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Earliest Intercept Geometry Guidance to Improve Mid-Course Guidance in Area Air-Defence

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Abstract

This paper describes a mid-course guidance strategy based on the earliest *intercept geometry* (EIG) guidance. An analytical solution and performance validation will be addressed for generalized mid-course guidance problems in area air-defence in order to improve reachability and performance. The EIG is generated for a wide range of possible manoeuvres of the challenging missile based on the guidance algorithm using differential geometry concepts. The main idea is that a mid-course guidance law can defend the area as long as it assures that the depending area and objects are always within the defended area defined by EIG. The velocity of *Intercept Point* in EIG is analytically derived to control the *Intercept Geometry* and the defended area. The proposed method can be applied in deciding a missile launch window and launch point for the launch phase.

Key words: Mid-course, Earliest *intercept geometry*, Differential geometry

1. Introduction

A full engagement of a missile against an attacking missile consists of three phases: launch, mid-course and terminal homing phase. For long or medium range engagement the missile generally cannot lock on to the target so that the mid-course guidance law will be applied to the missile. Mid-course guidance phase refers to the procedure that guides the missile to a terminal handover point in order to acquire the target motion through its on-board sensor. The mid-course guidance law differs significantly from the guidance logic for the terminal homing phase due to the inherited characteristic of the mid-course guidance: it must not only find the best condition for the terminal homing phase, but also deliver the missile to that condition using only the target motion data updated by the external source such as a plane, ship, or ground base. Although there are a number of objectives for the mid-course guidance, it is possible to classify them into two main categories.

Firstly, the onus is on the mid-course guidance to achieve the best terminal handover condition, i.e. to provide certainty that the missile can reach the target. For a target at a great distance, the mid-course guidance opts to maximize the terminal velocity and/or minimize the energy loss in order to gain high velocity and manoeuvrability advantages for the terminal homing guidance. Minimizing the flight time is commonly relevant to a close in target problem, since the missile should intercept the target before attacking its objective (Song and Tahk, 1998). The necessary conditions for an optimal mid-course guidance problem form a two-point boundary-value problem (Kirk, 1970). However, it is unfeasible not only to derive the analytical solution but also to find a numerical solution in real time by any present-day on-board computer.

Several approaches have been proposed to solve the optimal mid-course guidance problem. Song and Tahk

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(1998, 1999, 2001) have addressed artificial neural networks for the mid-course guidance algorithms to generate suboptimal guidance commands in a feedback fashion. Whilst this approach can overcome the difficulty in deriving an on-board optimal guidance law, it is difficult to cover all the region of the input vector for neural network training, and the stability cannot be guaranteed. To find a real time sub-optimal guidance algorithm, singular perturbation technology (SPT) (Cheng and Gupta, 1986; Dougherty and Speyer, 1997; Menon and Briggs, 1990) and linear quadratic regulator (LQR) (Imado and Kuroda, 2009; Imado et al., 1990) have been proposed. However, a guidance law based on SPT does not result in a true feedback control, and the LQR type guidance algorithm needs a large set of database.

Another important objective in the mid-course guidance is protecting and defending an area. For instance, two primary tasks of the maritime air defence area are protection of sea traffic or convoys and coast line protection against attacking weapon systems such as anti-ship missile (ASM), which has a high kill probability and low detectability. It is important that the mid-course guidance algorithm guides the missile to the acquiring point in order to guarantee reachability of the guidance in the terminal homing phase. However, the missile cannot guarantee to intercept and destroy the challenging missile before it has struck its target within the defending area.

Over the past decades, a range of missile genres have been developed and each of them has well published advantages and disadvantages. Yet there has been little research dealing with naval area defence and protection directly. For warships to continue their primary role, it is essential that air defence for a single unit, be it large or small, a close escort or a major surface group, provides a comprehensive defence against all types of air threat, from the sea skimmer to the high diving missile in addition to attack aircraft and weapon carrying unmanned aerial vehicles.

