Bull. Korean Math. Soc. **47** (2010), No. 4, pp. 701–714
DOI 10.4134/BKMS.2010.47.4.701

GEOMETRY OF SCREEN CONFORMAL REAL HALF LIGHTLIKE SUBMANIFOLDS

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Abstract. In this paper, we study the geometry of real half lightlike submanifolds of an indefinite Kaehler manifold. The main result is a characterization theorem for screen conformal real half lightlike submanifolds of an indefinite complex space form.

1. Introduction

It is well known that the radical distribution $\mathrm{Rad}(TM) = TM \cap TM^{\perp}$ of the lightlike submanifolds M of a semi-Rimannian manifold $(\bar{M}, \bar{g})$ of codimension 2 is a vector subbundle of the tangent bundle TM and the normal bundle $TM^{\perp}$, of rank 1 or 2. The codimension 2 lightlike submanifold (M, g) is called a *half lightlike submanifold* if $\mathrm{rank}(\mathrm{Rad}(TM)) = 1$. In this case, there exists two complementary non-degenerate distributions $S(TM)$ and $S(TM^{\perp})$ of $\mathrm{Rad}(TM)$ in TM and $TM^{\perp}$ respectively, called the *screen* and *co-screen distribution* on M. Then we have the following two orthogonal decompositions

$$TM = \mathrm{Rad}(TM) \oplus_{\mathrm{orth}} S(TM), \; TM^{\perp} = \mathrm{Rad}(TM) \oplus_{\mathrm{orth}} S(TM^{\perp}), \tag{1.1}$$

where the symbol $\oplus_{\mathrm{orth}}$ denotes the orthogonal direct sum. We denote such a half lightlike submanifold by $(M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M. Choose $L \in \Gamma(S(TM^{\perp}))$ as a unit vector field with $\bar{g}(L, L) = \epsilon = \pm 1$. Consider the orthogonal complementary distribution $S(TM)^{\perp}$ to $S(TM)$ in $T\bar{M}$. Certainly ξ and L belong to $\Gamma(S(TM)^{\perp})$. Hence we have the following orthogonal decomposition

$$S(TM)^{\perp} = S(TM^{\perp}) \;\oplus_{\mathrm{orth}}\; S(TM^{\perp})^{\perp},$$

where $S(TM^{\perp})^{\perp}$ is the orthogonal complementary to $S(TM^{\perp})$ in $S(TM)^{\perp}$. We known [3] that, for any smooth null section ξ of $\mathrm{Rad}(TM)$ on a coordinate

Received February 4, 2009.
2000 *Mathematics Subject Classification.* Primary 53C25, 53C40, 53C50.
Key words and phrases. real half lightlike submanifold, screen conformal, indefinite complex space form.

neighborhood $\mathcal{U} \subset M$, there exists a uniquely defined null vector field $N \in \Gamma(\mathrm{ltr}(TM))$ satisfying

$$\bar{g}(\xi, N) = 1,\ \bar{g}(N, N) = \bar{g}(N, X) = \bar{g}(N, L) = 0,\ \forall\, X \in \Gamma(S(TM)). \tag{1.2}$$

We call N, $\mathrm{ltr}(TM)$ and $\mathrm{tr}(TM) = S(TM^{\perp}) \oplus_{\mathrm{orth}} \mathrm{ltr}(TM)$ the *lightlike transversal vector field, lightlike transversal vector bundle* and *transversal vector bundle* of M with respect to the screen $S(TM)$ respectively. Therefore the tangent bundle $T\bar{M}$ of the ambient manifold $\bar{M}$ is decomposed as follows:

$$\begin{aligned} T\bar{M} &= TM \oplus \mathrm{tr}(TM) = \{\mathrm{Rad}(TM) \oplus \mathrm{tr}(TM)\} \oplus_{\mathrm{orth}} S(TM) \\ &= \{\mathrm{Rad}(TM) \oplus \mathrm{ltr}(TM)\} \oplus_{\mathrm{orth}} S(TM) \oplus_{\mathrm{orth}} S(TM^{\perp}). \end{aligned} \tag{1.3}$$

The objective of this paper is to study the geometry of real half lightlike submanifolds of an indefinite Kaehler manifold. First of all, we prove that such a real half lightlike submanifold M is a CR lightlike submanifold (Theorem 2.1) and if the induced structure tensor F on M is parallel, then M is locally a product manifold $M_2 \times M^{\sharp}$, where M_2 and $M^{\sharp}$ are leaves of some integrable distributions (Theorem 2.2). Next, we prove a characterization theorem for real half lightlike submanifolds M of an indefinite complex space form $\bar{M}(c)$: If M is screen conformal, then $c = 0$ (Theorem 3.5). Using this theorem, we prove several additional theorems for screen conformal real half lightlike submanifolds M of an indefinite complex space form $\bar{M}(c)$: If M is totally umbilical or an Einstein manifold, then M is Ricci flat (Theorems 4.3 and 4.4). If the conformal factor is a non-zero constant and the co-screen distribution is parallel, then M is locally a product manifold $M_2' \times M^{\hbar}$, where M_2' and $M^{\hbar}$ are leaves of some integrable distributions of M (Theorems 4.6 and 4.7).

Let $\bar{\nabla}$ be the Levi-Civita connection of $\bar{M}$ and P the projection morphism of $\Gamma(TM)$ on $\Gamma(S(TM))$ with respect to the decomposition (1.1). Then the local Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N + D(X, Y)L, \tag{1.4}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N + \rho(X)L, \tag{1.5}$$

$$\bar{\nabla}_X L = -A_L X + \phi(X)N, \tag{1.6}$$

$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi, \tag{1.7}$$

$$\nabla_X \xi = -A^*_{\xi} X - \tau(X)\xi \tag{1.8}$$

for all $X,\ Y \in \Gamma(TM)$, where ∇ and ∇^* are induced linear connections of M and on $S(TM)$ respectively, B and D are called the *local second fundamental forms* of M, C is called the *local second fundamental form* on $S(TM)$. A_N, A^*_{ξ} and A_L are linear operators on TM and τ, ρ and ϕ are 1-forms on TM. Since $\bar{\nabla}$ is torsion-free, ∇ is also torsion-free and both B and D are symmetric. From the facts $B(X, Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$ and $D(X, Y) = \epsilon \bar{g}(\bar{\nabla}_X Y, L)$, we know that B and D are independent of the choice of a screen distribution and

$$B(X, \xi) = 0, \quad D(X, \xi) = -\epsilon\phi(X),\ \forall\, X \in \Gamma(TM). \tag{1.9}$$

