and $\tau_2 = \{\bar{0}, \bar{1}, \bar{B}\}$ are WFI-fuzzy topologies on $\mathfrak{X}$. Let $f : (\mathfrak{X}, \tau_1) \to (\mathfrak{X}, \tau_2)$ be a function defined by $f(e) = e$, $f(a) = b$, $f(b) = a$, and $f(c) = c$. Then it is easy to verify that f is a WFI-homomorphism. Now we have

$$f^{-1}(\bar{B})(e) = \bar{B}(f(e)) = \bar{B}(e),$$
$$f^{-1}(\bar{B})(c) = \bar{B}(f(c)) = \bar{B}(c),$$
$$f^{-1}(\bar{B})(a) = \bar{B}(f(a)) = \bar{B}(b) = \bar{B}(a),$$
$$f^{-1}(\bar{B})(b) = \bar{B}(f(b)) = \bar{B}(a) = \bar{B}(b),$$

and so $f^{-1}(\bar{B}) = \bar{B} \notin \tau_1$. Hence f is not a WFI-fuzzy continuous mapping.

Theorem 3.17. *Let $(\mathfrak{X}, \tau_X)$ and $(\mathfrak{Y}, \tau_Y)$ be WFI-fuzzy topological spaces and assume that τ_X is discrete. Then every WFI-homomorphism $f : \mathfrak{X} \to \mathfrak{Y}$ is a WFI-fuzzy continuous mapping.*

Proof. Let $f : \mathfrak{X} \to \mathfrak{Y}$ be a WFI-homomorphism. Obviously, $f^{-1}(\bar{A}) \in \tau_X$ for all $\bar{A} \in \tau_Y$. Let $\bar{B}$ be a fuzzy subalgebra of $\mathfrak{Y}$ which is contained in τ_Y. For any $x, y \in X$, we have

$$\begin{aligned} f^{-1}(\bar{B})(x \ominus y) &= \bar{B}(f(x \ominus y)) = \bar{B}(f(x) \ominus f(y)) \\ &\geq \min\{\bar{B}(f(x)), \bar{B}(f(y))\} \\ &= \min\{f^{-1}(\bar{B})(x), f^{-1}(\bar{B})(y)\}, \end{aligned}$$

and hence $f^{-1}(\bar{B})$ is a fuzzy subalgebra of $\mathfrak{X}$. Therefore f is a WFI-fuzzy continuous mapping. □

The following example shows that the converse of Theorem 3.17 is not true in general.

Example 3.18. The mapping f in Example 3.11 is WFI-fuzzy continuous, but not a WFI-homomorphism since

$$f(a \ominus 1) = f(a) = x \neq y = x \ominus_Y e = f(a) \ominus_Y f(1).$$

References

[1] M. Caldas, G. Navalagi, and R. Saraf, *Weakly θ-closed functions between fuzzy topological spaces*, Mat. Vesnik **54** (2002), no. 1-2, 13–20.

[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.

[3] Y. B. Jun, *Weak fuzzy implication algebras*, Adv. Stud. Contemp. Math. (Kyungshang) **7** (2003), no. 1, 41–52.

[4] Y. B. Jun and M. Kondo, *On transfer principle of fuzzy BCK/BCI-algebras*, Sci. Math. Jpn. **59** (2004), no. 1, 35–40.

[5] Y. B. Jun, C. H. Park, and E. H. Roh, *Characterizations of filters and ideals on WFI-algebras*, Honam Math. J. **28** (2006), no. 4, 471–484.

[6] Y. B. Jun and S. Z. Song, *Simulative and mutant WFI-algebras*, Honam Math. J. **28** (2006), no. 4, 559–572.

[7] Z. Li and C. Zheng, *Relations between fuzzy implication algebra and MV algebra*, J. Fuzzy Math. **9** (2001), no. 1, 201–205.

[8] F. G. Lupiáñez, *C-scattered fuzzy topological spaces*, Appl. Math. Lett. **14** (2001), no. 2, 201–204.

[9] ———, *Fuzzy partitions of unity*, Mat. Vesnik **56** (2004), no. 1-2, 13–15.

[10] W. M. Wu, *Fuzzy implication algebras*, Fuzzy Systems Math. **4** (1990), no. 1, 56–63.

[11] L. A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338–353.

Department of Mathematics Education (and RINS)
Gyeongsang National University
Chinju 660-701, Korea
E-mail address: skywine@gmail.com