In our study represented in reference (Robb et al., 2005a, b), the earliest intercept line (EIL) concept is proposed for a mid-course guidance using shortest path technologies 'Dubins' (Dubins, 1957). The guidance law can improve reachability under consideration of lateral acceleration saturation and unpredictable target missile manoeuvre. Moreover, since the guidance algorithm is based on differential geometry, it is easy to show whether or not the missile can intercept the challenging missile while protecting the defending area. However, it is impossible to derive an analytical solution using the proposed algorithm and difficult to analyse the performance of the guidance algorithm mathematically.

This paper will describe an analytical solution and performance validation for a generalized mid-course guidance problem in area air-defence which improves reachability and overcomes the weakness of the previous study (Robb et al., 2005a, b). As will be seen, the proposed mid-course guidance strategy is derived based on the earliest *intercept geometry* (EIG) concepts. The EIG is generated for a wide range of possible manoeuvres of the challenging missile based on the guidance algorithm using differential geometry concepts (White et al., 2007). The main idea is that a mid-course guidance law can defend the area as long as it assures that the depending area is always inside the defended area defined by EIG. An analytical approach for controlling EIG and the defended area will be addressed to enhance reachability and performance of the guidance algorithm. The proposed method can be applied in deciding a missile launch window and launch point for the launch phase.

2. Earliest Intercept Geometry

The guidance algorithm described in this paper is a subset of possible solutions for a non-manoeuvring target, which have been proposed in our previous research (White et al., 2007) using different geometry concepts (Lipschutz, 1969; O'Neill, 1997). The algorithm is identical to the concept of proportional navigation guidance (PNG). Therefore, this gives solutions as robust as PNG algorithms, and allows for greater flexibility in the choice of trajectory which enhances reachability. Moreover, it allows predicting the precious *Intercept Geometry* which is vital in the area air-defence. The guidance geometry for a non-manoeuvring target and missile is shown in Fig. 1.

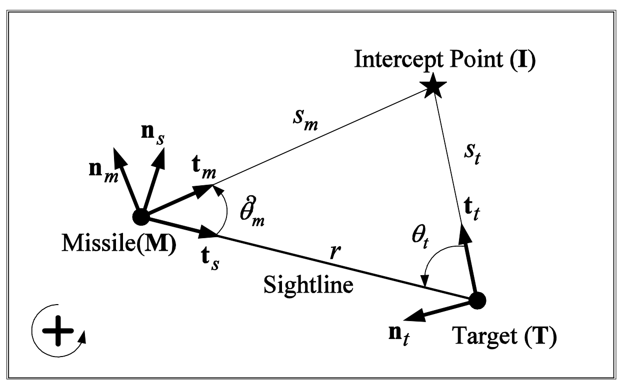


Fig. 1. Guidance geometry for direct intercept.

Figure 1 shows the case in which a target is flying in a straight line at constant velocity v_t. The missile is flying at velocity v_m, also in a straight line. Both trajectories are assumed to intercept at the *Intercept Point* I. The target and missile together with the *Intercept Point* form a triangle, which will be called the *Intercept Triangle*. In order to establish the matching condition for intercept, consider a time T such that the target has travelled in a straight line and at constant velocity from its initial position to the *Intercept Point* as shown in Figure 1. The length of this trajectory s_t will be:

$$s_t = v_t T \tag{1}$$

In order for the missile to arrive at the *Intercept Point*, it must travel a distance s_m in the same time *T*. i.e.:

$$s_m = v_m T \tag{2}$$

The ratio of the trajectory lengths is then given by:

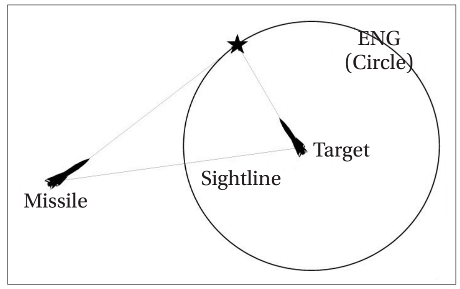
$$\frac{s_m}{s_t} = \frac{v_m}{v_t} = \gamma$$