The induced connection ∇ of M is not metric and satisfies

$$(\nabla_X g)(Y,Z) = B(X,Y)\,\eta(Z) + B(X,Z)\,\eta(Y) \tag{1.10}$$

for all X, Y, $Z \in \Gamma(TM)$, where η is a 1-form on TM such that

$$\eta(X) = \bar{g}(X,N),\ \forall\, X \in \Gamma(TM). \tag{1.11}$$

But the connection ∇^* on $S(TM)$ is metric. Above three local second fundamental forms of M and $S(TM)$ are related to their shape operators by

$$B(X,Y) = g(A^*_\xi X, Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{1.12}$$

$$C(X,PY) = g(A_N X, PY), \qquad \bar{g}(A_N X, N) = 0, \tag{1.13}$$

$$\epsilon D(X,PY) = g(A_L X, PY), \qquad \bar{g}(A_L X, N) = \epsilon\rho(X), \tag{1.14}$$

$$\epsilon D(X,Y) = g(A_L X, Y) - \phi(X)\eta(Y),\ \forall\, X, Y \in \Gamma(TM). \tag{1.15}$$

By (1.12) and (1.13), we show that A^*_ξ and A_N are $\Gamma(S(TM))$-valued shape operators related to B and C respectively and A^*_ξ is self-adjoint on TM and

$$A^*_\xi \xi = 0, \tag{1.16}$$

that is, ξ is an eigenvector field of A^*_ξ corresponding to the eigenvalue 0. But A_N and A_L are not self-adjoint on $S(TM)$ and TM respectively.

We denote by $\bar{R}$, R and R^* the curvature tensors of the Levi-Civita connection $\bar{\nabla}$ of $\bar{M}$, the induced connection ∇ of M and the induced connection ∇^* on $S(TM)$ respectively. Using the Gauss-Weingarten equations for M and $S(TM)$, we obtain the Gauss-Codazzi equations for M and $S(TM)$:

$$\begin{aligned} \bar{g}(\bar{R}(X,Y)Z, PW) &= g(R(X,Y)Z, PW) \\ &\quad + B(X,Z)C(Y,PW) - B(Y,Z)C(X,PW) \\ &\quad + \epsilon\{D(X,Z)D(Y,PW) - D(Y,Z)D(X,PW)\}, \end{aligned} \tag{1.17}$$

$$\begin{aligned} \bar{g}(\bar{R}(X,Y)Z,\, \xi) &= (\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) \\ &\quad + B(Y,Z)\tau(X) - B(X,Z)\tau(Y) \\ &\quad + D(Y,Z)\phi(X) - D(X,Z)\phi(Y), \end{aligned} \tag{1.18}$$

$$\begin{aligned} \bar{g}(\bar{R}(X,Y)Z,\, N) &= \bar{g}(R(X,Y)Z,\, N) \\ &\quad + \epsilon\{D(X,Z)\rho(Y) - D(Y,Z)\rho(X)\}, \end{aligned} \tag{1.19}$$

$$\begin{aligned} \bar{g}(\bar{R}(X,Y)\xi, N) &= g(A^*_\xi X, A_N Y) - g(A^*_\xi Y, A_N X) \\ &\quad - 2d\tau(X,Y) + \rho(X)\phi(Y) - \rho(Y)\phi(X), \end{aligned} \tag{1.20}$$

$$\begin{aligned} \bar{g}(R(X,Y)PZ,\, PW) &= g(R^*(X,Y)PZ,\, PW) \\ &\quad + C(X,PZ)B(Y,PW) - C(Y,PZ)B(X,PW), \end{aligned} \tag{1.21}$$

$$\begin{aligned} g(R(X,Y)PZ,\, N) &= (\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) \\ &\quad + C(X,PZ)\tau(Y) - C(Y,PZ)\tau(X). \end{aligned} \tag{1.22}$$

The *Ricci tensor*, denoted by $\bar{Ric}$, of $\bar{M}$ is defined by

$$\bar{Ric}(X,Y) = \text{trace}\{Z \to \bar{R}(Z,X)Y\}\,,\ \forall\ X,\, Y \in \Gamma(T\bar{M}). \tag{1.23}$$

In case Ricci tensor vanishes on $\bar{M}$, we say that $\bar{M}$ is *Ricci flat.* If $\dim \bar{M} > 2$ and $\bar{Ric} = \bar{\gamma} g$, where γ is a constant, then $\bar{M}$ is called an *Einstein manifold.* For $\dim \bar{M} = 2$, any $\bar{M}$ is Einstein but $\bar{\gamma}$ is not necessarily constant.

2. Real half lightlike submanifolds

Let $\bar{M} = (\bar{M}, J, \bar{g})$ be a real $2m$-dimensional indefinite Kaehler manifold, where $\bar{g}$ is a semi-Riemannian metric of index $q = 2v,\ 0 < v < m$ and J is an almost complex structure on $\bar{M}$ satisfying, for all $X,\, Y \in \Gamma(T\bar{M})$,

$$J^2 = -I, \quad \bar{g}(JX, JY) = \bar{g}(X,Y), \quad (\bar{\nabla}_X J)Y = 0. \tag{2.1}$$

Definition 1. Let $(M, g, S(TM))$ be a real lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. We say that M is a *CR-lightlike submanifold* [3] of $\bar{M}$ if the following two conditions are fulfilled:

(A) $J(\text{Rad}(TM))$ is a distribution on M such that

$$\text{Rad}(TM) \cap J(\text{Rad}(TM)) = \{0\}.$$

(B) There exist vector bundles H_o and H' over M such that

$$S(TM) = \{J(\text{Rad}(TM)) \oplus H'\} \oplus_{\text{orth}} H_o;\ J(H_o) = H_o;\ J(H') = K_1 \oplus_{\text{orth}} K_2,$$

where H_o is a non-degenerate almost complex distribution on M, and K_1 and K_2 are vector subbundles of $\text{ltr}(TM)$ and $S(TM^{\perp})$ respectively.