As proposed in our previous study (White et al., 2007), implementing the cosine rule to the *Intercept Triangle* gives:

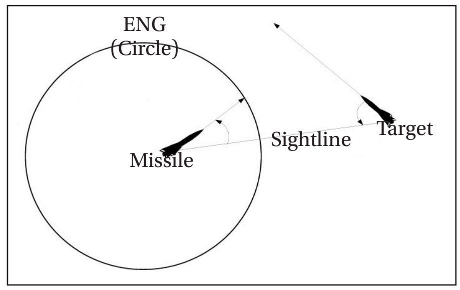
$$\begin{aligned} \frac{r}{s_t} &= \cos(\theta_t) + \gamma\cos(\hat{\theta}_m) \\ \cos(\hat{\theta}_m) &= \pm\frac{\sqrt{\gamma^2 - \sin^2(\theta_t)}}{\gamma} \end{aligned} \tag{3}$$

where θ_m and θ_t are the missile-to-sightline angle and the target-to-sightline angle, respectively; and, r represents the sightline range. Given that $\gamma > 1$, the solution for Eq. (3) must be positive. For γ smaller than or equal to unity, the determination of the sign in Eq. (3) will be addressed in this section.

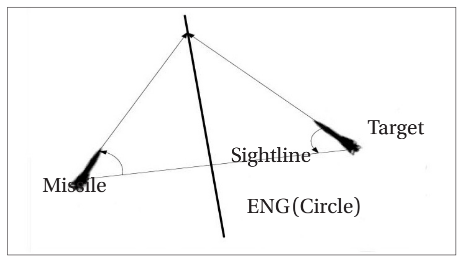
If a challenging missile heading angle and the range of *Intercept Triangles* are given, the *Intercept Geometry* is fixed. However, because the heading angle of the challenging missile can change and it is impossible to predict its lateral acceleration, the geometry cannot be determined. It is, therefore, possible to derive the locus of possible *Intercept Triangles*, i.e. the *Intercept Geometry*, by considering all viable headings of the challenging missile. The ratio of velocities γ is important to find the *Intercept Geometry* as, if the ratio falls below unity, the possible missile position is restricted to a position in front of the target. A range of velocity ratios can be classified by three main categories due to the significant difference of the *Intercept Geometry*: the velocity ratio greater than unity, less than unity and equal to unity. Their *Intercept Geometries* are shown in Fig. 2.



(a) $\gamma > 1$



(b) $\gamma < 1$



(c) $\gamma = 1$

Fig. 2. Locus of impact triangle.

From these figures it can be seen that a velocity ratio larger than unity produces possible missile positions for intercept both behind and in front of the target. The *Intercept Geometries* for velocity ratios less than or equal to unity show that there is a region that cannot produce an *Intercept Triangle*. A velocity ratio greater than unity is desirable to enable the target to be engaged from any position and with a reasonably small range to go before its strike. Current trends involve reducing the cost of missile systems, and so the speed advantage required for acquisition under all conditions will be lost as missiles with speed ranges comparable with the target are designed.

The *Intercept Geometry* is not unique; the only condition that is required for a non-manoeuvring intercept is that the ratio of the trajectories is the same as the ratio of the velocities. Figure 3 shows the *Intercept Geometry* that is required.

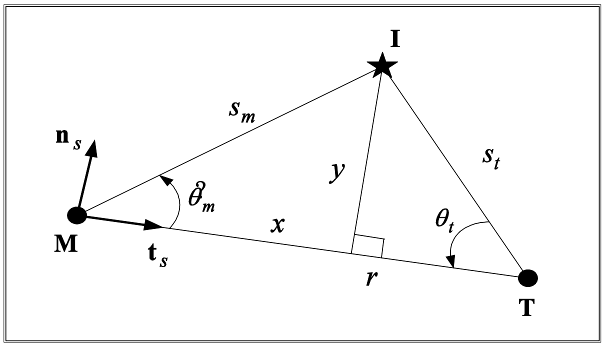


Fig. 3. Intercept geometry.

The geometry is drawn using the sightline axes centred in the missile, hence x is along the sightline and y is normal to it, with r as the sightline range between the missile and the target. As the target angle to the sight line θ_t varies, the *Intercept Point* **I** will change. The locus of the *Intercept Point* can be determined by using the Pythagoras Theorem on two triangles. The first is triangle is denoted as **M I N**, composed of the missile position **M**, the *Intercept Point* **I** and the intercept of the normal from the *Intercept Point* onto the sightline **N**. The second is triangle **T I N**, replacing the missile position with the target position **T**. Hence:

$$\begin{aligned} s_m^2 &= x^2 + y^2 \\ s_t^2 &= (r-x)^2 + y^2 \end{aligned} \qquad (4)$$

Using Eq. (3), this yields:

$$\left(x + \frac{\gamma^2 r}{1-\gamma^2}\right)^2 + y^2 = \frac{\gamma^2 r^2}{1-\gamma^2} + \frac{\gamma^4 r^2}{1-\gamma^2} = \left(\frac{\gamma r}{1-\gamma^2}\right)^2 \qquad (5)$$

This equation represents a circle with radius r_I and centre c_I with respect to the sightline axes, where:

$$\begin{aligned} r_I &= \frac{\gamma r}{\left|\gamma^2 - 1\right|} \\ c_I &= \frac{\gamma^2 r}{\gamma^2 - 1} \end{aligned} \qquad (6)$$

The speed ratio γ is key parameter of Eqs. (2-5). As shown in Eq. (6), the centre of the intercept circle c_I is greater than r for γ larger than unity, so that the centre is always closer to the target than the missile. Moreover, since subtracting the radius of the intercept circle from the distance between the centre and the target gives:

$$(c_I - r) - r_I = -\frac{\gamma + 1}{\gamma^2 - 1} r < 0 \qquad (7)$$

the target will be always located in the circle as shown in Fig. 2(a). However, if we have γ smaller than 1, there is the range of θ_t for which:

$$\sqrt{\gamma^2 - \sin^2(\theta_t)} < 0 \qquad \text{for } \gamma < 1 \qquad (8)$$

which implies that for some geometry, a real solution is not possible and the missile will not be able to intercept the target. Such a condition is shown in Fig. 2(b). For γ smaller than unity, the centre of the intercept circle is negative, i.e. the centre is placed along the sightline but on the opposite direction to the target. Furthermore, the radius of the circle is greater than the distance between the centre and the missile so that the missile will be always inside the circle. Therefore, if the ratio is greater than unity the defended area of the missile is outside the circle; however, the defended area is inside the circle for γ smaller than unity. This property can be used to allocate the missile in the position guaranteeing the area defence: for γ larger than unity, the missile has to launch when every defended object is located outside the circle, and the missile makes sure the defended objects are placed inside the circle where γ is smaller than 1

For γ smaller than unity, there are two matching conditions for the ratio r/s_r represented in Eq. (3) for one target heading angle: if the sign in Eq. (3) is positive, then the desired heading angle of the missile is an acute angle, otherwise it is an obtuse angle. In the SvA problem, since the SAM should protect the defended objects, it's better to choose an acute angle solution between two possible solutions for $\gamma < 1$. If the missile is heading for the *Intercept Point* along an obtuse heading angle under the assumption that the defended objects are only inside the *Intercept Geometry*, then missile cannot protect the defended objects; however, if the heading angle solution is an acute angle, the missile can likely defend the area. Therefore, the acute heading angle is the only angle considered for the solution angle.

For the case in which the missile and target are travelling at the same speed, we have γ equal to unity and the *Intercept Geometry* is no longer a circle. The matching condition now becomes:

$$\frac{r}{s_t} = \cos(\theta_t) \pm \cos(\theta_t) \qquad (9)$$

There is no longer an imaginary solution, and there are now two real solutions given by:

$$\frac{r}{s_t} = 2\cos(\theta_t) \quad or \quad 0 \qquad (10)$$

The first solution implies a geometry which produces an isosceles triangle solution as before, for S_t equal to s_m. The second solution implies that the ratio r/s_r is equal to zero, subsequently $S_t = \infty$ δand $S_m = \infty$. In this case, the missile cannot intercept the target and protect the defended objects. Therefore, the first solution is considered in this paper. Substituting $\gamma = 1$ into Eq. (4) yields:

$$x = \frac{r}{2} \tag{11}$$

Note that this means for γ equal to unity, the *Intercept Geometry* is a line as shown in Fig. 2(c).