An indefinite complex space form $\bar{M}(c)$ is a connected indefinite Kaehler manifold of constant holomorphic sectional curvature c such that

$$\begin{aligned} \bar{R}(X,Y)Z = \frac{c}{4}\{&\bar{g}(Y,Z)X - \bar{g}(X,Z)Y + \bar{g}(JY,Z)JX \\ &- \bar{g}(JX,Z)JY + 2\bar{g}(X,JY)JZ\},\ \forall\ X,\, Y,\, Z \in \Gamma(TM). \end{aligned} \tag{2.2}$$

Theorem 2.1. *Any real half lightlike submanifold $(M, g, S(TM))$ of an indefinite Kaehler manifold $\bar{M}$ is a CR-lightlike submanifold of $\bar{M}$.*

Proof. From the fact that $\bar{g}(J\xi, \xi) = 0$ and $\text{Rad}(TM) \cap J(\text{Rad}(TM)) = \{0\}$, the vector bundle $J(\text{Rad}(TM))$ is a subbundle of $S(TM)$ or $S(TM^{\perp})$ of rank 1. Also, from the fact that $\bar{g}(JN, N) = 0$ and $\bar{g}(JN, \xi) = -\bar{g}(N, J\xi) = 0$, the vector bundle $J(\text{ltr}(TM))$ is also a subbundle of $S(TM)$ or $S(TM^{\perp})$ of rank 1. Since $J\xi$ and JN are null vector fields satisfying $\bar{g}(J\xi, JN) = 1$ and both $S(TM)$ and $S(TM^{\perp})$ are non-degenerate distributions, we show that $\{J\xi, JN\} \in \Gamma(S(TM))$ or $\{J\xi, JN\} \in \Gamma(S(TM^{\perp}))$. If $\{J\xi, JN\} \in \Gamma(S(TM^{\perp}))$, as $J(\text{Rad}(TM)),\ J(\text{ltr}(TM))$ and $S(TM^{\perp})$ are non-degenerate of rank 1, we have $J(\text{Rad}(TM)) = J(\text{ltr}(TM)) = S(TM^{\perp})$. It is a contradiction. Thus we choose a screen distribution $S(TM)$ that contains $J(\text{Rad}(TM))$ and $J(\text{ltr}(TM))$. For $L \in \Gamma(S(TM^{\perp}))$, as $\bar{g}(JL, L) = 0,\ \bar{g}(JL, \xi) = -\bar{g}(L, J\xi) = 0$

and $\bar{g}(JL, N) = -\bar{g}(L, JN) = 0$, $J(S(TM^{\perp}))$ is also a vector subbundle of $S(TM)$ such that

$$J(S(TM^{\perp})) \oplus_{\text{orth}} \{J(\text{Rad}(TM)) \oplus J(\text{ltr}(TM))\}.$$

We choose $S(TM)$ to contain $J(S(TM^{\perp}))$ too. Thus the screen distribution $S(TM)$ is expressed as follow:

$$S(TM) = \{J(\text{Rad}(TM)) \oplus J(\text{ltr}(TM))\} \oplus_{\text{orth}} J(S(TM^{\perp})) \oplus_{\text{orth}} H_o, \tag{2.3}$$

where H_o is a non-degenerate distribution, otherwise $S(TM)$ would be degenerate. Moreover, by (2.3), we show that H_o is an almost complex distribution on M with respect to J, i.e., $J(H_o) = H_o$. Finally, denote $H' = J(\text{ltr}(TM)) \oplus_{\text{orth}} J(S(TM^{\perp}))$. Thus (2.3) gives $S(TM)$ as in condition (B) and $J(H') = K_1 \oplus_{\text{orth}} K_2$, where $K_1 = \text{ltr}(TM)$ and $K_2 = S(TM^{\perp})$. Hence M is a CR-lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. □

From Theorem 2.1, the general decompositions (1.1) and (1.3) reduce to

$$TM = H \oplus H', \quad T\bar{M} = H \oplus H' \oplus \text{tr}(TM), \tag{2.4}$$

where H is a 2-lightlike almost complex distribution on M such that

$$H = \text{Rad}(TM) \oplus_{\text{orth}} J(\text{Rad}(TM)) \oplus_{\text{orth}} H_o. \tag{2.5}$$

Consider the null and non-null vector fields $\{U, V\}$ and W such that

$$U = -JN, \quad V = -J\xi, \quad W = -JL. \tag{2.6}$$

Denote by S the projection morphism of TM on H. Then, by the first equation of (2.4) [denote (2.4)-1], any vector field on M is expressed as follows:

$$X = SX + u(X)U + w(X)W, \quad JX = FX + u(X)N + w(X)L, \tag{2.7}$$

where u, v and w are 1-forms locally defined on M by

$$u(X) = g(X, V), \quad v(X) = g(X, U), \quad w(X) = \epsilon\, g(X, W) \tag{2.8}$$

and F is a tensor field of type $(1, 1)$ globally defined on M by

$$FX = JSX, \quad \forall\ X \in \Gamma(TM).$$

Apply J to (2.7)-2 and using (2.1) and (2.8), we have

$$F^2X = -X + u(X)U + w(X)W\,; \tag{2.9}$$
$$u(U) = w(W) = 1,\ FU = FW = 0.$$

By using (1.9), (2.1), (2.7)-2 and (2.8) and Gauss-Weingarten equations for a half lightlike submanifold, we deduce

$$(\nabla_X u)(Y) = -u(Y)\tau(X) - w(Y)\phi(X) - B(X, FY), \tag{2.10}$$
$$(\nabla_X v)(Y) = v(Y)\tau(X) + \epsilon\, w(Y)\rho(X) - g(A_N X, FY), \tag{2.11}$$
$$(\nabla_X w)(Y) = -u(Y)\rho(X) - D(X, FY), \tag{2.12}$$
$$(\nabla_X F)(Y) = u(Y)A_N X + w(Y)A_L X - B(X, Y)U - D(X, Y)W. \tag{2.13}$$