The locus also has another interpretation. Given that the target is travelling at constant velocity in a fixed direction, as shown in Fig. 2, the locus represents the earliest intercept that the missile can achieve. This is based on the fact that the shortest distance between two points is a straight line. Hence the straight line *Intercept Geometry* in Fig. 2 represents the EIG.

3. Geometry Control

The EIG concept implies that if the target manoeuvres away from a straight line trajectory for striking its aiming object, then the missile travelling on a straight line trajectory can achieve the intercept outside the defended zone defined by the EIG. As mentioned in the previous section, the defended zone is outside the EIG for γ greater than or equal to unity and inside the EIG for γ smaller than unity. Conversely, if the missile deviates from a straight line trajectory, then the *Intercept Point* must lie within the defended zone. Therefore, the proposed mid-course guidance strategy is to keep all the defended object and objects within the defended area. This interpretation is useful in capturing the area in which the target can evade intercept and the missile can successfully defend.

Due to physical and operational conditions of the missiles, the missile cannot always protect the defended objects or area. Figure 4 represents one of these cases when the missile velocity is greater than that of the challenging.

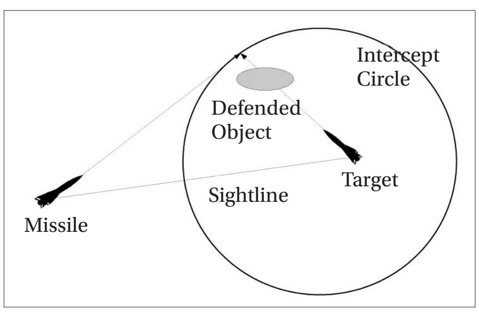


Fig. 4. Failure geometry.

If it is possible to control the EIG to allocate the defended area within updated defended zone, the problem can be resolved. Therefore, it is needed to control the EIG in order to improve reachability and performance of the guidance algorithm. This can be achieved to some extent by examination of the impact triangle shown in Fig. 1.

The *Intercept Point* I is stationary for the missile and target velocity vectors lying along the impact triangle sides for a non-manoeuvring target. In order to see this, consider the velocity of the impact point v_I in the inertial reference frame. The position vector of the *Intercept Point* I can be represented with respect to the target T:

$$\mathbf{r}_I = \mathbf{r}_t + s_t \mathbf{t}_t \tag{12}$$

where $\mathbf{r}_I$ and $\mathbf{r}_t$ are the position vectors of the *Intercept Point* and the target, and $\mathbf{t}_t$ is the unit vector tangent to the target velocity vector. Furthermore, from the first time derivative of the position vector, we have:

$$\mathbf{v}_I = \mathbf{v}_t + \dot{s}_t \mathbf{t}_t + s_t \dot{\mathbf{t}}_t = v_t \mathbf{t}_t + \dot{s}_t \mathbf{t}_t - s_t \dot{\theta}_{ft} \mathbf{n}_t \tag{13}$$

where θ_{ft} is the heading angle of the challenging missile. The intercept condition must obey the equation:

$$r = \left[\cos(\theta_t) + \sqrt{\gamma^2 - \sin^2(\theta_t)}\right] s_t \tag{14}$$

Differentiating the above equation with respect to time gives:

$$\dot{r} = \left[\cos(\theta_t) + \sqrt{\gamma^2 - \sin^2(\theta_t)}\right]\dot{s}_t - \left[\sin(\theta_t) + \frac{\sin(\theta_t)\cos(\theta_t)}{\sqrt{\gamma^2 - sin^2(\theta_t)}}\right]\dot{\theta}_t s_t \tag{15}$$

Then, $\dot{s}_t$ can be derived as:

$$\dot{s}_t = \frac{1}{\cos(\theta_t) + \sqrt{\gamma^2 - \sin^2(\theta_t)}}\dot{r} + \frac{\sin(\theta_t)}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} s_t \dot{\theta}_t \tag{16}$$

In Fig. 1, the sightline-to-missile angle is the desired angle to intercept a non-manoeuvring target. However, it can be different from the desired angle for a deliberate purpose, or due to the missile's physical constraints as shown in Fig. 5. To derive the range rate and the line-of-sight (LOS) angle rate, consider a guidance geometry of a challenging missile and a defending missile whose heading angle is either the same as or different from the desired angle.

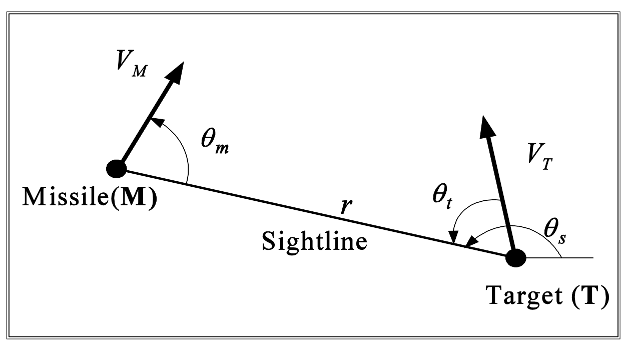


Fig. 5. Guidance geometry.

From the guidance geometry, the first time derivatives of the range and LOS are given by:

$$\begin{aligned}\dot{r} &= -\left[v_m \cos(\theta_m) + v_t \cos(\theta_t)\right] \\ &= -\left[\gamma \cos(\theta_m) + \cos(\theta_t)\right] v_t\end{aligned} \quad (17)$$

$$\begin{aligned}\dot{\theta}_s &= -\frac{v_m \sin(\theta_m) - v_t \sin(\theta_t)}{r} \\ &= -\frac{\gamma \sin(\theta_m) - \sin(\theta_t)}{r} v_t\end{aligned} \quad (18)$$

where θ_m is the real sightline-to-missile angle. Substituting Eq. (17) into Eq. (16) yields:

$$\dot{s}_t = -\frac{\gamma \cos(\theta_m) + \cos(\theta_t)}{\cos(\theta_t) + \sqrt{\gamma^2 - \sin^2(\theta_t)}} v_t + \frac{\sin(\theta_t)}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} s_t \dot{\theta}_t \quad (19)$$

Moreover, substituting this equation into Eq. (13) gives:

$$\begin{aligned}\mathbf{v}_I &= \left[1 - \frac{\gamma \cos(\theta_m) + \cos(\theta_t)}{\cos(\theta_t) + \sqrt{\gamma^2 - \sin^2(\theta_t)}}\right] v_t \mathbf{t}_t + \frac{\sin(\theta_t)}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} s_t \dot{\theta}_t \mathbf{t}_t - s_t \dot{\theta}_{ft} \mathbf{n}_t \\ &= \frac{\cos(\hat{\theta}_m) - \cos(\theta_m)}{\cos(\theta_t) + \sqrt{\gamma^2 - \sin^2(\theta_t)}} \gamma v_t \mathbf{t}_t + \frac{\sin(\theta_t)}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} s_t \dot{\theta}_t \mathbf{t}_t - s_t \dot{\theta}_{ft} \mathbf{n}_t\end{aligned} \quad (20)$$

This equation implies that the *Intercept Geometry* can be controlled.