Differentiating (2.6) with X and using (1.5), (1.7), (2.1) and (2.9), we have

$$B(X,U) = v(A^*_\xi X) = u(A_N X) = C(X,V); \tag{2.14}$$

$$C(X,W) = v(A_L X) = \epsilon\, w(A_N X) = \epsilon\, D(X,U); \tag{2.15}$$

$$B(X,W) = u(A_L X) = \epsilon\, w(A^*_\xi X) = \epsilon\, D(X,V); \tag{2.16}$$

$$\nabla_X U = F(A_N X) + \tau(X)U + \rho(X)W, \tag{2.17}$$

$$\nabla_X V = F(A^*_\xi X) - \tau(X)V - \epsilon\,\phi(X)W, \tag{2.18}$$

$$\nabla_X W = F(A_L X) + \phi(X)U. \tag{2.19}$$

Lemma 1. *Let $(M, g, S(TM))$ be a real half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. If F is parallel with respect to the induced connection ∇, i.e., $(\nabla_X F)Y = 0$ for all X, $Y \in \Gamma(TM)$, then we have*

$$B(X,Y) = u(Y)B(X,U), \quad D(X,Y) = w(Y)D(X,W), \tag{2.20}$$

$$\begin{aligned} B(X,V) &= B(X,W) = C(X,U) = C(X,W) \\ &= D(X,V) = D(X,U) = \phi(X) = \rho(X) = 0. \end{aligned} \tag{2.21}$$

Proof. If F is parallel with respect to the induced connection ∇, then, taking the scalar product with V, W, U and N at (2.13) by turns, we have

$$B(X,Y) = u(Y)u(A_N X) + w(Y)u(A_L X), \tag{2.22}$$

$$D(X,Y) = u(Y)w(A_N X) + w(Y)w(A_L X), \tag{2.23}$$

$$\begin{aligned} &u(Y)v(A_N X) + w(Y)v(A_L X) = 0, \\ &w(Y)g(A_L X, N) = 0, \quad \forall\, X,\, Y \in \Gamma(TM). \end{aligned} \tag{2.24}$$

Replace Y by V, U and W in (2.22), (2.23) and (2.24), Y by ξ in (2.23) and Y by W in the last equation, we obtain (2.21). From (2.21), (2.22) and (2.23), we have (2.20). □

Theorem 2.2. *Let $(M, g, S(TM))$ be a real half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. If F is parallel with respect to the induced connection ∇ on M, then the distributions H and H' are integrable and parallel with respect to ∇ and M is locally a product manifold $M_2 \times M^\sharp$, where M_2 is a leaf of H' and $M^\sharp$ is a leaf of H.*

Proof. Using (1.4), (1.7), (1.12) and (2.1), we derive

$$\begin{aligned} &g(\nabla_X \xi, V) = -B(X,V), && g(\nabla_X \xi, W) = -B(X,W), \\ &g(\nabla_X V, V) = 0, && g(\nabla_X V, W) = -\phi(X), \\ &g(\nabla_X Y, V) = B(X, JY), && g(\nabla_X Y, W) = \epsilon\, D(X, JY) \end{aligned} \tag{2.25}$$

for any $X \in \Gamma(H)$ and $Y \in \Gamma(H_o)$. Since F is parallel with respect to the induced connection ∇, we have $B(X,V) = B(X,W) = \phi(X) = 0$. Take $Y \in \Gamma(H_o)$ in two equations of (2.20), we have $B(X,Y) = 0$ and $D(X,Y) = 0$ for all $X \in \Gamma(TM)$ respectively. Thus we have $B(X,JY) = D(X,JY) = 0$ due to $JY \in \Gamma(H_o)$. Thus H is integrable and parallel with respect to ∇.

Using the Gauss-Weingarten formulas, (1.12)∼(1.15) and (2.1), we derive

$$\begin{aligned} &g(\nabla_Z W, N) = \epsilon D(Z, U), \qquad &&g(\nabla_Z U, N) = C(Z, U),\\ &g(\nabla_Z W, U) = -\epsilon \rho(Z), &&g(\nabla_Z U, U) = 0,\\ &g(\nabla_Z W, Y) = -\epsilon D(Z, JY), &&g(\nabla_Z U, Y) = -C(Z, JY) \end{aligned} \tag{2.26}$$

for any $Z \in \Gamma(H')$ and $Y \in \Gamma(H_o)$. Since F is parallel with respect to ∇ and $Z = U$ or W, we have $D(Z, U) = D(Z, JY) = C(Z, U) = C(Z, JY) = 0$ and $\rho(Z) = 0$. Thus the distribution H' is also integrable and parallel with respect to the induced connection ∇. From this result, we have our theorem. □

The *type numbers* $t_{_N}(p)$ and $t_{_L}(p)$ of M at a point $p \in M$ is the rank of the shape operators A_N and A_L at p respectively. By the equation (2.13) it follows that $A_N X = B(X, U)U$ and $A_L X = D(X, W)W$. Thus we have:

Theorem 2.3. *Let $(M, g, S(TM))$ be a real half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. If F is parallel with respect to the induced connection ∇ on M, then the type numbers of M satisfy $t_{_N}(p) \leq 1$ and $t_{_L}(p) \leq 1$ for any $p \in M$.*

3. Screen conformal half lightlike submanifolds

A half lightlike submanifold $(M, g, S(TM))$ of a semi-Riemannian manifold $(\bar{M}, \bar{g})$ is *screen conformal* [1] if the shape operators A_N and A^*_ξ of M and $S(TM)$ respectively are related by $A_N = \varphi\, A^*_\xi$, or equivalently

$$C(X, PY) = \varphi\, B(X, Y), \quad \forall\; X,\, Y \in \Gamma(TM), \tag{3.1}$$

where φ is a non-vanishing smooth function on a neighborhood $\mathcal{U}$ in M. In particular, if φ is a non-zero constant, M is called *screen homothetic* [4].

Note 1. For a screen conformal half lightlike submanifold M, the second fundamental form C is symmetric on $S(TM)$. Thus $S(TM)$ is an integrable distribution and M is locally a product manifold $L_\xi \times M^*$ where L_ξ is a null curve tangent to $\mathrm{Rad}(TM)$ and M^* is a leaf of $S(TM)$ [3].

From (2.14), (2.15), (2.16) and (3.1), we obtain

$$h(X,\, U - \varphi V) = 0, \quad \forall\, X \in \Gamma(TM), \tag{3.2}$$

where $h(X, Y) = B(X, Y)N + D(X, Y)L$ is the global second fundamental form tensor of M. Thus we have:

Theorem 3.1. *Let $(M, g, S(TM))$ be a screen conformal real half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then the non-null vector field $U - \varphi V \neq 0$ is conjugate to any vector field on M. In particular, $U - \varphi V$ is an asymptotic vector field.*

Corollary 1. *Let $(M, g, S(TM))$ be a screen conformal real half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then the second fundamental form h on M (consequently, C on $S(TM)$) is degenerate on $S(TM)$.*

Proof. Since $h(X, U - \varphi V) = 0$ for all $X \in \Gamma(S(TM))$ and $U - \varphi V \in \Gamma(S(TM))$, the second fundamental form tensor h is degenerate on $S(TM)$. □

Theorem 3.2. *Let $(M, g, S(TM))$ be a screen conformal real half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. If M is totally umbilical, then M and $S(TM)$ are totally geodesic.*

Proof. If M is totally umbilical, then there exists a smooth transversal vector field $\mathcal{H} \in \Gamma(\mathrm{tr}(TM))$ such that

$$h(X, Y) = \mathcal{H}\, g(X, Y), \ \forall\ X,\ Y \in \Gamma(TM).$$

From this fact and the equation (3.2), we have

$$\mathcal{H}\, g(X,\, U - \varphi V) = 0, \ \forall\, X\ \in \Gamma(TM).$$

Replace X by V in this equation, we have $\mathcal{H} = 0$. Thus $h = 0$. It follow that $B = D = 0$ and $C = 0$. Consequently, M and $S(TM)$ are totally geodesic. □

Theorem 3.3. *Let $(M, g, S(TM))$ be a screen conformal real totally umbilical half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. Then H is an integrable and parallel distribution with respect to ∇ and M is locally a product manifold $L_u \times L_w \times M^{\sharp}$, where L_u and L_w are null and non-null curves tangent to $J(\mathrm{ltr}(TM))$ and $J(S(TM^{\perp}))$ respectively and $M^{\sharp}$ is a leaf of H.*

Proof. Since M is totally umbilical, both M and $S(TM)$ are totally geodesic and $B = D = C = \phi = 0$. All equations of (2.25) are zero. Thus H is an integrable and parallel distribution with respect to ∇. Also, $J(\mathrm{ltr}(TM))$ and $J(S(TM^{\perp})$ are integrable distributions. Thus we have our theorem. □

Theorem 3.4. *Let $(M, g, S(TM))$ be a screen conformal real totally umbilical half lightlike submanifold of an indefinite Kaehler manifold $\bar{M}$. If U or W is parallel with respect to the induced connection ∇ on M, then the distributions H and H' are integrable and parallel with respect to ∇ and M is locally a product manifold $M_2 \times M^{\sharp}$, where M_2 is a leaf of H' and $M^{\sharp}$ is a leaf of H.*

Proof. As M is totally umbilical, M and $S(TM)$ are totally geodesic and all of (2.25) and (2.26) are zero except only $g(\nabla_Z JL,\, JN) = -\epsilon\, \rho(Z)$. If U is parallel, applying J to (2.17) and using (2.6) and (2.9), we obtain

$$A_N X = u(A_N X)U + w(A_N X)W\,; \quad \tau(X) = \rho(X) = 0,\ \forall\, X \in \Gamma(TM).$$

If W is parallel, applying J to (2.19) and by using (2.6) and (2.9), we obtain

$$A_L X = u(A_L X)U + w(A_L X)W\,; \quad \phi(X) = 0,\ \forall\, X \in \Gamma(TM).$$

From the last equation, we have $\rho(X) = \epsilon\, g(A_L X,\, N) = 0$. Thus H and H' are integrable and parallel distributions on M. We have our theorem. □

Theorem 3.5. *Let $(M, g, S(TM))$ be a screen conformal real half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$. Then we have $c = 0$.*

Proof. By using (1.18) and (2.2), we have

$$\begin{aligned}&\frac{c}{4}\{u(X)\bar{g}(JY,Z)-u(Y)\bar{g}(JX,Z)+2u(Z)\bar{g}(X,JY)\}\\ &=(\nabla_X B)(Y,Z)-(\nabla_Y B)(X,Z)+B(Y,Z)\tau(X)\\ &\quad -B(X,Z)\tau(Y)+D(Y,Z)\phi(X)-D(X,Z)\phi(Y)\end{aligned}$$

for all $X,\,Y,\,Z\in\Gamma(TM)$. Using this, (1.19), (1.22) and (3.1), we obtain

$$\begin{aligned}&\frac{c}{4}\{g(Y,PZ)\eta(X)-g(X,PZ)\eta(Y)+v(X)\bar{g}(JY,PZ)\\ &-v(Y)\bar{g}(JX,PZ)+2v(PZ)\bar{g}(X,JY)\}\\ &=\{X[\varphi]-2\varphi\tau(X)\}B(Y,PZ)-\{Y[\varphi]-2\varphi\tau(Y)\}B(X,PZ)\\ &\quad+\{\varphi\phi(Y)+\epsilon\rho(Y)\}D(X,PZ)-\{\varphi\phi(X)+\epsilon\rho(X)\}D(Y,PZ)\\ &\quad+\frac{c}{4}\varphi\{u(X)\bar{g}(JY,PZ)-u(Y)\bar{g}(JX,PZ)+2u(PZ)\bar{g}(X,JY)\}.\end{aligned}$$

Replacing PZ by μ in the last equation and using (3.2), we obtain

$$\frac{c}{2}\{2\varphi\,g(X,JY)+(v(X)-\varphi\,u(X))\eta(Y)-(v(Y)-\varphi\,u(Y))\eta(X)\}=0.$$

Taking $X=V,\ Y=\xi$ in this equation, we obtain $c=0$. □

Corollary 2. *There exist no screen conformal real half lightlike submanifolds M of indefinite complex space form $\bar{M}(c)$ with $c\neq 0$.*