For analysis of the guidance geometry, consider the direct intercept guidance for either a manoeuvring or non-manoeuvring challenging missile under the assumption that the missile can change its heading angle instantaneously; the missile is always heading for the *Intercept Point* derived using a geometrical matching condition, i.e.

$$\theta_m = \hat{\theta}_m \quad (21)$$

For a direct intercept guidance:

$$\dot{\theta}_t = \dot{\theta}_s - \dot{\theta}_{ft} \quad (22)$$

Substituting these two equations and into Eq. (20) yields:

$$\mathbf{v}_I = \frac{\sin(\theta_t)}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} s_t (\dot{\theta}_s - \dot{\theta}_{ft}) \mathbf{t}_t - s_t \dot{\theta}_{ft} \mathbf{n}_t \quad (23)$$

From the intercept matching condition, we have:

$$\gamma \sin(\theta_m) - \sin(\theta_t) = 0 \quad (24)$$

Then, the *Intercept Point* velocity becomes:

$$\begin{aligned}\mathbf{v}_I &= -\frac{\sin(\theta_t)}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} s_t \dot{\theta}_{ft} \mathbf{t}_t - s_t \dot{\theta}_{ft} \mathbf{n}_t \\ &= -\left[\sin(\theta_t) \mathbf{t}_t + \cos(\theta_t) \mathbf{n}_t\right] \frac{s_t}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} \dot{\theta}_{ft}\end{aligned} \quad (25)$$

Note that since:

$$r_I = \frac{s_t}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} \quad (26)$$

and the tangent vector, **t**, at the *Intercept Point* can be represented as:

$$\mathbf{t} = -\left[\sin(\theta_t) \mathbf{t}_t + \cos(\theta_t) \mathbf{n}_t\right] \quad (27)$$

the *Intercept Point* velocity is tangent to the intercept circle. For a non-manoeuvring challenging missile, as its challenging missile heading angle rate is zero, we have:

$$\mathbf{v}_I = 0 \quad (28)$$

and the *Intercept Point* is stationary in space.

Now, consider a non-manoeuvring target, but the missile-to-sightline angle θ_m is different from the desired angle θ_m for the intercept on the purpose controlling the *Intercept Geometry*. As represented in Eq. (20), in order to change the position of the impact point, the missile-to-sightline angle θ_m should be set to give a change rate of the *Intercept Point*. In a scenario that the challenging missile can hit one of the defended objects, the worst scenario for the missile is that the challenging missile is heading directly to the defended object as shown in Fig. 4. In this scenario, the heading angle of the challenging missile is locked on its target and its rate is zero:

$$\dot{\theta}_{tf} = 0 \quad (29)$$

Furthermore, the target-to-sightline angle rate is the same as that of the sightline:

$$\dot{\theta}_t = \dot{\theta}_s \quad (30)$$

Substituting these two equations and Eq. (18) into Eq. (20) yields:

$$\mathbf{v}_I = \frac{s_t}{r} \gamma v_t \left[\cos(\hat{\theta}_m) - \cos(\theta_m) - \frac{\sin(\theta_m) - \sin(\hat{\theta}_m)}{\sqrt{\gamma^2 - \sin^2(\theta_t)}} \sin(\theta_t)\right] \mathbf{t}_t \quad (31)$$

Note that the equation shows that it is possible to control the *Intercept Point* along the target tangent vector $\mathbf{t}_t$ by manipulating the sightline-to-missile angle. For further analysis, the angle is represented as:

$$\theta_m = \hat{\theta}_m + \alpha \quad (32)$$

Then, the *Intercept Point* velocity v_I along $\mathbf{t}_t$ can be derived:

$$v_I = \left\{\left[1 - \cos(\alpha) - \frac{\sin\alpha \sin(\theta_t)}{\gamma \cos(\hat{\theta}_m)}\right] \cos(\hat{\theta}_m) \left[\sin(\alpha) + \frac{1 - \cos(\alpha)}{\gamma \cos(\hat{\theta}_t)} \sin(\theta_t)\right]\right\} \frac{s_t}{r} \gamma v_t \quad (33)$$

Since the *Intercept Point* velocity can be derived as:

$$v_I = \frac{1-\cos(\alpha)}{\cos^2(\hat{\theta}_m)}\frac{s_t}{r}\gamma v_t \tag{34}$$

the *Intercept Point* velocity is positive or equal to zero.