4. Induced Ricci curvatures

Let $R^{(0,2)}$ denote the induced Ricci type tensor of M given by

$$R^{(0,2)}(X,Y)=\mathrm{trace}\{Z\to R(Z,X)Y\} \tag{4.1}$$

for all $X,\,Y,\,Z\in\Gamma(TM)$. Substituting the Gauss-Codazzi equations (1.17) and (1.19) in (1.23), then, using the relations (1.12)∼(1.15), we obtain

$$\begin{aligned}R^{(0,2)}(X,Y)&=\bar{Ric}(X,Y)+B(X,Y)\mathrm{tr}A_N+D(X,Y)\mathrm{tr}A_L\\ &\quad-g(A_NX,\,A^*_\xi Y)-\epsilon\,g(A_LX,\,A_LY)+\rho(X)\phi(Y)\\ &\quad-\bar{g}(\bar{R}(\xi,Y)X,\,N)-\epsilon\,\bar{g}(\bar{R}(L,Y)X,\,L).\end{aligned}$$

A tensor field $R^{(0,2)}$ of M is called its *induced Ricci tensor* if it is symmetric. In the sequel, a symmetric $R^{(0,2)}$ tensor will be denoted by *Ric*.

If $\bar{M}$ is an indefinite complex space form $\bar{M}(c)$, using (2.2), we have

$$\begin{aligned}R^{(0,2)}(X,Y)&=\frac{c}{4}\{(2m+1)g(X,Y)-u(X)v(Y)-2v(X)u(Y)\}\\ &\quad+B(X,Y)\mathrm{tr}A_N+D(X,Y)\mathrm{tr}A_L-g(A_NX,A^*_\xi Y)\\ &\quad-\epsilon\,g(A_LX,\,A_LY)+\rho(X)\phi(Y).\end{aligned} \tag{4.2}$$

Moreover, if M is a screen conformal, then (4.2) reduces to

$$R^{(0,2)}(X,Y)\ =\varphi\{B(X,Y)\mathrm{tr}A^*_\xi-g(A^*_\xi X,A^*_\xi Y)\} \tag{4.3}$$

$$+ \ D(X,Y)\mathrm{tr}A_L - \varepsilon g(A_L X, A_L Y) + \rho(X)\phi(Y).$$

From (1.20) and (4.3), we have the following assertions:

Theorem 4.1. *For a screen conformal half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$, the following assertions are equivalent*;

(1) *The Ricci type tensor $R^{(0,2)}$ is a symmetric Ricci tensor.*
(2) *each 1-form τ is closed, i.e., $d\tau = 0$ on any $\mathcal{U} \subset M$.*
(3) $\rho(X)\phi(Y) = \rho(Y)\phi(X)$ *for all* $X, Y \in \Gamma(TM)$.

Theorem 4.2. *Let $(M, g, S(TM))$ be a screen conformal real half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$ with a parallel co-screen distribution. Then $R^{(0,2)}$ is a symmetric Ricci tensor and*

$$R^{(0,2)}(X,Y) = \varphi\{B(X,Y)\mathrm{tr}A^*_\xi - g(A^*_\xi X, A^*_\xi Y)\}. \tag{4.4}$$

Theorem 4.3. *Let $(M, g, S(TM))$ be a screen conformal real half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$. If M is totally umbilical, then M is Ricci flat.*

Proof. From Theorem 3.2, M is totally geodesic. Thus, from (1.12) and (1.14), we have $B = D = A^*_\xi = \phi = 0$ and $A_L X = \epsilon\rho(X)\xi$. Therefore, using (4.3), we obtain $R^{(0,2)}(X,Y) = 0$ and $R^{(0,2)} = Ric$. □

As $\{U, V\}$ is a basis of $\Gamma(J(\mathrm{Rad}(TM)) \oplus J(\mathrm{ltr}(TM)))$, the vector fields

$$\mu = U - \varphi V, \qquad \nu = U + \varphi V \tag{4.5}$$

form also a basis of $\Gamma(J(\mathrm{Rad}(TM)) \oplus J(\mathrm{ltr}(TM)))$. From (3.2), we have

$$g(A^*_\xi \mu, X) = 0, \quad g(A^*_\xi \mu, N) = 0, \qquad A^*_\xi \mu = 0, \tag{4.6}$$

$$g(A_L \mu, X) = 0, \quad g(A_L \mu, N) = \epsilon\rho(\mu), \quad A_L \mu = \epsilon\rho(\mu)\xi, \tag{4.7}$$

due to $\phi(\mu) = -\epsilon\, D(\mu, \xi) = 0$. Thus μ is an eigenvector field of A^*_ξ on $S(TM)$ corresponding to the eigenvalue 0. From (2.15), (3.1), (4.5) and the linearity of F, for all $X \in \Gamma(TM)$, we have

$$\nabla_X \mu = \tau(X)\nu - X[\varphi]V + (\rho + \epsilon\varphi\phi)(X)W, \tag{4.8}$$

$$\nabla_X \nu = 2F(A_N X) + \tau(X)\mu + X[\varphi]V + (\rho - \epsilon\varphi\phi)(X)W. \tag{4.9}$$

Theorem 4.4. *Let $(M, g, S(TM))$ be a screen conformal real half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$ with a symmetric Ricci tensor. If M is an Einstein manifold, then M is Ricci flat.*

Proof. If M is a screen conformal real half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$ with a symmetric Ricci tensor, then $c = 0$ and $R^{(0,2)} = Ric$. Let M be an Einstein manifold, that is, $R^{(0,2)} = \gamma g$. Replacing X and Y by V and μ in (4.3) respectively and using (4.6) and (4.7), we obtain $\gamma = 0$. Thus M is Ricci flat. □

Note 2. Suppose $R^{(0,2)}$ is symmetric, since $d\tau = 0$, there exists a pair $\{\xi, N\}$ on $\mathcal{U}$ such that the corresponding 1-form τ vanishes. We call such a pair the *distinguished null pair* of M.

Although $S(TM)$ is not unique, it is canonically isomorphic to the factor vector bundle $TM^* = TM/\mathrm{Rad}\,TM$ considered by Kupeli [7]. Thus all $S(TM)$ are isomorphic. For this reason, we consider only screen homothetic real half lightlike submanifolds equipped with the distinguished null pairs.

Theorem 4.5. *Let $(M, g, S(TM))$ be a half lightlike submanifolds of a semi-Remanning manifold $(\bar{M}, \bar{g})$. Then the co-screen distribution $S(TM^{\perp})$ is parallel with respect to the connection $\bar{\nabla}$ if and only if $A_L = 0$ on $\Gamma(TM)$.*

Proof. If the co-screen distribution $S(TM^{\perp})$ is parallel with respect to the connection $\bar{\nabla}$, then, from (1.6), we have $A_L X = \phi(X)N$ for all $X \in \Gamma(TM)$. Taking the scaler product with ξ and N to this equation, we obtain $\phi = 0$ and $\rho = 0$ respectively. Consequently, we obtain $A_L = 0$ and $D = 0$. Conversely, if $A_L X = 0$ for all $X \in \Gamma(TM)$, then, from (1.14), we have $D = \rho = 0$. From (1.9), we obtain $\phi = 0$. Thus L is parallel with respect to the connection $\bar{\nabla}$. □

Theorem 4.6. *Let $(M, g, S(TM))$ be a screen homothetic real half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$ with a parallel co-screen distribution. Then M is locally a product manifold $M_2' \times M^{\hbar}$, where M_2' and $M^{\hbar}$ are some leaves of integrable distributions of M.*

Proof. Let $\mathcal{H}' = \mathrm{Span}\{\mu, W\}$. Then $\mathcal{H} = H_o \oplus_{\mathrm{orth}} \mathrm{Span}\{\xi, \nu\}$ is a complementary subbundle to $\mathcal{H}'$ in TM and we have the decomposition

$$TM = \mathcal{H}' \oplus_{\mathrm{orth}} \mathcal{H}. \tag{4.10}$$

Using (1.8), (2.19), (4.8) and (4.9), for $X \in \Gamma(\mathcal{H})$ and $Y \in \Gamma(H_o)$, we derive

$$\begin{aligned} &g(\nabla_X Y, \mu) = 0, && g(\nabla_X Y, W) = -g(F(A_L X), Y), \\ &g(\nabla_X \nu, \mu) = X[\varphi] - 2\varphi\tau(X), && g(\nabla_X \nu, W) = (\epsilon\rho - \varphi\phi)(X), \\ &g(\nabla_X \xi, \mu) = -B(X, \mu) = 0, && g(\nabla_X \xi, W) = -B(X, W). \end{aligned} \tag{4.11}$$

If L is parallel, then we have $A_L = D = \phi = \rho = 0$. From (2.16), we get $B(X, W) = 0$. Thus all of the equation (4.11) are 0. Thus $\mathcal{H}$ is parallel with respect to ∇ and $\mathcal{H}$ is an integrable distribution.

Also, using (2.19) and (4.9), for $X \in \Gamma(\mathcal{H}')$ and $Y \in \Gamma(H_o)$, we derive

$$\begin{aligned} &g(\nabla_X \mu, \xi) = 0, && g(\nabla_X W, \xi) = 0, \\ &g(\nabla_X \mu, \nu) = -X[\varphi] + 2\varphi\tau(X), && g(\nabla_X W, \nu) = -(\epsilon\rho - \varphi\phi)(X), \\ &g(\nabla_X \mu, Y) = 0, && g(\nabla_X W, Y) = g(F(A_L X), Y). \end{aligned} \tag{4.12}$$

If L is parallel, then all of the equation (4.12) are 0. Thus $\mathcal{H}'$ is parallel with respect to ∇ and $\mathcal{H}'$ is an integrable distribution. Thus we have our theorem. □

Let $\mathcal{H}' = \text{Span}\{\mu, W\}$. Then $\mathcal{G} = H_o \oplus_{\text{orth}} \text{Span}\{\nu\}$ is a complementary vector subbundle to $\mathcal{G}$ in $S(TM)$. From (4.11) and (4.12), we show the distributions $\mathcal{H}'$ and $\mathcal{G}$ are integrable and we have the following decomposition

$$S(TM) = \mathcal{H}' \oplus_{\text{orth}} \mathcal{G}. \tag{4.13}$$

Theorem 4.7. *Let $(M, g, S(TM))$ be a screen homothetic Einstein real half lightlike submanifold of an indefinite complex space form $\bar{M}(c)$ of index 2. If the co-screen $S(TM^{\perp})$ is a parallel distribution, then M is locally a product manifold $L_\xi \times M_2' \times M^{\text{ß}}$ or $L_\xi \times M_2' \times (M^{\text{ß}} = L_\alpha \times M^0)$, where L_ξ and L_α are null and spacelike curve, M_2' is a hyperbolic plane, and both $M^{\text{ß}}$ and M^0 are Euclidean spaces.*

Proof. By Theorem 4.4 and the equation (4.4), we have

$$g(A_\xi^* X, A_\xi^* Y) - \text{tr}A_\xi^*\, g(A_\xi^* X, Y) = 0. \tag{4.14}$$

From (1.12) and (2.16), we obtain $A_\xi^* W = 0$. Thus ξ, μ and W are eigenvector fields of A_ξ^* corresponding the eigenvalue 0. Let $\mu = \frac{1}{\sqrt{2\epsilon_1 \varphi}}\{U - \varphi V\}$ where $\epsilon_1 = \text{sgn}\varphi$. μ is a timelike vector field and $\mathcal{G}$ is an integrable Riemannian distribution. Since A_ξ^* is $\Gamma(\mathcal{G})$-valued real symmetric operator due to $g(A_\xi^* X,\, N) = g(A_\xi^* X,\, \mu) = g(A_\xi^* X,\, W) = 0$, A_ξ^* have $(2m - 5) \equiv n$ real orthonormal eigenvector fields in $\mathcal{G}$ and is diagonalizable. Consider a frame field of eigenvectors $\{\mu, W, e_1, \ldots, e_n\}$ of A_ξ^* on $S(TM)$ such that $\{e_1, \ldots, e_n\}$ is an orthonormal frame field of A_ξ^* on $\mathcal{G}$. Then $A_\xi^* e_i = \lambda_i e_i\, (1 \leq i \leq n)$. Put $X = Y = e_i$ in (4.14), λ_i is a solution of equation