4. Numerical Examples

Surface to air missile versus Anti ship missile (SvA) problems are considered to verify the proposed missile mid-course algorithm in area air-defence. Two SvA scenarios are classified by different anti ship missile (ASM) lateral acceleration: the acceleration is zero in the first scenario, and $10g$ in the second scenario. The initial conditions of ASM and surface to air missile (SAM) are the same in two scenarios except the ASM lateral acceleration and given in Table 1.

Note that it is possible to notice that the speed ratio γ is 2 from Table 1. In the simulations, it is assumed that two missiles are lag free systems and can change their flight heading angle instantaneously. Simulation results are represented in Figs. 6 and 7.

Table 1. Initial conditions

	ASM	SAM
Position (km, km)	(15, 5)	(0, 0)
Velocity (m/s)	300	600
Heading angle (deg)	0	-100

ASM: anti ship missile, SAM: surface to air missile.

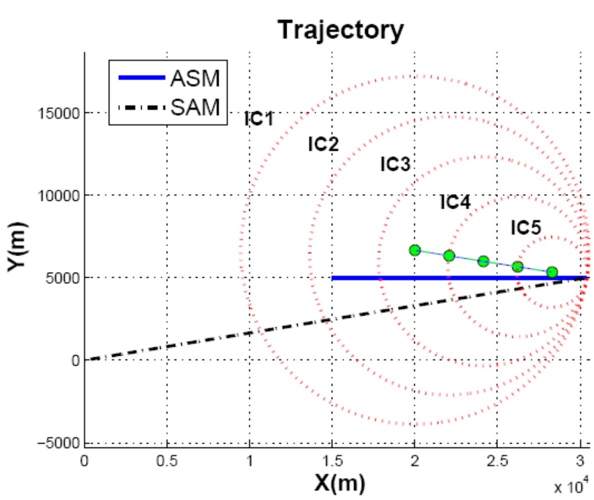


Fig. 6. Guidance geometry change of scenario 1.

ICs in the figures represent the intercept circle. As shown in Fig. 6, the *Intercept Geometry* does not change when the missile can instantaneously follow the desired heading angle for the non-manoeuvring challenging missile. Figure 7 shows that if the manoeuvres of the challenging missile (move?) away from a straight line trajectory to get to the intercept with the circular locus, then the missile can achieve the intercept within the circle that contains the ASM.

5. Conclusions

In this paper, the missile mid-course guidance law in area air-defence is proposed using the EIG concepts and differential geometry to resolve single-to-single problem. Its characteristics are mathematically analysed under several assumptions to improve reachability and overcome the drawbacks in our previous studies (Robb et al., 2005a, b). Furthermore, its performance is examined by using some numerical examples. The proposed method can be used in the launch phase because it provides the defended zone defined by the EIG. A launch window and launch point can be determined by finding a moment and position that the guidance algorithm assures all the defended area and objects are inside the defended zone.

For the next step of in the series of the mid-course guidance law in area air-defence, physical and operational constraints of the missiles will be considered to analyse the effect on the EIG. Furthermore, the multiple defending missiles versus multiple challenging missiles problem will be considered in order to develop cooperative missile guidance techniques and improve global defence performance.

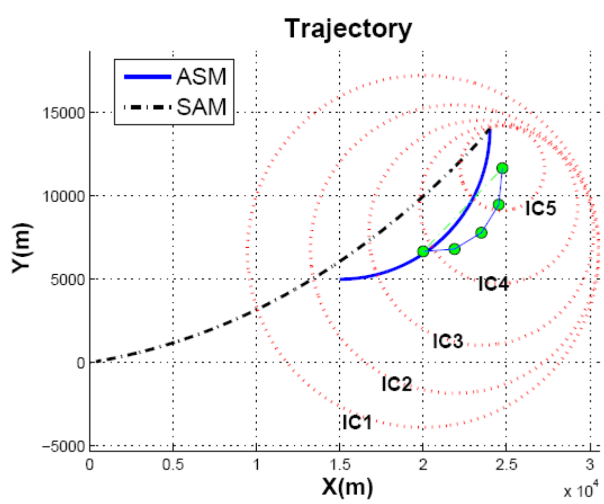


Fig. 7. Guidance geometry change of scenario 2.

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