$$x(x - \alpha) = 0,$$

where $\alpha = \text{tr}A_\xi^*$. This equation has at most two distinct solutions 0 and α on $\mathcal{U}$. Assume that there exists $p \in \{0, \ldots, n\}$ such that $\lambda_1 = \cdots = \lambda_p = 0$ and $\lambda_{p+1} = \cdots = \lambda_n = \alpha$, by renumbering if necessary, then we have

$$\alpha = \text{tr}A_\xi^* = (n - p)\alpha\,.$$

If $\alpha = 0$, then $A_\xi^* X = 0$ for all $X \in \Gamma(TM)$. Thus M is a totally geodesic and $S(TM)$ is also totally geodesic. From (1.17) and (1.21), we have $R^*(X, Y)Z = \bar{R}(X, Y)Z = 0$ for all $X,\, Y,\, Z \in \Gamma(S(TM))$. Thus M is locally a product manifold $L_\xi \times (M^* = M_2' \times M^{\text{ß}})$, where L_ξ is a null curve tangent to $\text{Rad}(TM)$, the leaf M^* of $S(TM)$ is a Minkowski space, M_2' is a hyperbolic plane and $M^{\text{ß}}$ is a Riemannian manifold. Since $\nabla_X \mu = \nabla_X W = 0$ and $g(\nabla_X^* Y, \mu) = g(\nabla_X^* Y, W) = 0$ for all $X,\, Y,\, Z \in \Gamma(S(TM))$, we have $\nabla_X^* Y \in \Gamma(\mathcal{G})$ and $R^*(X, Y)Z \in \Gamma(\mathcal{G})$. This imply $\nabla_X^* Y = Q(\nabla_X^* Y)$, that is, $M^{\text{ß}}$ is a totally geodesic and $R^*(X, Y)Z = Q(R^*(X, Y)Z) = 0$, where Q is a projection morphism of $S(TM)$ on $\mathcal{G}$ with respect to the decomposition (4.13). Thus $M^{\text{ß}}$ is a Euclidean space.

If $\alpha \neq 0$, then $p = n-1$, i.e.,

$$A^*_\xi = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & 0 & \\ & & & \alpha \end{pmatrix}. \tag{4.15}$$

Consider the following two distributions E_0 and E_α on $\mathcal{G}$;

$$\Gamma(E_0) = \{X \in \Gamma(\mathcal{G}) \mid A^*_\xi X = 0\}, \quad \Gamma(E_\alpha) = \{X \in \Gamma(\mathcal{G}) \mid A^*_\xi X = \alpha X\}.$$

From (4.15), we know that the distributions E_0 and E_α are mutually orthogonal non-degenerate subbundle of $\mathcal{G}$, of rank $(n-1)$ and 1 respectively, satisfy $\mathcal{G} = E_0 \oplus_{\text{orth}} E_\alpha$. From (4.14), we get $A^*_\xi(A^*_\xi - \alpha Q) = 0$. Using this equation, we have $\text{Im}A^*_\xi \subset \Gamma(E_\alpha)$ and $\text{Im}(A^*_\xi - \alpha Q) \subset \Gamma(E_0)$. For any X, $Y \in \Gamma(E_0)$ and $Z \in \Gamma(\mathcal{G})$, we get $(\nabla_X B)(Y, Z) = -g(A^*_\xi \nabla_X Y, Z)$. Use this and the fact $(\nabla_X B)(Y, Z) = (\nabla_Y B)(X, Z)$, we have $g(A^*_\xi[X, Y], Z) = 0$. If we take $Z \in \Gamma(E_\alpha)$, since $\text{Im}A^*_\xi \subset \Gamma(E_\alpha)$ and E_α is non-degenerate, we have $A^*_\xi[X, Y] = 0$. Thus $[X, Y] \in \Gamma(E_0)$ and E_0 is integrable. Thus M is locally a product manifold $L_\xi \times (M^* = M'_2 \times L_\alpha \times M^0)$, where L_α is a spacelike curve and M^0 is an $(n-1)$-dimensional Riemannian manifold satisfy $A^*_\xi = 0$. From (1.17) and (1.21), we have $R^*(X, Y)Z = \bar{R}(X, Y)Z = 0$ for all X, Y, $Z \in \Gamma(E_0)$. Since $g(\nabla^*_X Y, \mu) = g(\nabla^*_X Y, W) = 0$ and $g(\nabla^*_X Y, e_n) = -g(Y, \nabla_X e_n) = 0$ for all X, $Y \in \Gamma(E_0)$ because $\nabla_X e \in \Gamma(E_\alpha)$ for $X \in \Gamma(E_0)$ and $e \in \Gamma(E_\alpha)$. In fact, from (1.18) such that $D = c = \tau = 0$, we get

$$g(\{(A^*_\xi - \alpha Q)\nabla_X e - A^*_\xi \nabla_e X\}, Z) = X[\varphi]\, g(e, Z)$$

for all $X \in \Gamma(E_0)$, $e \in \Gamma(E_\alpha)$ and $Z \in \Gamma(\mathcal{G})$. Using the fact that $\mathcal{G}$ is non-degenerate distribution, we have $(A^*_\xi - \alpha Q)\nabla_X e = A^*_\xi \nabla_e X + X[\varphi]e$. Since the left term of this equation is in $\Gamma(E_0)$ and the right term is in $\Gamma(E_\alpha)$ and $E_0 \cap E_\alpha = \{0\}$, we have $(A^*_\xi - \alpha Q)\nabla_X e = 0$ and $A^*_\xi \nabla_e X = -X[\varphi]e$. This imply that $\nabla_X e \in \Gamma(E_\alpha)$. Thus $\nabla^*_X Y = \pi \nabla^*_X Y$ for all X, $Y \in \Gamma(E_0)$, where π is the projection morphism of $\Gamma(S(TM))$ on $\Gamma(E_0)$ and $\pi\nabla^*$ is the induced connection on E_0. This imply that the leaf M^0 of E_0 is totally geodesic. As $g(R^*(X, Y)Z, \mu) = g(R^*(X, Y)Z, W) = 0$ and $g(R^*(X, Y)Z, e_n) = 0$ for all X, Y, $Z \in \Gamma(E_0)$, we have $R^*(X, Y)Z = \pi R^*(X, Y)Z \in \Gamma(E_0)$ and the curvature tensor πR^* of E_0 is flat. Thus M^0 is a Euclidean space. □

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